

GEOMETRY IN EARLY MUSLIM ARCHITECTURE

c. A.D. 692-1125

by

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ABSTRACT

Proportional systems in the Muslim decorative arts are often manifest and easier to detect than those which may underlie their architectural forms. In order to study the plans of early Muslim architecture and gain understanding of the method by which they were generated, fourteen examples of early Muslim buildings, dating from AD 692 to 1125, have been geometrically analysed using measured plans and actual dimensions. These examples cover a vast area of the Muslim world, from Syria and Iraq in the north, to Egypt and Córdoba in the west. The results of the analysis demonstrate that a proportional system, familiar to most ancient cultures and based on a circle inscribed within a square and subdivided to create a set of grid lines, relating in a proportion of 1:2 and $1:\sqrt{2}$, proves to be of notable relevance

The thesis initially explores the nature of the evidence and the limits of inference in relation to the subject of geometry in early Muslim architecture, then focuses on the architectural planning of the Near East prior to the coming of Islam and on the work of medieval Muslim scholars. The thesis examines the concept of squaring the circle, its components, and its role as a proportional system in two-dimensional Islamic architectural decoration.

Chapter Four ("Geometric Analysis of Individual Buildings"), is the main part of the thesis, setting out an investigation of the proportional system of structures varying from mosques and palaces to cupolas and *mihrābs*. It is intended to be read in conjunction with vol. II of the thesis, which contains the analytical drawings and proposed geometric schemes.

By pursuing two separate lines of enquiry on each of the selected buildings, the first based on geometrical lines superimposed on published plans, the second through mathematical calculation using actual dimensions, it is proved beyond doubt that early Muslim buildings were in fact geometrically organized.

Finally, the thesis concludes by considering alternative systems of analysis in order to compare its findings and establish the validity of its proposed geometric schemes for the examples selected.

This study of geometry in early Muslim architecture is greatly aided by the pioneering work done by Issam El-Said and Ayşe Parman in the field of two-dimensional geometric design in Islamic architecture, and by the painstaking surveys and beautiful drawings prepared by K.A.C. Creswell.

ACKNOWLEDGMENTS

I wish to express my sincere thanks and deep gratitude to Prof. Robert Hillenbrand whose insight and knowledge of Islamic architecture have been an inspiration. Without his interest, support, encouragement, and criticism, this work would not have been possible. My deep gratitude also goes to Prof. Eric Fernie who, through our discussions, helped me to clarify and organize many of the ideas in this research.

I am grateful to my teachers in architecture, David Ellis who long ago led me to the path I am now pursuing, and Alex Barr who never doubted my abilities. I am also indebted to my secondary school teacher *Ustādh* Muḥammad Zāyid (may he rest in peace), who instilled in me a sense of self-assurance.

I would like to acknowledge the initial encouragement given me by the late Anthony Hutt who also introduced me to Prof. Hillenbrand. My thanks are due also to Prof. Barry Jones of the Dept. of Archaeology at Manchester University for his help regarding Ajdabiya. The work on Qayrawān was made feasible through the co-operation and assistance of Dr. Murād al-Ramāḥ of the Dept. of Antiquities in Qayrawān, who put at my disposal all the help I needed

to survey parts of the Great Mosque.

Grateful acknowledgment must also be made to Lisa Golombek, Ron Lewcock, Mustafa ben Yunis, and David Brady.

I would like to acknowledge that this work could not have materialized without the meticulous and painstaking groundwork pioneered by K.A.C. Creswell whose plans formed the solid foundation for most of the geometrical analyses presented in this thesis.

I am indebted to my family for their continuous support and commitment, and finally I would like very much to dedicate this work to my father Col. Muḥammad bin Muşbaḥ who, despite the limited educational prospects in Libya of the 1950s, aspired to educate me to attain the highest goals. I hope I have not disappointed him.

CHAPTER ONE

CHAPTER ONE

METHODOLOGY

1.1 Introduction

1.2 The Nature of the Evidence

1.3 The Limits of Inference

the following formula:

1.1 Introduction

The purpose of this thesis is to examine the planning methods used in early Muslim architecture (AD 692–1125) and to determine whether geometric systems predominated in their construction.

This work is about architecture and its design principles, and, although the subject matter is fixed in a historic context, i.e. a particular period of Islamic art, its relevance to the present is intriguing. It is also about those who conceived these buildings and their aspirations to achieve, through the use of geometry (*ilm al-miqdār*), forms of harmony and proportion.

In the present study, geometry is viewed as a tool which enabled the architect to realize his vision, a system which helped him to transmit complex plans from paper to reality with extreme efficiency, maintaining their proportions, from an overall concept of inner space and volume to the smallest details. Mathematics is a means of translating geometry into another language. For example, any figure involving an octagon is full of irrational measurements which, as Vitruvius was aware,¹ cannot be expressed in round numbers. Today, however, we are able to express the diameter between the sides of the octagon in the following formula:

1. *Architecture*, IX, Introduction (cited by J. Wilkinson, *Levant*, XIII, 160, n. 11).

$$\frac{2 \times (4 + 3 \sqrt{2})}{4 + 2 \sqrt{2}}$$

Vitruvius's solution to the problem was to set out the geometrical figure at full size on the ground and then measure it.

If we take as an example the Dome of the Rock, which is an obvious octagon in shape, and then prove arithmetically that it is so, are we in fact checking the margin of error in the implementation of the octagon in built form, or are we translating the characteristics of the octagon into numerals? If the former is the case, the margin of error will always relate to many factors, such as the method of marking out on the ground (e.g. the use of cord and pegs, shadow casting, or steel chains) or the quality of the labour involved, as well as the ground conditions or site restrictions.

Vitruvius admitted that architects were 'and perhaps still are' very different from pure mathematicians.¹ If the architect used geometric principles in marking the ground to solve the problem, the resulting building should *reproduce* fairly closely the same geometric proportion derived from the diagram used.

The methodology of the present research is based on the following

1. *Op. cit.*, I. 2. 12-16.

principles. Firstly, a brief examination is made of the historical sources relating to the use of geometry as a basis for generating and setting out building plans. This includes studying available information on medieval treatises.

Secondly, by examining particular examples of geometric architectural constructions, it is shown that ancient methods, e.g. the use of the square and circle, were used to establish complex geometric proportions and that these proportions were then used in transferring the exact layout to the full-scale, which might be in built form or in decorative motifs or artefacts.

Thirdly, by analysing plans of early Muslim architecture, i.e. of buildings constructed between AD 692 and 1125, use is made of the principle of 'squaring the circle' in reverse, that is to say, superimposing a geometric grid of squared circles on the plans of buildings as they now stand, in order to establish if the main proportions of the building relate to the proportions of the geometric pattern. Towards the end of each analysis, conclusions are drawn and observations recorded, usually backed by historical evidence, especially that in support of extensions carried out on these buildings which appear to relate closely to the same geometric principle that generated the original plan.

It is necessary to investigate the possible existence of such proportions

by means of calculations,¹ using measurements derived from the building or of the site itself. *This I have tried to achieve with every building analysed, where exact measurements have been published or where it has been possible to take such measurements personally.*² The following are the buildings whose analyses, contained in Chapter Four, are based on actual measurements and are not merely the result of imposing diagrams on plans: the Dome of the Rock, Qubbat al-Barūdiyyīn, Ajdābiyyah Palace, Ajdābiyyah Mosque, the Great Mosque of Córdoba, the Great Mosque of Qairawan, Ukhayḍir, the Great Mosque of Damascus, and the Mosque of al-Aqmar. I have also included a number of buildings which, although examined here, still require further investigation. Although a full complement of actual dimensions for these are lacking, they nevertheless show interesting possibilities (4.10.1–4.10.5).

The fourth stage of the research method consists of an examination of alternative systems of analysis and different mathematical interpretations of geometry which have been used to explain certain medieval Islamic buildings. These findings are then compared with the principle³ adopted in the thesis as a gauge for detecting the presence of systems of proportion.

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1. See E. Fernie, "A Beginner's Guide to the Study of Architectural Proportions and Systems of Length", in E. Fernie & P. Crossley (eds.), *Medieval Architecture and its Intellectual Concepts: Studies in Honour of Peter Kidson* (London, 1990) 229–238.
 2. E.g. of the Great Mosque of Qairawan.
 3. I.e. the principle of squaring the circle, used in Chapter Four to examine a number of buildings constructed between AD 692 and 1125.

Finally, conclusions are drawn in the course of summarizing and assessing the results of the research in the hope that new light may be thrown on the subject of geometry in early Muslim architecture.

1.2 The Nature of the Evidence

The subject of this thesis is fairly wide, both in geographical terms and in time span. It deals with a number of buildings of different functions and patronage, some of which still exist while of others only excavated traces remain.

We are also dealing with a politically sensitive area of the world, where free access to carry out the necessary study is limited and where accurate surveys are not readily available. Happily, K.A.C. Creswell's *Early Muslim architecture* and *The Muslim architecture of Egypt* provide a solid basis for such studies.

As evidence we have the following:

1. Published plans, some with dimensions, some without. In some of the former cases, the dimensions are clear, extensive, and detailed, including column widths and interstitial dimensions, while in other cases the dimensions relate to the outer walls alone.

2. Mathematical calculations, which have been carried out in all cases to prove, if possible, the existence of a proportional system. *The calculations are based on exact published dimensions and not on tracings over plans or sketches.* The diagrams accompanying the thesis (vol. II) are indicative of the work done in this connection and serve as illustrations of the argument put forward here.
3. Measurements and surveys undertaken by myself for the present study. In some cases, e.g. the Great Mosque of Qairawan, I was able to personally check some dimensions and measure new ones. Use has also been made of the recent surveys carried out by UNESCO in Ajdābiyyah.
4. References in historical sources to building extensions or related events. For instance, in the case of Córdoba, we find reference to an 'expansion twice or three times [the size of] the original mosque'.

1.3 The Limits of Inference

This is a very important aspect of the research, as it sets the parameters of the analysis of the buildings and their plans. After all, we are dealing with a large number of buildings greatly dispersed, from Iraq and Palestine in the Middle East, Libya and Tunisia in North Africa, to Córdoba in Spain. We are also spanning a period of over four hundred years

Before setting the limits of our use of inference, we ought to ask the preliminary question, why set such limits? Is it to prove the accuracy of the actual building and determine how close the building as ultimately completed is to the geometric framework assumed to have been intended? Or are the limits to be used as a scientific barometer and a tool enabling us to explore beyond the external appearance? Thus, for example, in the Dome of the Rock, the geometric framework as shown in our analysis went beyond the obvious octagon and became part of a complex diagram which appears to have influenced the height, structural supports, and the position of the openings?

If we concentrate on looking for accuracy in buildings, we are in fact searching for tolerances, either those that can be expected in machines and which are measured in microns, or those of an artistic creation in which the sum is more important than each part viewed separately. For example, are the joints between the mosaic tiles of the Alhambra Palace all perfect, or is it the overall effect which gives the tiles their appeal? What tolerance do we work in when examining an ancient or medieval building? Do we take into account optical correction and cultural or religious beliefs? If we examine the buildings to determine how close they are to an intended geometric system, we need to look at the evidence, not only to prove such intentions but also to disprove them. If the evidence points towards a symmetrical building with major elements of design that are hard to imagine and that are organized completely haphazardly and as the result of odd chance, then the inference limit is not as important as the intention.

What is truly essential is the evidence we can find of a 'geometric process' which appears to have been applied through the historical development of the site.¹ In some cases, modern scholars dismiss fairly sophisticated medieval buildings from being classified as square because their dimensions are not those of an exact square. Creswell, for example, denied that Ukhayḍir was square in shape because it measured 174.74 x 169.05 x 175 x 163 m.,² but who, it may be asked, would intend to build a rectangular palace with such dimensions?

The principle of the 'intention' is very well illustrated in the study of the towers of Laon Cathedral carried out by Eric Fernie, who demonstrated that both sides – the believers led by Ueberwasser and the sceptics led by Branner – are right.³ Branner had cast doubt on Ueberwasser's theory because of the presence of contradictory construction lines. Here, I believe, the evidence is overwhelming that the designer intended an octagon in the centre of the tower. Why should anyone have drawn and built a shape that is *almost* an octagon if the octagonal shape had not been intended? I would not go so far as to accuse Ueberwasser of having fudged his diagram, but accept that he drew his diagram to demonstrate

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1. See the case of the Great Mosque of Qairawan, where even the latest extension appears to relate to a geometric grid system very apparent in the original mosque. See also the case of the Great Mosque of Córdoba where, to the last wall built, one is able to relate it to the first four walls envisaged.
 2. Creswell, *Early Muslim Architecture*, vol. II, p. 52.
 3. *Op. cit.*, 230 f.

the 'intention'.

In some of the cases I have examined, the limit of the inference is very narrow. For example, in the Qairawan and Damascus Mosques, it is to one or two decimal points. In other cases, however, e.g those listed in § 4.10, the limit of inference is higher, ranging between 3-4 %.

Having stated the methodology of the thesis and having discussed the issue of inference, we next explore architectural planning in the Near East prior to the coming of Islam and by examining the work of medieval Muslim scholars.

2.1 Introduction: Architectural Planning in the Near East prior to the Coming of Islam

From the time that the Muslims entered the Levantine region of Syria and Egypt, with their "Greek culture and Roman government", it was inevitable that they should become acquainted with the classical laws of geometry and design taught in the well-flourishing Greek schools of that region.

CHAPTER TWO

MEDIEVAL TREATISES

2.1 Introduction: Architectural Planning in the

Near East prior to the Coming of Islam

2.2 Al-Tūsī

2.3 Abū al-Wafā' al-Būzjānī

2.4 The *Siddhantā*

As the Qur'ān provided a stable nucleus for society, Pythagorean and Platonic concepts of ideal and absolute beauty gave Islam an organized structure. Further, they provided a system of basic principles that could suggest a refined level of decorative perfection. The amount of much of Arabic geometric knowledge can be directly traced to the metaphysical ideas of Pythagoras and his school.

2.1 Introduction: Architectural Planning in the Near East prior to the Coming of Islam

From the time that the Muslims overran the Byzantine provinces of Syria and Egypt, with their 'Greek culture and Roman government', it was inevitable that they should become acquainted with the classical ideas of philosophy and science taught in the still flourishing Greek schools of this region. An immediate interest in what they found culminated in the establishment, by the Baghdad Caliphs, of schools of translation. These soon brought most of the scientific and philosophic literature of the Greeks into Arabic, which proved to be a particularly adaptable language for scientific purposes by virtue of its use of short abstract forms. The Caliphs despatched emissaries as far as Byzantium itself to buy further works. Greek mathematical knowledge, which was primarily concerned with geometry, was studied, together with the philosophy from which it is inseparable. In fact, these configurations of straight lines and circles may be seen as an extension of the Platonic search for a form of beauty that is not relative, but absolute.

As the Shari'ah provided a stable structure for society, Pythagorean and Platonic concepts of ideal and absolute beauty gave Islam an equivalent structure for art; they provided a system of basic principles that could support a refined form of aesthetic perfection. The source of much of Arabic geometric knowledge can be directly traced to the metaphysical ideas of Pythagoras and his school.

The Pythagoreans taught that the structure of the universe was to be found in mathematics and ascribed mystical properties to numbers and geometric figures. Plato was greatly influenced by their ideas and believed, like them, that the key to the universe was to be found in number and form. His metaphysical speculations on this relationship between the world of ideas and the material world are given in his dialogue *Timaeus*. In this work he proposed that four 'basic solids' correspond to the four elements of earth, air, fire, and water, thought by the Greeks to constitute all matter (see fig. 2.1.1).

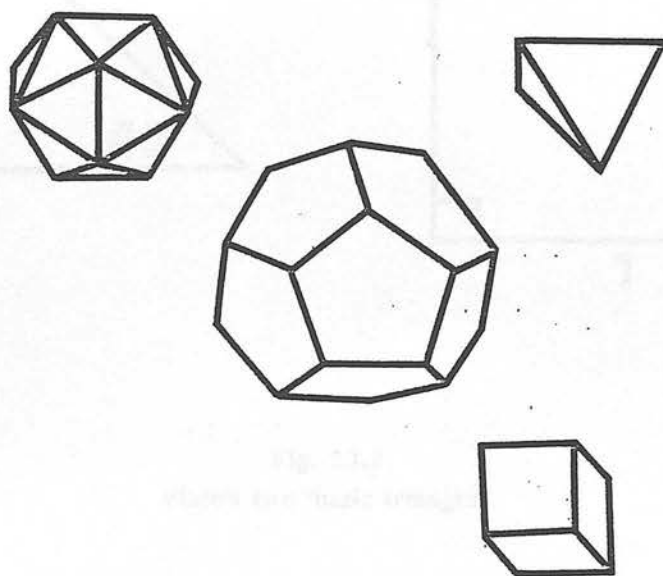


Fig. 2.1.1

These solids – the pyramid, the octahedron, the cube, and the icosahedron – were further reduced to two component right-angled 'basic triangles' whose sides are in the ratio of 1 to 1 to the square root of 2, and 1 to 2 to the square root of 3 (see fig. 2.1.2).

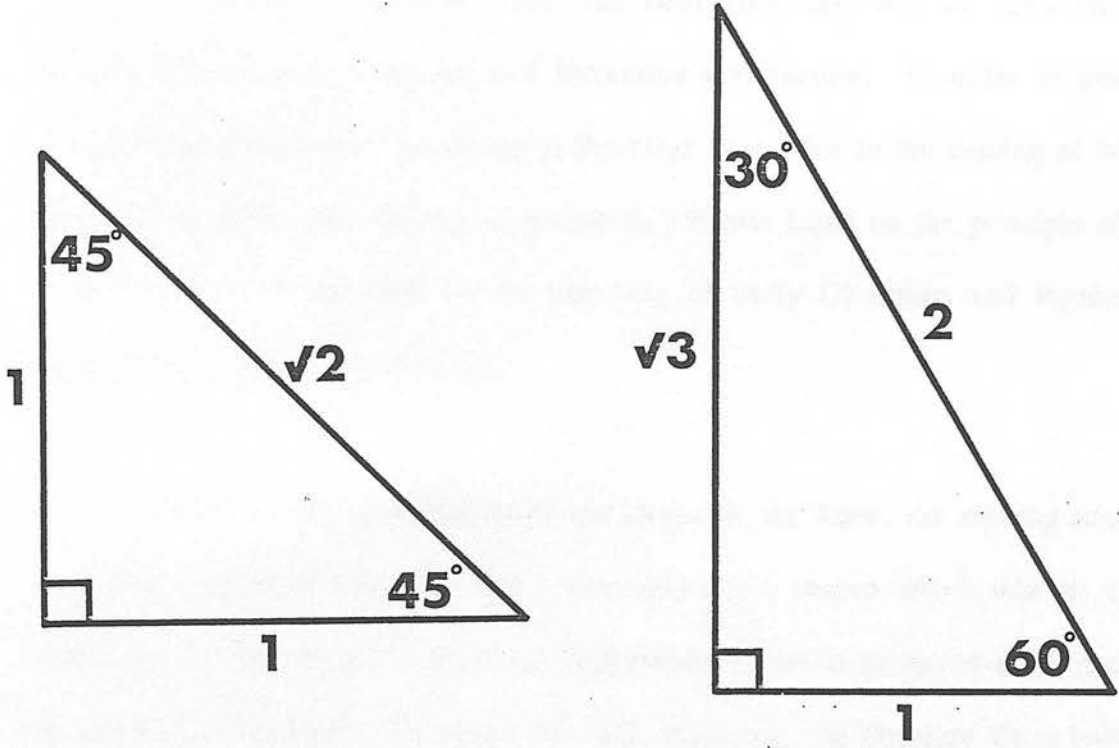


Fig. 2.1.2
Plato's two 'basic triangles'

By the time of the founding of Plato's Academy, all the ordinary figures of plane geometry, including the pentagon, had been identified and their construction was known; the same goes for arcs, sectors, and segments as conditions of the circle. The one aim of this geometry was the study of relations in order to grasp the fundamental realities of the universe. Other important discoveries, relevant to this study of the construction of geometric ornament, were the geometry of plane

division and of proportional systems.

Byzantine Palestine (5th–7th centuries AD) was an area rich in monuments of early Christian and Byzantine architecture. In order to gain an insight into architectural planning in the Near East prior to the coming of Islam, we shall examine the evidence of geometric schemes based on the principle of the squared circle being used in the planning of early Christian and Byzantine monuments in Byzantine Palestine.

Prior to the construction of the Dome of the Rock, the existing octagon was well known in the Near East. One may cite a church which was set up c. 450 over the 'House of St. Peter' at Capernaum, which in course of time came to be used as a baptistery. In about AD 485, moreover, the Emperor Zeno built an octagonal church on the summit of Mount Gerizim. These latter two monuments are obviously to be compared with the Dome of the Rock.

Craftsmen working in the Holy Land prior to the construction of the Dome of the Rock had already displayed their skill in handling octagonal designs in the time of Herod the Great, as we can see from the decoration of the ceiling of the Double Gate at the Haram al-Sharif (fig. 2.1.3).

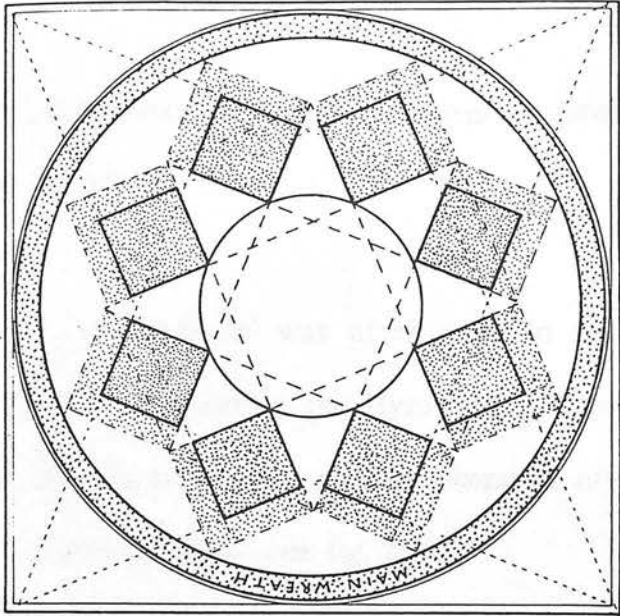


Fig. 2.1.3

At first sight, the design contains no octagons, but one could evidently be constructed between the points of the eight-rayed star which divides the small squares, as shown in fig. 2.1.4.

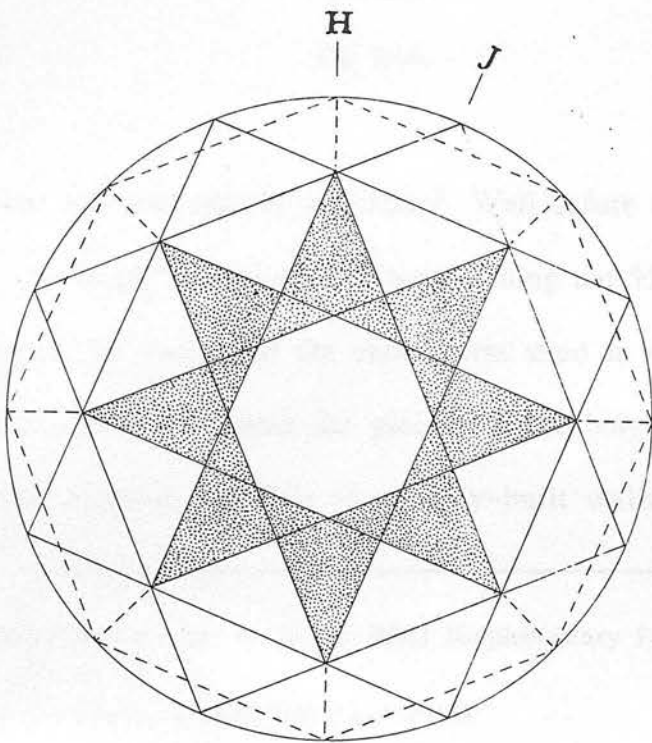


Fig. 2.1.4

By extending the sides of this octagon, two squares are formed, which determine the radius of the outer circle.

The same 'star diagram' was often used in a simpler way in late antiquity and, in particular, during the Byzantine period. One of the many possible examples is to be seen in the mosaic decoration of a floor made in the sixth century for a church at Jerash¹ (see fig. 2.1.5).

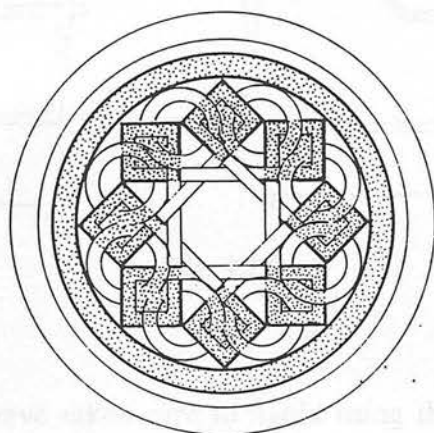


Fig. 2.1.5

The star diagram was also used by architects.² Well before the octagonal church was built at Capernaum, Christians had been visiting the 'House of St. Peter' to pray in one particular room and the church was sited in such a way that the central feature of its plan marked the position of the holy place. But it was a complicated site, since it was full of roughly-built walls (see fig. 2.1.6), as

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1. See J.W. Crowfoot, *Churches at Jerash*, BSAJ Supplementary Papers 3 (London, 1931) p. 15 f. and plate IV (b).
 2. See J. Wilkinson, *Levant*, vol. XIII (1981) p. 159-163.

Wilkinson has shown.

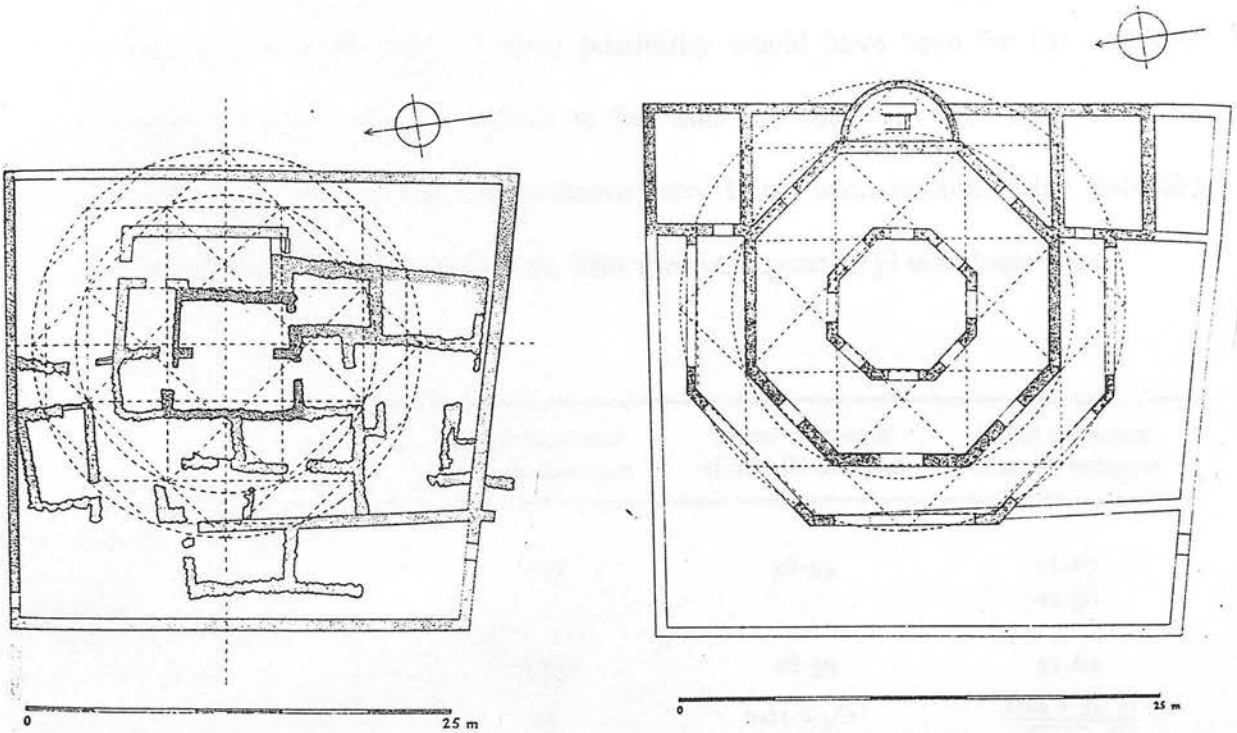


Fig. 2.1.6

The architect seems to have taken care to avoid using these as foundations for the walls of the new church. New foundations were laid for all but 15% of the length of the new walling.

How did the Capernaum architect deal with the irrational numbers resulting from an octagonal plan? Did he set out the geometrical figure at full size on the ground and measure off from it the lengths he wished to use? This might have been possible, as long as he had ordered enough masonry to enable him to adjust the figure on the site. Alternatively, he may have calculated the values of the irrational quantities to the nearest sixteenth of a foot, which was the

smallest unit he could specify. However, this would have demanded considerable mathematical calculations. A third possibility would have been for the architect to produce simple working figures in feet and half-feet.¹ In the table presented by Wilkinson, we see that the evidence from Capernaum is compatible with the third method (arithmetic) as well as with the first (geometry) (see table 2.1.1).

	Inner diameter of inner octagon	Outer diameter of middle octagon	Inner diameter of outer octagon
A. True measurements in metres			
maximum	6.78	16.53	21.67
minimum			21.56
B. Geometrical values when factor in 2nd column = 16.53	6.85 ^a	16.53	21.60
factors	2x	$2x(1 + \sqrt{2})$	$\frac{2x(4 + 3\sqrt{2})}{\sqrt{4 + 2\sqrt{2}}}$
C. Working figures: feet of 30.82 cm. metric equivalent	22 6.78	53.5 16.49	70 21.57

Table 2.1.1

Another monument examined by Wilkinson gives further insight into the planning method used in the Near East prior to the coming of Islam. The Church on Mount Gerizim was far more sophisticated than the one at Capernaum and was precisely built. Wilkinson takes us through the development of its geometrically based plan stage by stage as follows (see fig. 2.1.7):

1. See N. Spremo, *Proportions architecturales des basiliques* (Belgrade, 1971), cited in Wilkinson, *op. cit.*

- (a) First draw a circle with an octagon surrounding it, and radii which extend to touch the corners [see fig. 2.1.7].
- (b) Draw a square in the circle and, where the radii intersect the square, mark out a second octagon. Its position is emphasized in the figure by the circle drawn round it.
- (c) Join the corners of the smaller octagon to form a star, and note that some of the lines which form it are extended outwards. Construct a third octagon whose sides pass through the points from which the rays of the star diverge. In fact this last octagon could equally well have been constructed by drawing a larger star joining the corners of the outer octagon, but the smaller star is needed in order that some of its lines can be extended.
- (d) Remove the northern and southern parts of the outer octagon which lie outside the intersections marked. Now on the inner octagon draw a circle, and round this circle a square. Draw the octagon which touches the corners of this square.
- (e) Inside the last octagon draw a circle, and taking the eastern point on its circumference as centre, draw another circle of the same radius to determine the projection of the apse.

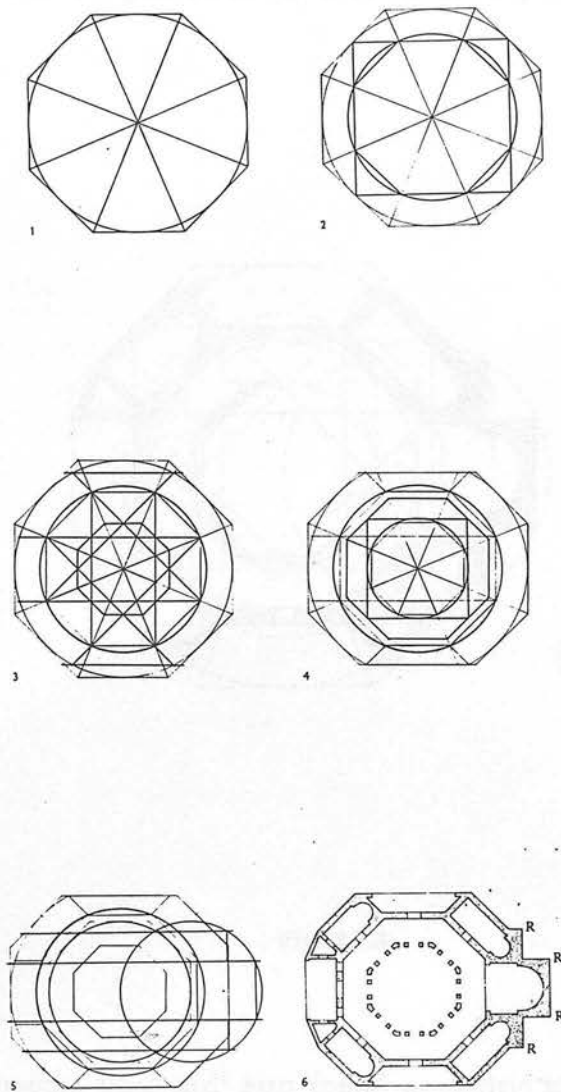


Fig. 2.1.7

Such steps were surely involved both in drawing the plan and in the setting out on site. As Wilkinson demonstrates in his final plan (fig. 2.1.8), the lines of the finished building correspond very closely with those of the geometrical diagram. For example, the true measurement in metres of the outer face of the inner

octagon is 5.8 m. and the geometric value of the corresponding line is 5.86 m.¹

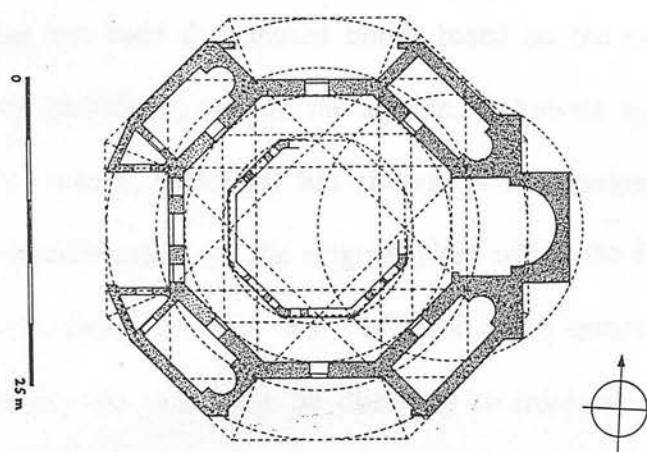


Fig. 2.1.8

In St. Simeon, Ecochard² also found geometric schemes, within the actual building, involving fundamental circles with a radius of 26.868 m., i.e. roughly 7 m. less than that of the Dome of the Rock. Similarly, in S. Vitale in Ravenna, the octagonal central core is engendered by a circle having a radius of 11.13 m., which circumscribes two squares disposed at an angle of 45°, whose angles are connected, while a circle with radius of 20.56 m. inscribes the outer octagon and

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1. For further examination, see table 2 (Mount Gerizim) in Wilkinson, *op. cit.*
 2. A. Papadopoulo, *Islam and Muslim art* (London, 1980) p.240.

a circle of 26.875 m. in radius determines the exterior dimension of the narthex. The plan of the Church of the Ascension in Jerusalem is likewise engendered by two squares whose angles are on the same circle with a radius of 26.875 m. The same appears to apply in the ruined Cathedral of Bosra, though its precise arrangement has not been determined but is based on the same generating circle with a radius of 26.875 m., and on the square. Likewise again, in the church at Seleucia Pievia,¹ where, Ecochard has shown, if the contours established by the surveyors are superimposed on the original plan, which the fifth-century architect must have formulated on the basis of this same geometrical scheme, the difference between the two must be due only to minimal errors in pegging out the ground plan. In the Rotunda of the Holy Sepulchre in Jerusalem, the earliest of the Christian monuments considered by Ecochard, the actual measurements show that the sides of the squares must have been 38.5 m., instead of 38 m. as elsewhere in the building, and the radius of the circle was likewise slightly longer.

This constant use of what could be considered the same plan and proportion appears again in the monument built on exactly half those dimensions and discussed by Ecochard, i.e. the Church of St. Donat (900-950) in Zadar, former Yugoslavia.² What is extraordinary in this enumeration of monuments cited by Ecochard is that not only the geometrical scheme but also the measurements utilized were rounded off and the unit of measurement was in all

1. M. Ecochard, "A propos du Dome", *Bulletin d'etudes orientales* (Damascus, 1973) p.37-45.

2. *Filiation de monuments grecs, byzantins, et islamiques: une question de géometrie* (Paris, 1977) p.1-21.

cases the same.

This generating schema was employed in pre-Islamic times not only for sanctuaries but also for civil edifices, especially the octagonal or dodecagonal aedicules so often found in the courts of Roman markets, as in Leptis Magna in Libya (see fig. 2.1.9) and Pompeii.

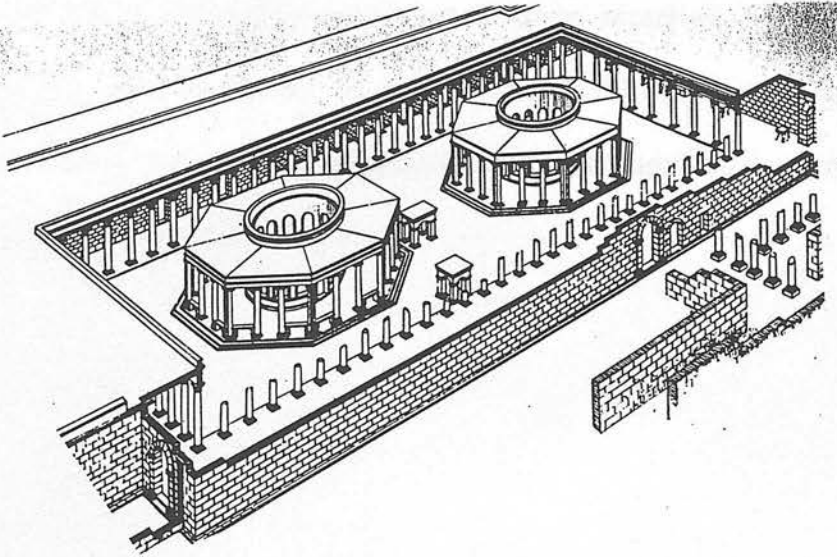


Fig. 2.1.9

It is therefore reasonable to conclude that this was a standard method of construction based on squares inscribed within a circle, the squares being of greater or lesser number to give figures with eight, twelve, sixteen, twenty, or

twenty-four. According to Vitruvius,¹ the procedure was known to the Greeks and used by them in designing their theatres. This geometrical conception of architecture and the idea of utilizing a square inscribed within a circle, rotated at angles of 45°, 30°, or 15° to make different polygons, goes back to Pythagorean and Platonic speculations on numbers and geometrical figures. Ever since antiquity, squares and regular polygons inscribed in circles appear to have been commonly fraught with philosophical and symbolical connotations, which, in the course of further development, had become interlinked with and inspired Neo-Pythagoreanism among Christian scholars. The Muslims appear to have inherited this entire intellectual current and it became an integral part of their theology, philosophy, and science.

1. Cited by A. Papadopoulos, *Islam and Muslim Art* (Paris, 1976).

2.2 Al-Ṭūsī

In order to examine early Muslim architecture in terms of ordered preconceived plans prior to construction, one needs to study medieval Muslim work dealing with the subject of geometry as a means of organizing built forms. The question here is, if the square and the circle played a major part in the proportional systems of Islamic architecture (which, as we will see later, they did), were medieval Muslim architects aware of such geometry and, if they were, where did this influence come from?

In order to obtain precious manuscripts, the Muslim caliphs, we are told, did not even hesitate to address themselves to the Byzantine Emperors. Al-Manṣūr wrote to the Christian sovereign asking him for works treating the pure sciences and in return he received the books of Euclid, as well as some works on physics and other sciences.

With regard to *'ilm al-handasa*, the name given by the Arabs to geometry, their acquaintance with pure theoretical geometry came through the *Elements* of Euclid which was first translated by al-Ḥajjāj ibn Yūsuf ibn Maṭar in A.D. 790, whereas the Arabs' acquaintance with applied geometry, which was paramount in the practice of architecture, appears to have reached them through the applied geometry of Archimedes, Hero of Alexandria, and the compendium

of Indian texts known as the *Siddhantā*.¹

‘Geometry,’ wrote the *Ikhwān al-Ṣafāʾ*, ‘has as its principal field of application the measurements of surfaces, measurement which is essential for surveyors, accountants, tax agents, landowners in their various operations, or transactions such as the collection of property tax, the drainage of water courses, the postal service, etc.’²

Arab scholars’ familiarity with pure geometry through their acquaintance with classical works may have been an indirect means of learning about the applied geometry of the Indian *Siddhantā*. Thus, al-Kindī (A.D. 800–873) in his work relating to the translation of Euclid, *Risālah fī Aghrāḍ Ūklīdīs* (‘Epistle on the Objectives of Euclid’), comments that Euclid’s work is in reality a compendium of ancient knowledge set in order and annotated by Euclid.³

One particular medieval Muslim accomplishment most relevant to this thesis was that of the thirteenth-century polymath Nāṣir al-Dīn al-Ṭūsī. Archimedes’s treatise on squaring the circle (*Fī Taksīr al-Dā’ira, Tarbī’ al-Dā’ira*) was first translated by Thābit ibn Kurra (A.D. 834–901)⁴ and by Ḥunayn ibn Ishāq. This was later revised by Nāṣir al-Dīn al-Ṭūsī.

1. Art. “Ilm al-Handasa”, *ET*², vol.II, p.414.

2. *Ibid.*

3. *Ibid.*, p.413.

4. *Ibid.*, p.412.

The following section is an attempt to translate al-Ṭūsī's introduction to his work on Archimedes's book ('The Sphere and the Cylinder'),¹ and a small part of the treatise on squaring the circle. Although it is possible to understand the overall argument. The absence of clear divisions or breaks between sentences makes it hard to establish where one sentence ends and another begins.

Nāṣir al-Dīn al-Ṭūsī's Introduction to his Work on Archimedes's Treatise

'In the Name of Allāh, the Most Compassionate,

the Most Merciful,

O Allāh, you have bestowed, may this

continue.

I say

After thanking God and glorifying him, praise be on Muḥammad and his family the chosen ones. For a long time I have been seeking to pause at some matters which were included in Archimedes's book *The Sphere and the Cylinder* due to their great demand in the use of the various honourable geometrical needs.² I stumbled on the famous copy of the book which was revised by Thābit, despite all its shortcomings due to difficulties of translation into Arabic. I have studied this document which was weak in parts. I filled in the gaps as much as possible

1. *Maqālah fī Taksīr al-Dārah li-Arkīmīdīs*, Bibliothèque Nationale de Paris, MS. arabe no.2467.

2. Arabic: *al-maṭālib al-sharīfiyyah al-handasiyyah*.

and endeavoured to investigate the issues contained in it until I reached the second treatise. I also discovered what Archimedes had ignored in the way of introductions, as well as constructing some of what was required. I investigated this and as my eagerness to obtain more information increased, I was able to obtain an ancient notebook containing the explanations by al-‘Asqalānī on the problems of this book,¹ which Ishāq had translated accurately into Arabic. This notebook contained the text of the book, from its introduction to the last figure (number 14), from the first treatise also translated by Ishāq. All that was recorded has been honestly represented. So I found in this notebook what I was searching for and I have sought to revise this book in an ordered sequence, summarizing its meanings and clarifying its origins which can only be explained through geometric principles. I have produced the necessary introduction and provided explanations of what I illustrate from what is described in, or from what I have learnt from, other books produced by experts in this field. I have differentiated clearly between what is found in the text and what is not. I have confirmed the number of figures on the margins, for the figures in the first treatise by Thābit amount to 48 and in the copy by Ishāq they amount to 43. I have finally appended Archimedes’s treatise on squaring the circle, for it was built on certain conclusions mentioned in this book. I ask Allāh’s help to attain that which appeases him. He is the best to give guidance and help.’

1. This is presumably Archimedes’s book.

Nāṣir al-Dīn al-Ṭūsī's Version of Archimedes's Treatise on Squaring the Circle

In studying this treatise (see fig. 2.2.1 a & b) one can establish the following:

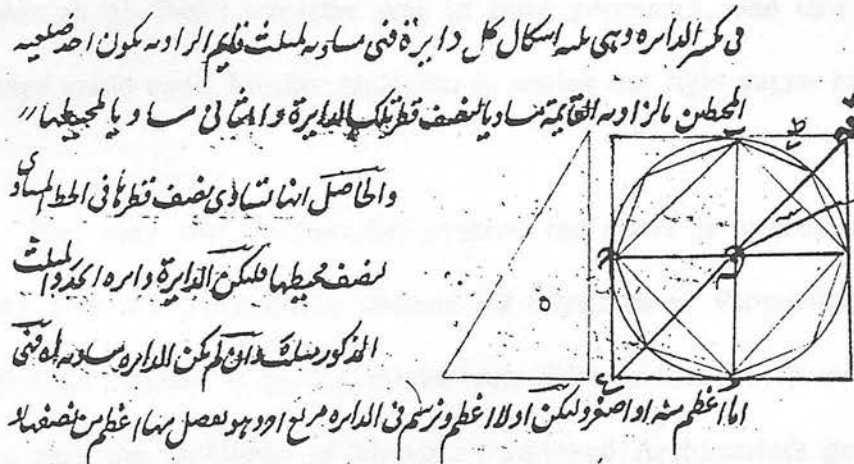
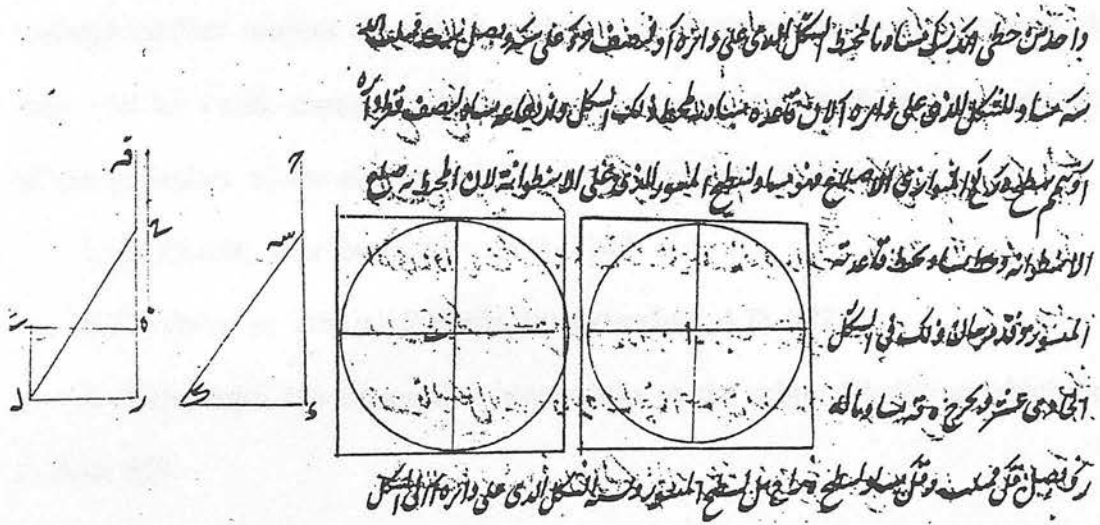


Fig. 2.2.1 a & b

A. It appears that through works such as al-Ṭūsī's, the Arabs were familiar with

the geometric term 'squaring the circle' (*tarbī' al-dā'ira*), although al-Ṭūsī does not say whether earlier scholars in their translations of Archimedes¹ did actually attempt the section on squaring the circle.

B. The argument that the Arabs' early contact with practical geometry had to be through earlier sources than the translation of Archimedes (early 11th cent. A.D.) may still be valid, especially in view of the strong evidence (noted in chapter 3) of the geometry of the square and the circle in sites such as:

1. Qayrawān, first extended c. A.D. 724.²
2. Cordoba, in 'Abd al-Raḥmān II's extension, A.D. 822.
3. Kūfa, with the major alteration works to the original building which began c. A.D. 638.

C. Although al-Ṭūsī's exercise was in pure geometry, one can see how his knowledge could assist Muslim architects in setting out right angles on site.

The way that Archimedes squared the circle is interestingly different from the way we proposed in chapter 2.2 ('Systems of Proportion'). Although Archimedes's method is purely theoretical, there is some evidence in al-Ṭūsī's writings that the architects of his time employed Archimedes's geometry. This must, however, be considered an exception to our general thesis that practical

1. The earliest work of Archimedes to appear in Arabic was the treatise on "The Cylinder and the Sphere" translated by the Banū Mūsā, c. A.D. 827-873.

2. Creswell, *Early Muslim Architecture*, vol.II, p.211.

geometry reached the Arabs through Indian writings rather than via translations of Greek works.

To square the circle according to the method advocated by Archimedes,

1. Line A-C can be drawn on the ground using stretched rope and peg. This line often indicated east-west alignment.¹ B-D is drawn at right angle to A-C by drawing arcs from A and C with a radius of more than half the length of C-A (see fig. 2.2.2 a).

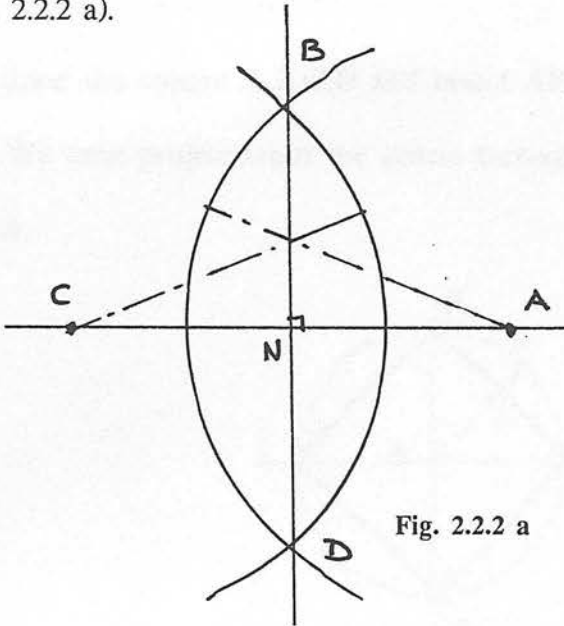


Fig. 2.2.2 a

2. We draw a circle with centre N, cutting the horizontal and vertical axes at A,B, as shown in fig. 2.2.2 b.

1. G. Michell, *Architecture of the Islamic world* (London, 1978) p.131.

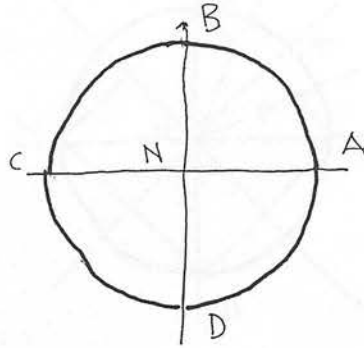


Fig. 2.2.2 b

3. We draw the square A,B,C,D and bisect AB at F, and so on for the remaining sides. We now project from the centre through F and cut the circle at T, as in fig. 2.2.3

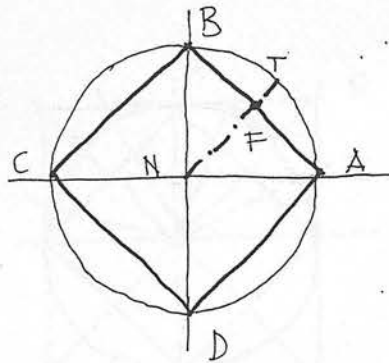


Fig. 2.2.3

4. We now connect the chords of the circle, for example AT-TB, and so on for the rest of the sides. We further bisect AT at Z and project a line from centre N to Z and beyond. See fig. 2.2.4

may also do so from N-B, N-A, N-C, and N-D. We project these lines at either end and where they intersect the square E,F,G,H is formed. However, Archimedes's method in adding a further tangent at 45° (N-T) makes the achieving of a perfect square on site more possible.

2.3 Abu'l-Wafā' al-Būzjānī

Another medieval source relevant to this thesis and connected to applied geometry is a work by Abu'l-Wafā' al-Būzjānī. Born in A.D. 940 in Būzjān, a small town in Khurāsān, he soon emigrated to Baghdad where from 959 to 998, according to many modern historians, he played a most important part in furthering Arabic mathematics and astronomy.

Most of Abu'l-Wafā's work is no longer to be found. The extant material has survived in both Arabic and in a Persian version produced by one of his pupils. It contains a large number of geometrical problems, from the fundamental constructions of plane geometry to the construction of the corners of a regular polyhedron on the circumscribed sphere. A number of these problems are solved by a single span of the compasses. Also many problems appear to show pronounced Indian influence.¹

One of his treatises, *Fīmā yahtāju ilayh al-Ṣānī' min A'māl al-Handasa*, is a practical handbook on what the craftsman should know of geometry. This has fortunately been made more widely available through the

1. Art. "Handasa", *El²*, vol.II, p.257. For the Indian connection, see F. Sezgin, *Geschichte des arabischen Schrifttums*, Bd.V (Leiden, 1974) p.321f.

efforts of Krasnova and Bulatov.¹ The work, which does not appear to dwell long on the complex mathematical aspects of geometry, was translated into Persian soon after it appeared. The translation begun by Najm al-Dīn Muḥammad was completed by Abū Ishāq ibn ‘Abd Allāh Yazīd on the instructions of his teacher the *muhandis* Shams al-Dīn Abū Bakr, whose grandfather was called Abū Bannā.

In a paper entitled "Text, Plan and Building: on the Transmission of Architectural Knowledge",² Renata Holod explored the various possibilities for interpreting the phrase found in a historical text: "...and a *ṭarḥ* was sent". *Ṭarḥ* is here taken to mean a plan or drawing. A scenario was constructed beginning with the initial idea for a building as conceived by the patron and continuing to the building itself with special reference to the types of information which were available, proposing a model for how architectural knowledge was to be transmitted. Holod uses the building of Madīnat al-Zahra as a model on the basis of several documents such as Ibn Ghālib's *Farḥat al-Anfus* (6th cent. A.H.) quoting an earlier text. Holod concludes this section by pointing out the nature of the building process in the Muslim world: '...where change could be rapid and transregional change is on a slower time scale, where replication is prized and where transmission is mostly by example or by gesture and less by architectural

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1. S.A. Krasnova, *Geometricheskie Preobrozovaniya* (Moscow, 1966); Bulatov, *Geometricheskaia Garmonizatsia v Arkhitekture Srednei Azii ix-xv v.v.* (Moscow, 1978).
 2. Extracts published under the title *Theories and Principles of Design in the Architecture of Islamic Societies* (Aga Khan Programme for Islamic Architecture, 1988).

notation and description.'

Another text examined by Holod is Abu'l-Wafā's *Fīmā yaḥtāju ilayh al-Ṣānī' min A'māl al-Handasa*. In Holod's opinion, the text offers a constructive 'hands-on' version of practical geometry explained in a straightforward language. In explaining Abu'l-Wafā's work dealing with the problem of dropping a perpendicular line onto a plane, Holod inserts '(or, as the author says, to a flat wall, a piece of land or to a roof).' The phrase 'a piece of land' in this insertion may be of importance in examining whether Muslims, in constructing right angles on site, applied the same principles of squaring the circle adopted by the Indians (see chapter VII on squaring the circle).

The Istanbul copy of Abu'l-Wafā's treatise *Fīmā yaḥtāju ilayh al-Ṣānī' min A'māl al-Handasa* apparently came from the library of Ulugh Beg in Samarqand. The copy of the Persian translation found in the Bibliothèque Nationale in Paris is also from the Timurid period. Krasnova's work presents one of the two known Arabic copies of the manuscript which is in Istanbul. Bulatov's later works are based on the known Arabic and Persian copies. The Istanbul copy and the Persian version in the Bibliothèque Nationale were made accessible to Bulatov through the translations of Krasnova and Viblanava. In his study of the Samanid mausoleum, Bulatov proposes harmony theories based on drawings

from the work of al-Būzjānī and the Ikhwān al-Ṣafā'.¹

Although Bulatov's book *Geometricheskaia Garmonizatsia* refers to Abu'l-Wafā' and his mathematical formulae, it fails to present any evidence in the way of diagrams which must have appeared in the original medieval text. These would have helped us to understand the basis of Bulatov's analysis and its mathematical interpretations. We shall discuss Bulatov's work in more detail in the next chapter.

In order to determine for oneself the significance of any original drawings or diagrams, it becomes apparent that one needs to see a copy of the treatise itself. Having spent many months to no avail in search of the elusive *Fīmā yaḥtāju ilayh al-Ṣānī' min A'māl al-Handasa* by Abu'l-Wafā', we must be content here to use Woepcke's examination² of the Persian copy of Abu'l-Wafā's treatise. In this Woepcke discusses Abu'l-Wafā's trigonometry as well as his theories on geometry. The contents of the Persian text examined by Woepcke read as follows.

بسم الله الرحمن الرحيم وبه نستعين

الحمد لله رب العالمين والصلوة على خير خلقه محمد وآله اجمعين اما بعد
اين ترجمه كتاب ابي الوفاء محمد بن محمد البوزجاني است در اعمال

1. Holod, "Text, Plan and Building", in *Theories and Principles of Design*, p.1-3.

2. "Recherches sur l'histoire des sciences mathématiques chez les Orientaux, d'après des traités inédits arabes et persans", *Journal asiatique* (April 1855) p.309-359.

هندسیة مشتمل بر مقدمه ودوازده باب بر سبیل اختصار
مقدمه در مسطره وپرگار وگونیا

فهرست الابواب

الباب الاول در چیزها که اصل باشد وپیشد باید داشت

الباب الثاني در بیان شکلهای يك اندازه پهلوي

الباب الثالث در عمل شکلها در دائرها

الباب الرابع در عمل دائره بر اشكال

الباب الخامس در عمل دائره در اشكال مذكوره

الباب السادس در ساختن بعضي اشكال در بعضي

الباب السابع در قسمت مثلثات

الباب الثامن در قسمت مربعات

الباب التاسع در قسمت دوائر

الباب العاشر در رفع طرق

الباب الحادي عشر در قسمت کردن مربعها بچند مربع وترکیب کردن

مربعي از چند مربع

الباب الثاني عشر در قسمت کرها وانواع اشكال که بر کره واقع تواند شد

Chapter I: on matters which form the elements and which are basic

Chapter II: on equilateral figures (regular polygons)

Chapter III: on the construction of figures inscribed in a circle

Chapter IV: on the construction of a circle superimposed on figures

Chapter V: on the construction of a circle inscribed within figures previously mentioned

Chapter VI: on the construction of figures superimposed upon each other

Chapter VII: on the division of triangles

Chapter VIII: on the division of squares

Chapter IX: on the division of circles

Chapter X: on road survey/construction

Chapter XI: on the division of a square within a square and the composition of a square from

several squares

Chapter XII: on the division of the sphere and its different types.

Chapters 3, 4, and 9 are all to do with divisions of circles or geometric shapes within circles, but unfortunately Woepcke does not give any diagrams. Chapter 8 appears to deal with divisions of the square, but again Woepcke fails to present any drawings. Finally, chapter 11, which appears to deal with divisions of a square within a square, within a square, sounds very much like the geometric principle adopted in this thesis of alternate concentric squares, but once again there are no drawings to illustrate the process.

Thus, further careful research into Abu'l-Wafā's contributions to practical geometry and their relevance to the present work is needed, but at present the means for this research are not available to us.

2.4 The *Siddhantā*

If the architecture of early Islam is found to display similar proportioning systems to that used in ancient India, how and when did this transfer of knowledge occur?

Many scholars may argue that the bulk of Muslims' experience in practical geometry did not reach them through the translation of Greek writings

but rather through the Indian *Siddhantā*.¹ We are also told that on purely practical geometry, Arab mathematicians did not as a rule write special treatises,² but discussed such problems in their work on the construction and use of the astrolabe and quadrant. To see how the pieces of the Indian-Muslim jigsaw puzzle may fit together, we may refer especially to F. Sezgin's summary of two main issues involved: the Indian-Muslim link through the *Siddhantā*, and the role of the medieval geometer Abu'l-Wafā' al-Būzjānī. Sezgin states that although some scholars doubt the sources which refer to the first translation of Indian medical books into Arabic in the second half of the eighth century A.D. during the reign of the 'Abbāsid Caliph Hārūn al-Rashīd, fortunately mathematical historians have not in the least been influenced by this stand.³ In fact, there are objections to the reports that in the first half of the ninth century A.D. Indian scholars were either invited or summoned to Baghdad and that the *Siddhantā* compiled in A.D. 628 by Brahmagupta was submitted through this delegation to be translated by the but rather through the Indian *Siddhantā*.⁴ We are also told that on purely practical geometry, Arab mathematicians did not as a rule write special treatises,⁵ but discussed such problems in their work on the construction and use of the astrolabe and quadrant. To see how the pieces of the Indian-Muslim jigsaw puzzle may fit together, we may refer especially to F. Sezgin's summary of two main issues involved: the Indian-Muslim link through the *Siddhantā*, and the role

1. *ET*², vol.II, p.257. See also *ET*² Suppl., fasc. 5-6 (1982), p.411f., art. "Ilm al-Handasa".

2. *ET*², vol.II, p.258.

3. *Geschichte des arabischen Schrifttums*, Bd. V (Mathematik) (Leiden, 1974) p. 191.

4. *ET*², vol.II, p.257. See also *ET*² Suppl., fasc. 5-6 (1982), p.411f., art. "Ilm al-Handasa".

5. *ET*², vol.II, p.258.

of the medieval geometer Abu'l-Wafā' al-Būzjānī. Sezgin states that although some scholars doubt the sources which refer to the first translation of Indian medical books into Arabic in the second half of the eighth century A.D. during the reign of the 'Abbāsīd Caliph Hārūn al-Rashīd, fortunately mathematical historians have not in the least been influenced by this stand.¹ In fact, there are objections to the reports that in the first half of the ninth century A.D. Indian scholars were either invited or summoned to Baghdad and that the *Siddhantā* compiled in A.D. 628 by Brahmagupta was submitted through this delegation to be translated by the mathematicians and astrologers.

Sezgin observes that sources confirm how such scholars as al-Fazārī and Ya'qūb ibn Tāriq among others of their contemporaries were completely devoted to the content of the *Siddhantā* and its widespread promulgation. With this point in mind, C. Nallino in 1910 collected fragments of these two scholars' works, revealing in detail their importance to the history of Arabic mathematics and astronomy. D. Pingree and S. Kennedy cite further pieces of evidence to conclude that through learning the contents of the *Siddhantā*, which was composed of 24 parts, the Arabs during the reign of al-Manṣūr must have gained advanced knowledge in mathematics and mathematical terminology. The first part of the *Siddhantā* is on arithmetic, calculation, and mensuration, while the

1. *Geschichte des arabischen Schrifttums*, Bd. V (Mathematik) (Leiden, 1974) p. 191.

eighteenth part is on the theory of numbers, algebra, etc.¹

Sezgin next notes how Kennedy and Pingree have concentrated mainly on the contributions which al-Fazārī and Ya'qūb ibn Ṭāriq have made to Arabic astronomy and have left the reader to evaluate for himself how far the above two authors have contributed to stifling the false conception of the late arrival of Arabic science.²

We must once again emphasize that the *Siddhantā* and other similar Indian works which were translated into Arabic were not all books on mathematics but contained mathematical sections. However, the translations of these mathematical sections must presuppose prior mathematical knowledge and hence give rise to certain questions. Did the mathematics of the Arabs begin with the translation of these books? If so, how could the first translators such as al-Fazārī have understood the contents of such complex works? Did they create the terminology necessary to translation? How could their contemporaries such as Ya'qūb ibn Ṭāriq, after the first translation, write more books in the same area? This fact alone forces us to accept that a certain development in Arabic mathematics had already taken place. But we know from al-Bīrūnī that the earlier translations are sometimes difficult to understand as they contain numerous Indian words left untranslated, in accordance with the common practice in early

1. Sezgin, *op. cit.*, p.192.

2. *Ibid.*, p.193.

stages of translation from one language into another.

Sezgin also mentions another interesting piece of the jigsaw puzzle, i.e. the role of the Persians in the link between India and early Arabic sources. The translation of books which deal with mathematics appears to go back to the early times of Arabic science. What interests us above all is the fact that a great deal of the Indian calculation methods appear to reach Arab scholars via Persian works. Muslim Arab scholars and caliphs were quite attentive to the existence and importance of sciences in India. An Indian scholar aroused the lively interest of the Caliph al-Manṣūr in the importance of the *Siddhantā*, which led to the translation activity already spoken of. One must remember that the presence of the scholars in the court of Baghdad was a result of the Muslim desire for knowledge. The Arabs knew certain Indian works through the translations of Pahlavi writings, of which the oldest translation into Arabic was handed down in the first century A.H. The early intermediate role of Persian writings between Indian and Arabic sciences in mathematics and astronomy has been demonstrated by Pingree and Kennedy. The perimeter of the planet equation and methods of Middle Persian (*Zīj al-Shahriyār*) do not differ from those in the *Siddhantā* because the Persians must have taken it from the Indians. The Arabs appear to have received this knowledge first through the Persians and then directly from the Indians. Al-Bīrūnī reports that Persian editions and *Zīj al-Arkand* contain

additions of Indian origins.¹

Regrettably many Arabic works linked to the Indian *Siddhantā* are no longer extant. Therefore we do not know what stand these authors took in the mathematical part of their works (which al-Bīrūnī occasionally called *maqālāt al-ḥisāb*). There is no doubt that the *Siddhantā* and other translated books are of great importance to the early period of Arabic mathematics, although many writings that have survived show no direct relation between the Indian and Arabic algebra in the early period. In the area of trigonometry, on the other hand, the effect is striking, as the chord was replaced by the sine.

Indian mathematics has from the beginning influenced Arabic thinking by different processes. It is certain that the Arab mathematicians were aware of numbering systems known only among the Indians which they called *al-ḥisāb al-Hindī* and on which they wrote many books. It should be emphasized, however, that this does not mean that the knowledge of this system first arrived through the translation of Indian writings or directly from the Indians themselves. We must see it as synonymous with the decimal positive value system which the Arabs probably found in the Hellenistic East.

Sezgin relates the popular tale of how the modern trigonometric term 'sine' was derived. He states that although the geometry of the Indians appears to

1. Sezgin, *op. cit.*, p.193 f.

have left few traces among the Arabs, it is interesting that al-Bīrūnī commented on the state of the Indian knowledge and its diversity, 'The crystals are mixed with ordinary stone.' Sezgin here refers to al-Bīrūnī's *Tahqīq mā li'l-Hind*. Motivated by the wish to spread scientific knowledge, al-Bīrūnī has in his work about the Indians made the Arab world aware of the many achievements of the Indian scholars, while on the other hand he undertook to achieve the translation of the *Elements* of Euclid and the *Almagest* of Ptolemy into Sanskrit.¹

In view of the above discussion, the achievement of the Indian development of trigonometry must be acknowledged. As mentioned above, their replacement of the chord by the sine meant operating with the half chord of the double angle instead of the whole. The present term 'sine' is a translation of the Arabic word *jayb* ('pocket'), for the Arabs rendered the trigonometric term *jīva* (meaning 'sector') as *jayb*, which was then translated into Latin by the word for 'pocket': *sinus*. In the early works the word *ardajīva* for 'half chord' was later shortened to *jīb* (for 'sine').²

Having examined geometric planning in the Near East prior to and during the early period of Islam, the principle of squared circles subdivided to

1. *Ibid.*, p.195 f.

2. See Ya'qūb ibn Ṭāriq, *Taqī' Kardaajā al-Jīb*; and in addition al-Bīrūnī's three books on the mathematical section of the *Siddhantā*, produced in A.D. 777:

(1) *Risālah Rāshikāt al-Hind*; (2) *Risālah fī Kayfiyyat Rusūm al-Hind fī ta'allum al-Ḥisāb*, *al-Jawābāt 'an al-Masā'il al-'Ashr al-Kashmīriyya*; and (3) *Risālah fī anna Ra'y al-'Arab fī Marātib al-'Adad Aṣwab min Ra'y al-Hind fihā*.

create a guiding system for harmony and proportion is a subject which one needs to consider in detail.

CHAPTER THREE

SHAPING THE DESIGN

3.1 Introduction

3.2 The Components

3.3 Aids to Design

3.4 Systems for Analysis

3.5 Two-dimensional Architectural Design

3.6 The Plan and the Elevation

CHAPTER THREE

SQUARING THE CIRCLE

3.1 Introduction

3.2 The Components

3.3 Aid to Design

3.4 System for Analysis

3.5 Two-dimensional Architectural Design

3.6 The Plan and the Elevation

3.1 Introduction

Make a round circle of the man and the woman, and draw out of this a square, and out of the square a triangle. Make a round circle and you will have the stone of the philosophers.¹

Metaphorically, the circle often represented infinity, whereas the square reflected the finite, in particular, the four corners of the earth. In order for a limit to be put on an infinite number of permutations, a square needed to be drawn round the circle.

The square and the circle are universal figures. Each possesses unique properties and carries an immemorial esoteric symbolism. These basic shapes, from which, many believe, all of the diversity of structures in the universe are composed, require no use of measurement in their construction.²

There are numerous diagrams and charts in the literature of sacred geometry all related to the single idea of 'squaring the circle'. This practice, which employs only the usual compass and straight-edge, constructs a square which virtually encloses a given circle. Squaring the circle is of great importance to the geometer, for if the circle is assumed to represent pure, unmanifested spirit

1. Cited in N. Pennick, *Geomancy*, p.123. See also fig. 3.1.1.

2. R. Lawlor, *Sacred Geometry*, p. 6-8.

and limitless space, the square represents the manifest and comprehensible world.¹ When when an edge is drawn around the circle, the infinite is able to express its limit through the finite.

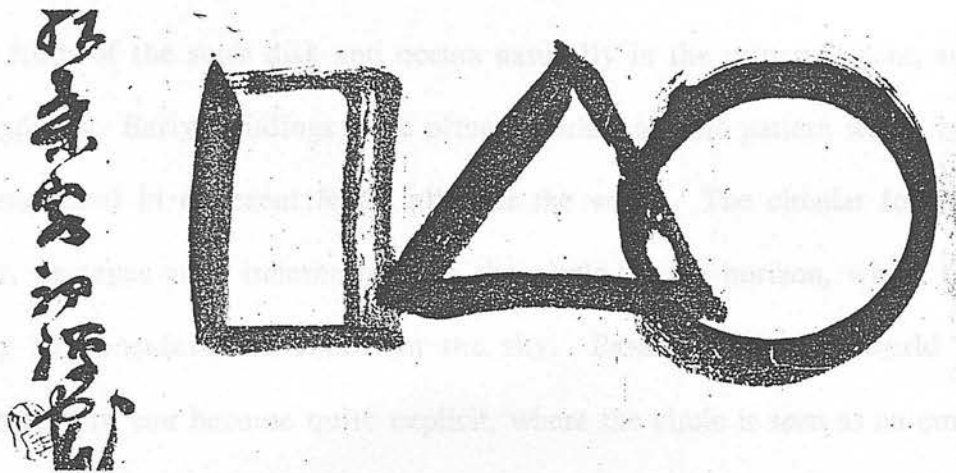


Fig. 3.1.1: Japanese Zen calligraphic drawing showing ‘creation’² through the simple progression from the unity of the circle, through the triangle, to the square

Among early Islamic authors, Ibn Khaldūn (1332–1406) refers to the use of geometry in architecture as follows:

It requires either a general or a specialized knowledge of proportion and measurement, in order to bring the forms of things *from potentiality to actuality* in the proper manner, and for the knowledge of proportions one must have recourse to the geometrician.³

1. G. Michell, *Architecture of the Islamic World* (London, 1978) p. 132.
2. R. Lawlor, *Sacred Geometry*, p. 13.
3. Michell, *loc. cit.*

3.2 The Components

3.2.1 The Circle

The circle is one of the most ancient of the symbols used by humanity. It is also perhaps the easiest geometric shape to construct. It is seen each day in the form of the sun's disk and occurs naturally in the mineral, plant, and animal kingdoms. Early buildings were often circular, a basic pattern which can still be encountered in different forms all over the world. The circular form of a hut, yurt, or tepee may inherently echo the circle of the horizon, where the ceiling may be considered to represent the sky. Parallels with the world in sacred architecture can become quite explicit, where the circle is seen as an embodiment of the universal whole, representing perfection and completeness.

In ancient Rome the universal nature of the circle was deemed apt for the shrine of all the gods. The Pantheon, which was constructed as a vast semicircular dome, is one example. From the megalithic era onwards, round edifices have repeatedly been erected in Europe. Wooden structures like the so-called Woodhenge and the Overton Hill sanctuary (which are known solely from their post-hole remains) paralleled the structure and perhaps the function of the later North American medicine lodges.¹ Stonehenge is another such structure. Circular temples were also relatively common in ancient Greece, e.g. at Delphi. The pattern was also found in Rome and in the characteristic fashion of that

1. N. Pennick, *Geomancy*, p.121. See also Michell, *Architecture of the Islamic World*, p.132; El-Said, *Geometric Concepts*, p.3.

empire. The Roman round temples at Tivoli and Spalato have survived beyond their era. The former was a basic rotunda surrounded externally by columns in the Greek manner, whilst the latter, which was part of the palace complex constructed by the emperor Diocletian, was the forerunner of later structures.¹

In sacred architecture, perhaps the most comprehensive yet simple form for defining a sacred area is the circle. This 'opening to the transcendental' is achieved either as the result of setting out the parts of the temple as a whole or is literally symbolized by a physical feature such as a central pole or the smoke hole above the altar fire. The word 'temple' apparently has its roots in the Ancient Greek *temenos*, to cut off a sacred area, as well as in the Latin *templum* (space), which has survived in the modern 'template', and *tempus* (time). Thus the word implies both time and space. Sacred space is an area set aside from worldly concerns for religious matters.

3.2.2 The Square

In many cosmologies, the world was represented by a square. Like the circle, the square is a symbol with unique properties, but unlike the fluid and somewhat indefinite nature of the circle, the square is rigid and unyielding, symbolic of permanence and inflexibility. Just as the circle can be said to represent the heavenly plane of the spirits, so the square may symbolize the stable plane of matter.

1. N. Pennick, *loc. cit.*

The square has been mystically linked with the solidity of the earth, and, almost invariably, square buildings erected to geomantic principles are orientated 'foursquare'; each side faces a cardinal point which, if determined by astronomical observation, can be very near to true.¹ The Egyptian pyramids are the most spectacular monuments possessing square ground plans. They were orientated towards true north-south, according to many scientists with astonishing precision, and were constructed to rigorous standards of accuracy.²

Although the square is a relatively simple figure to lay out, it still has remarkable geometrical properties. It is capable of precise division by two and multiples of two, without any need for measurement. The holy or cosmic city layout is thus derived simply by bisecting the sides, and by drawing vertical and horizontal axes which divide the area into four.

From the square is derived the system used by medieval masons as well as Muslim craftsmen and architects (*ad quadratum*), one square being superimposed upon another whose orientation differs by 45°, thus producing the octagram (see fig. 3.2.2). This is again easy to lay out with string and compasses, the method adopted by ancient surveyors.³

1. *Ibid.*, p.123. See also K. Vatsyayan, *The Square and the Circle of the Indian Arts*, p.75.

2. See E. Raymond, *The Great Pyramid Decoded*, p. 49-53.

3. Pennick, *op. cit.*, p.125.

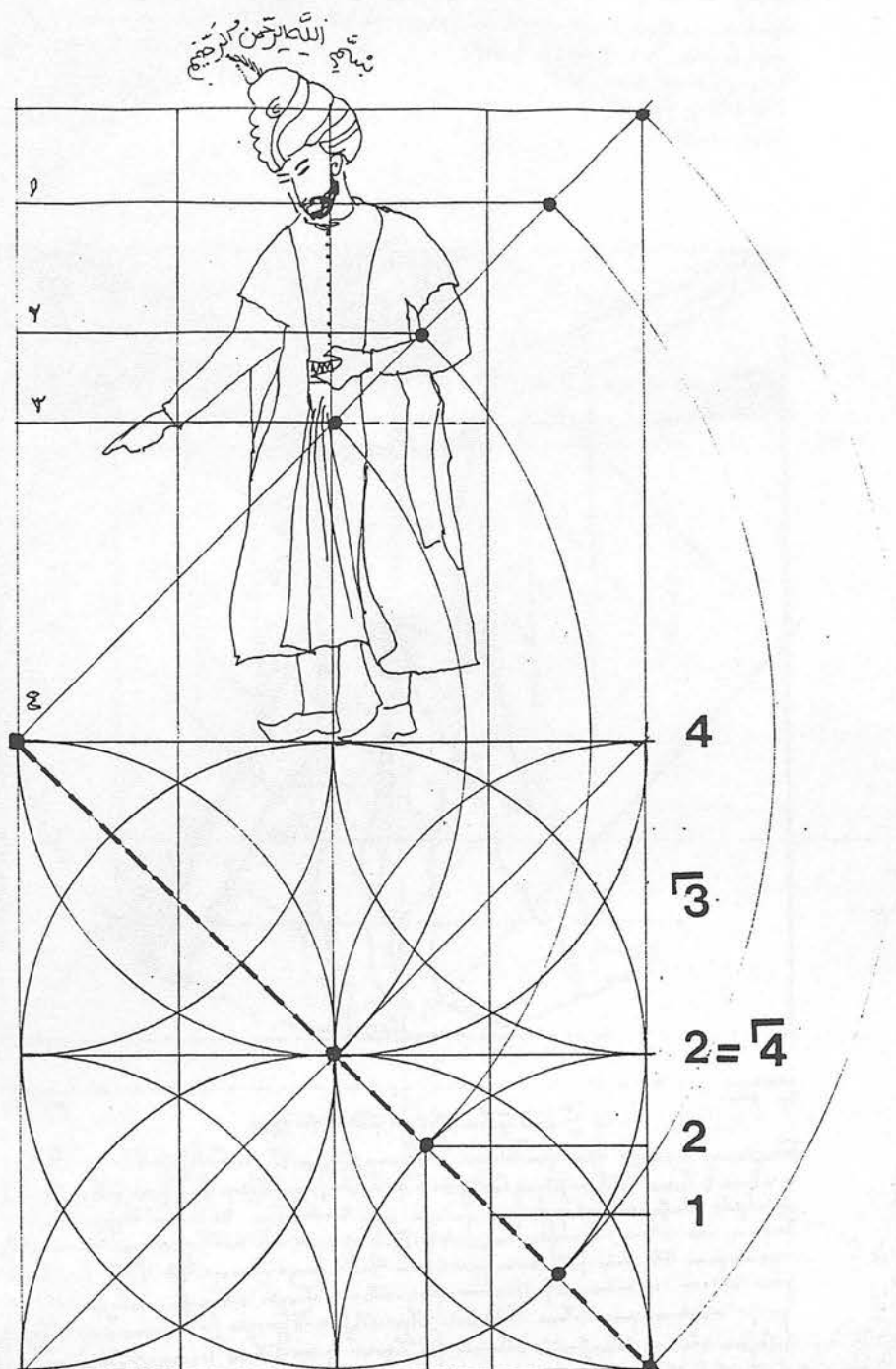


Fig. 3.2.2: Interpretation of Ikhwān al-Ṣafā, *Rasā'il*, "Risāla 2: al-Handasa"

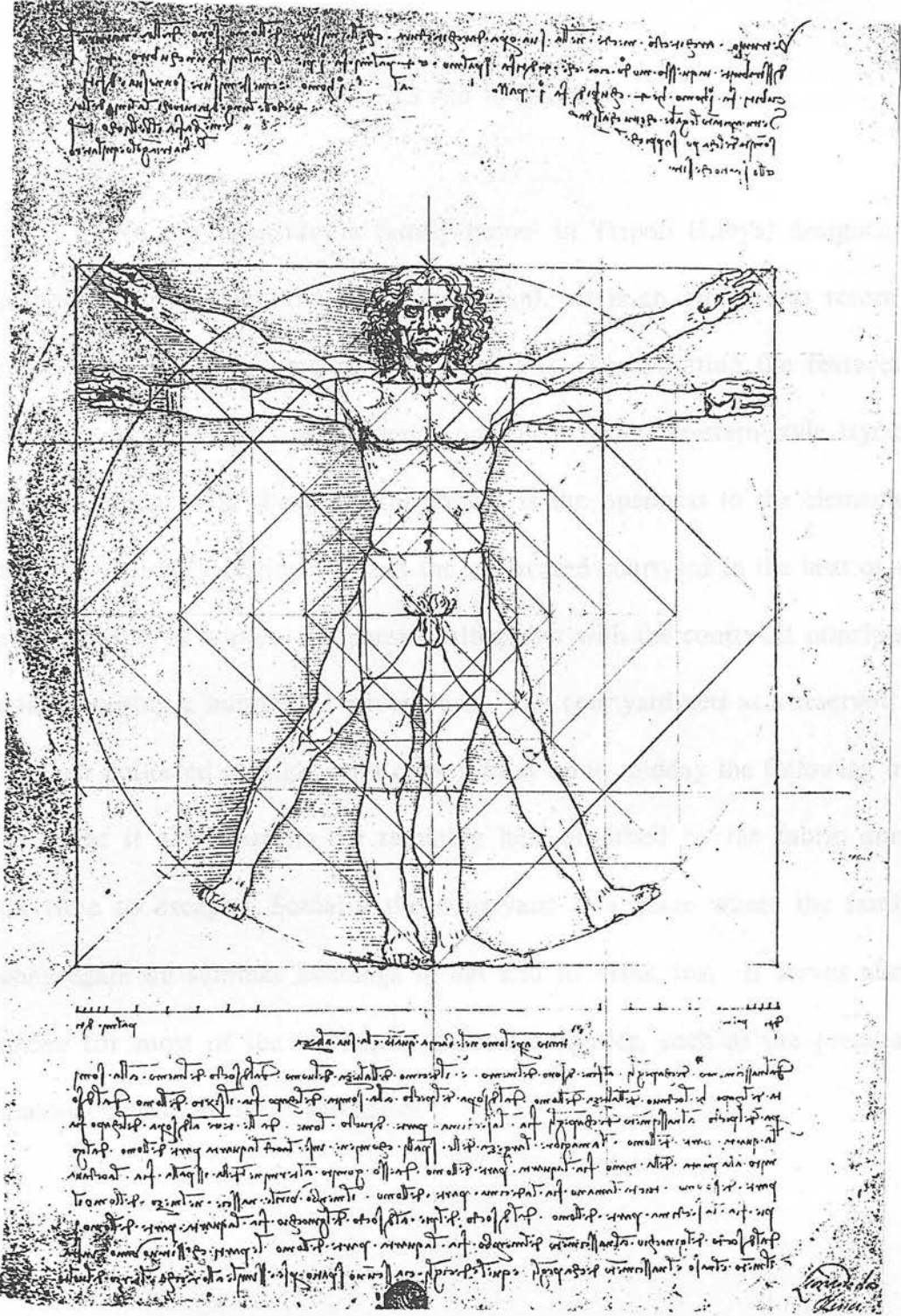


Fig. 3.2.3

Leonardo da Vinci's universal man, adhering to the principle of the square and the circle

3.3 Aid to Design

We may consider a family home¹ in Tripoli (Libya) designed on the principle of squaring the circle (see plan). It is an attempt to return to the traditional Arabic courtyard house but also incorporating the features which modern Arab dwellers have found convenient in the Western-style layout. The main problem with the old arrangement is the openness to the elements which meant the family having to cross the uncovered courtyard in the heat of summer and the cold of winter. Dispensing altogether with the courtyard principle meant compromising a number of advantages. The courtyard acts as a reservoir for the cool air collected at night, and conserves it up to midday the following morning. At night it also permits the radiating heat absorbed by the fabric during the daytime to escape. Socially the courtyard is a place where the family may congregate on summer evenings to eat and to drink tea. It serves also as the arena for most of the household general activities, such as the preparation of seasonal foods like grain and spices.

1. At present under construction.

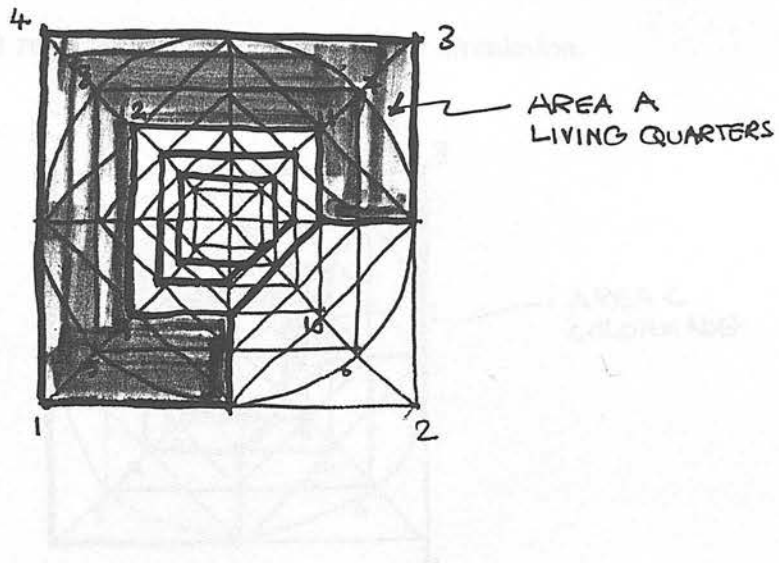


Fig. 3.3.1

In this fig. 3.3.1, area A, i.e. the space between square 9,10,11,12 and square 1,2,3,4, contains the living quarters.

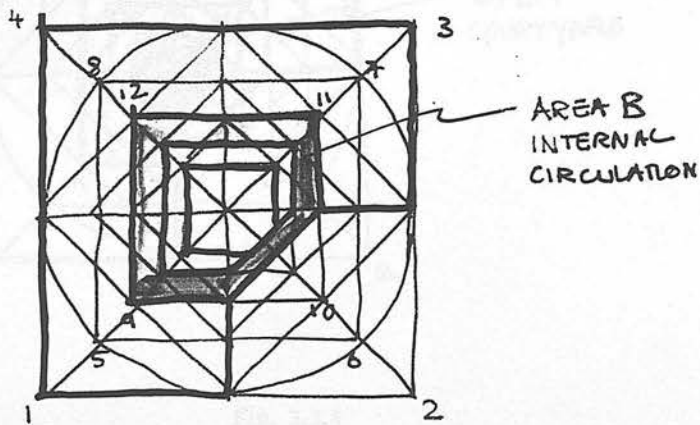


Fig. 3.3.2

In fig. 3.3.2 area B represents the internal covered circulation.

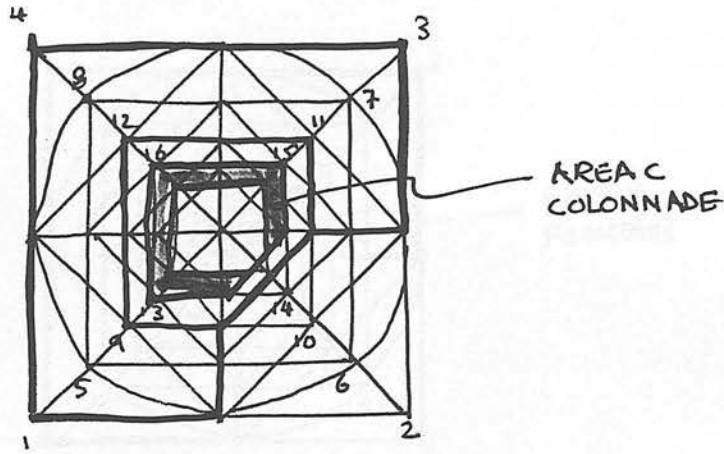


Fig. 3.3.3

In fig. 3.3.3 area C is a semi-covered external colonnade, a transition space linking the external to the internal.

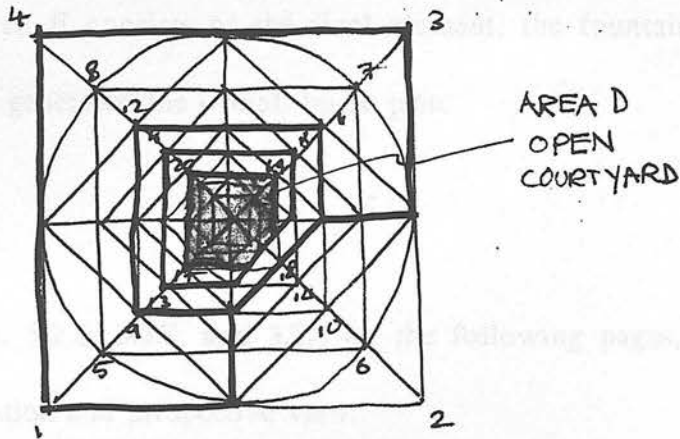


Fig. 3.3.4

In fig. 3.3.4 area D is the open courtyard which, by a diagonal approach, can be reached directly from outside.

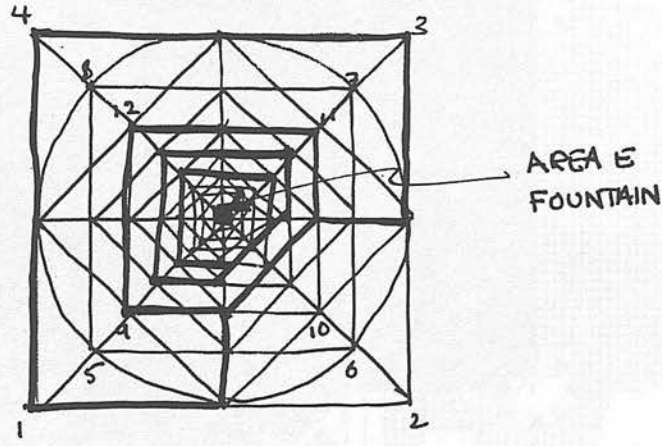


Fig. 3.3.5

In fig. 3.3.5 area E consists of the final element, the fountain expressing the geometry which generated the overall house plan.

In figs. 3.3.6, 3.3.7, and 3.3.8 on the following pages, we present the house plan elevation and prospective view.

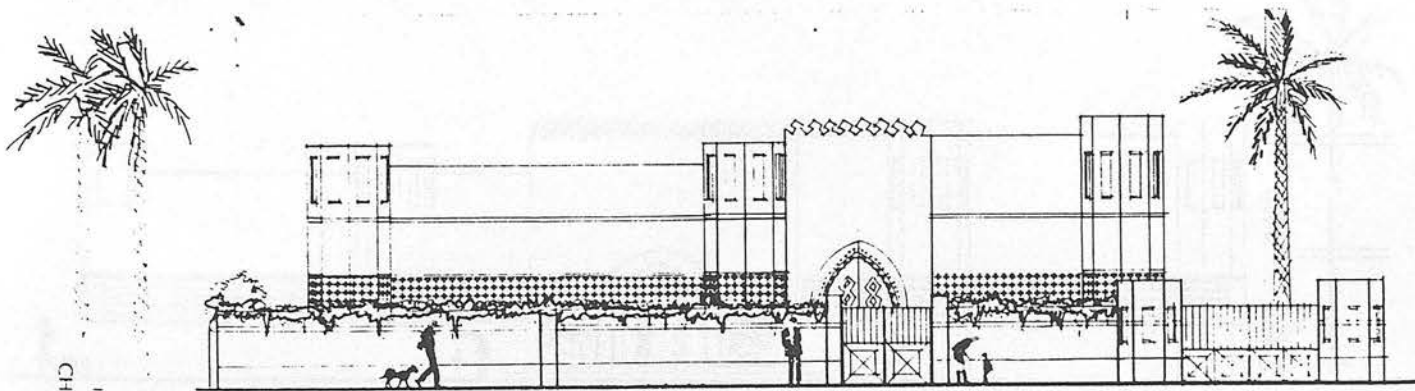
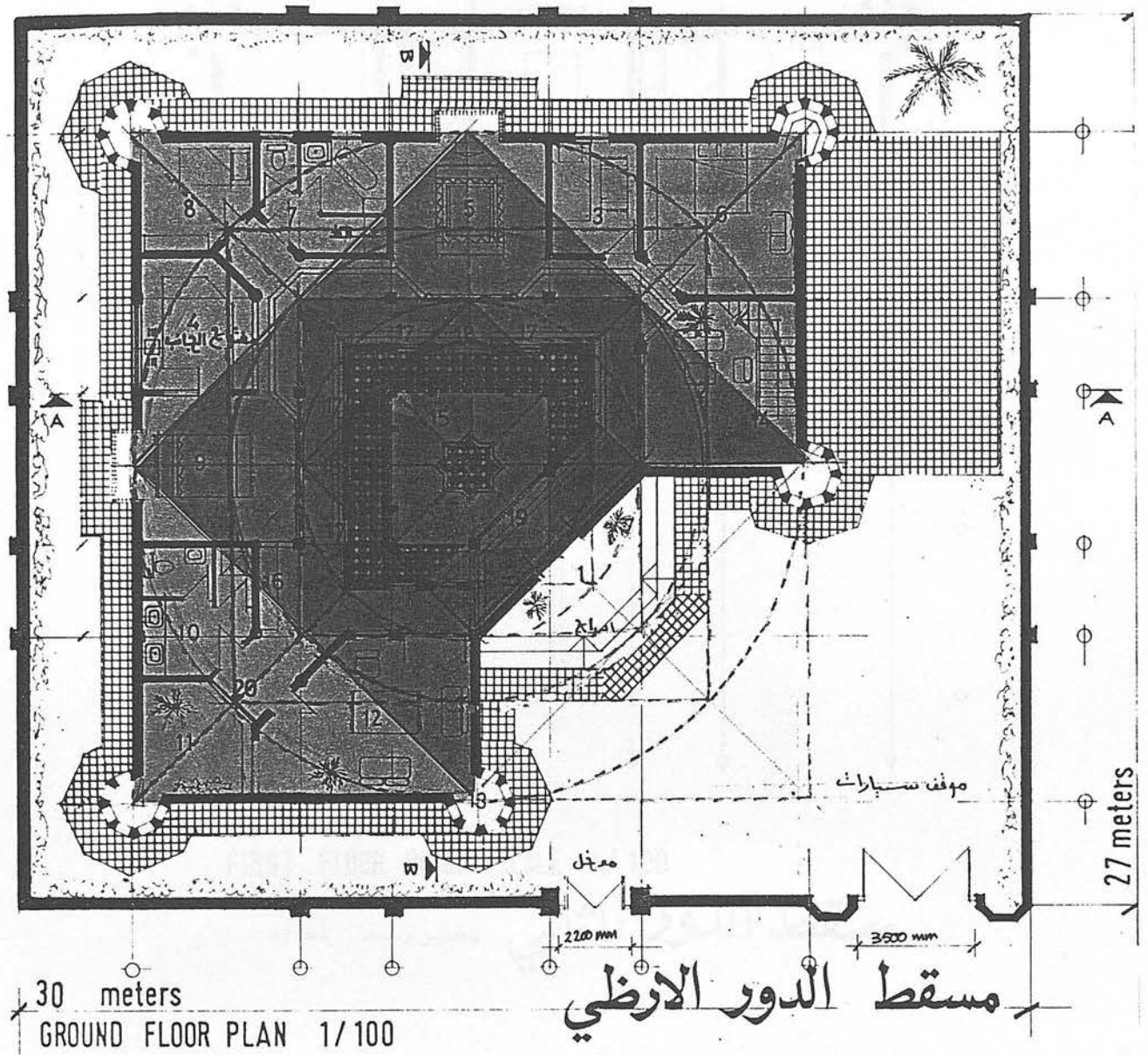
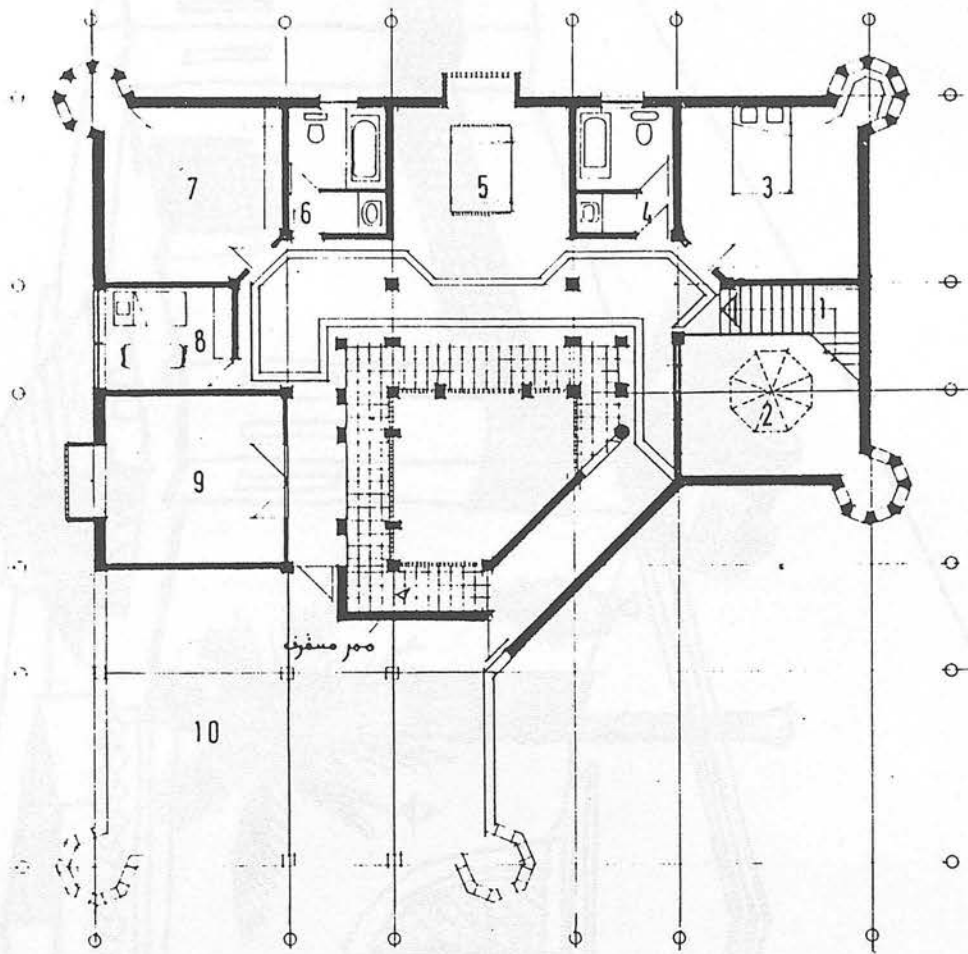


Fig. 3.3.6



FIRST FLOOR PLAN . SCALE 1/100

مسقط الدور الثاني
مقياس ١/١٠٠

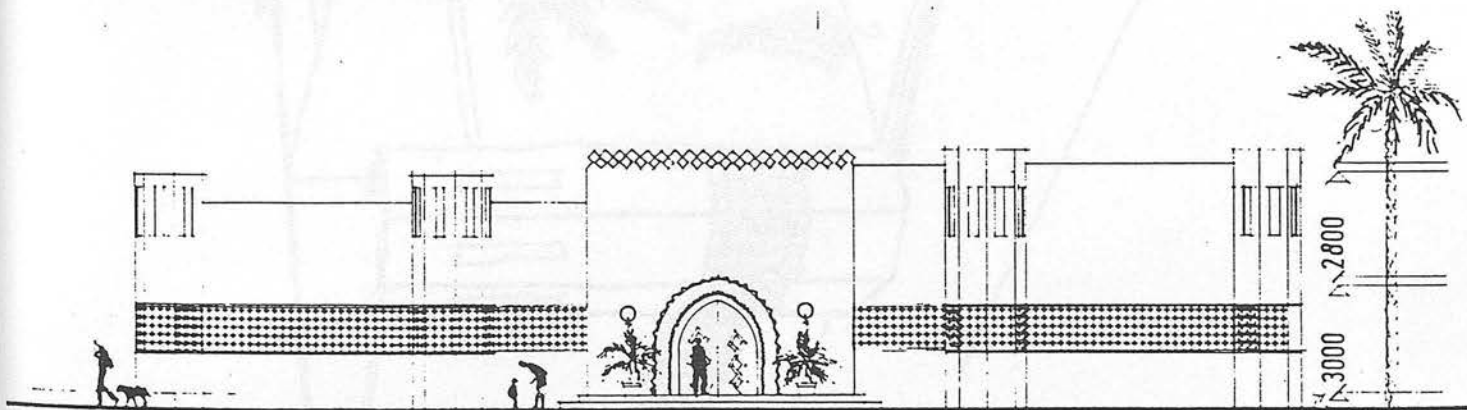


Fig. 3.3.7

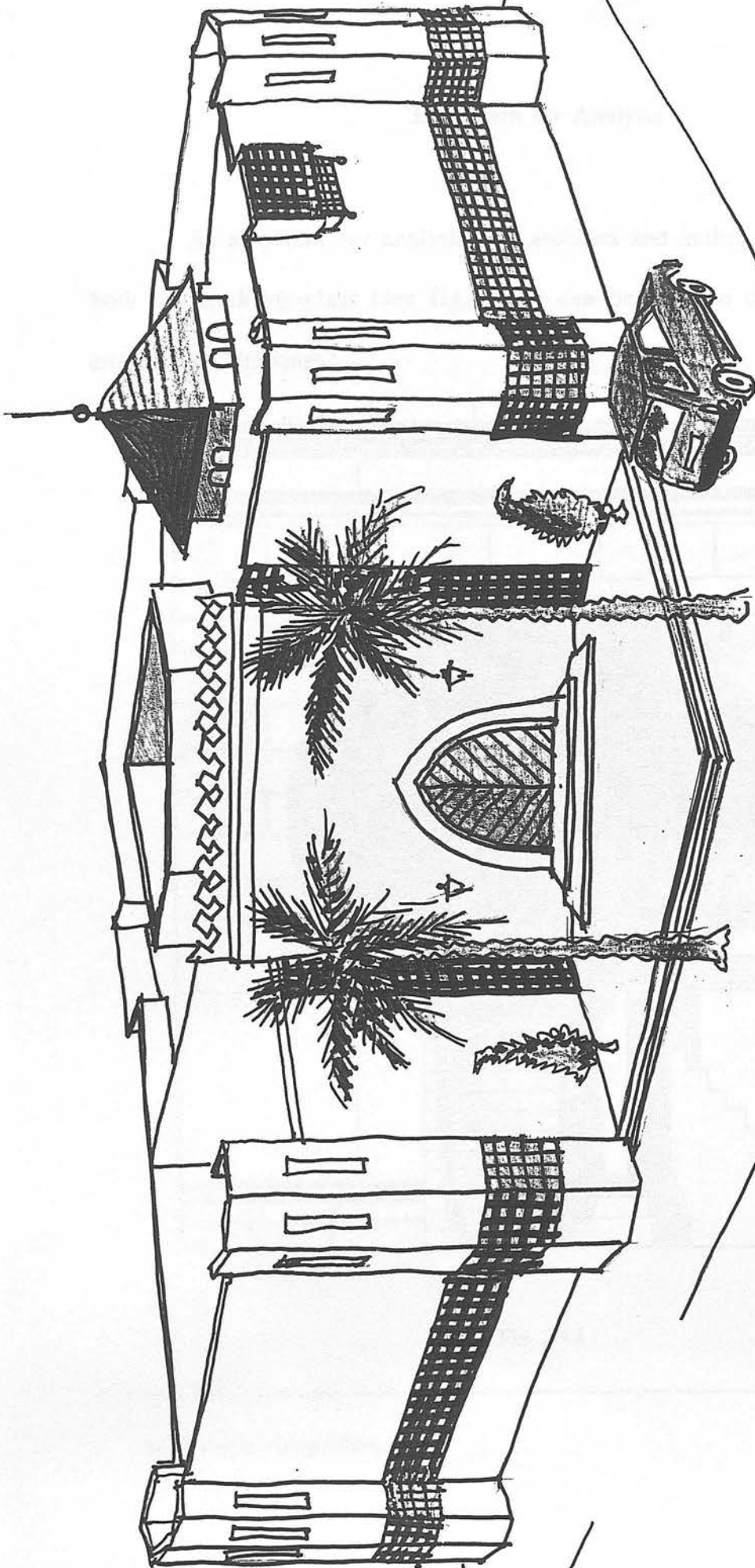


Fig. 3.3.8

3.4 System for Analysis

As a system for analysis, the architect and author Rob Krier illustrates how his looking-glass (see fig. 3.4.1) can be used to detect proportion in architectural drawings.¹

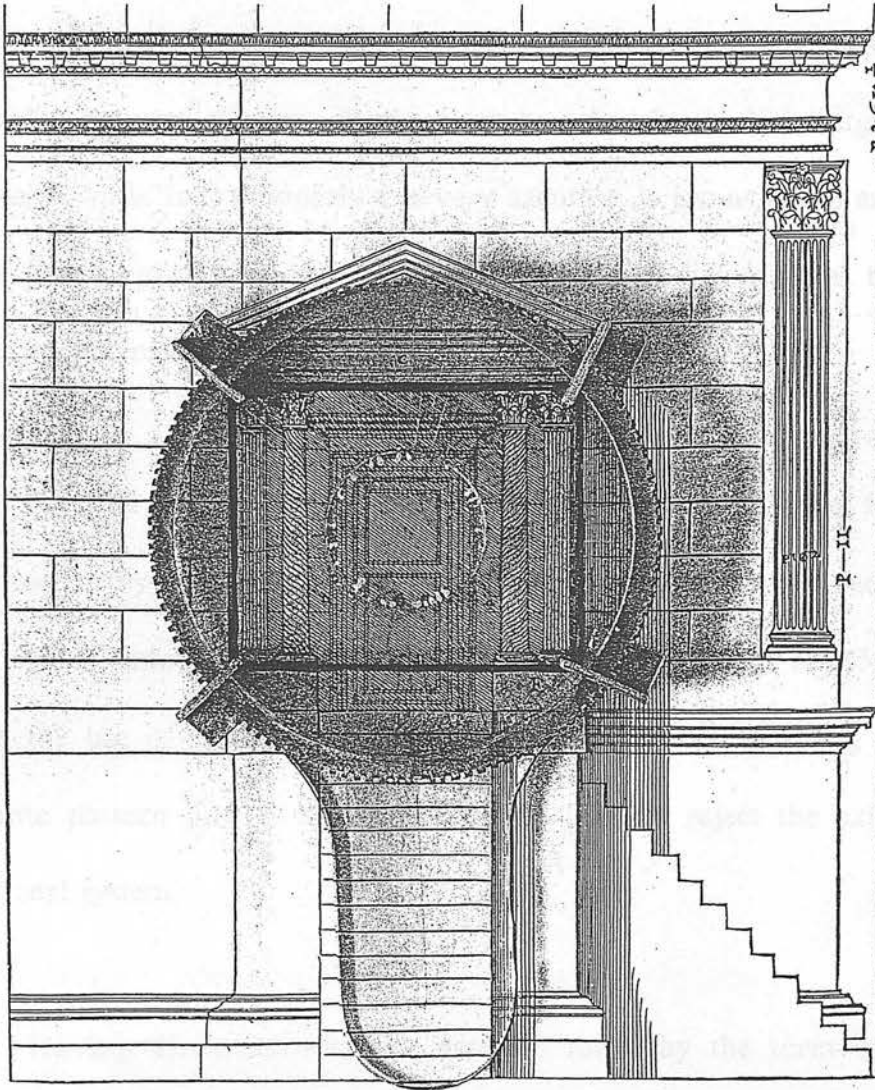


Fig. 3.4.1

1. *Architectural Composition*, p.181.

Krier built his useful little instrument, as he calls it, in order to observe proportions during his architectural excursions. It consists of two clear plates of plexiglass with their planes parallel to each other. Expandable rubber pads sit on each one of the four corners.¹ The distance between the two perspex plates is variable, according to the focal length of the envisaged object. The distance between the two plates is measured and, with a slide rule or calculator, the relationship between the two distances can be ascertained. Krier argues that this instrument, while it is obviously not very accurate, is just as useful as pacing out for the first exploration of a building plan. He also hopes that this type of exploration will encourage the direct study of architecture.

Another way of using the square and the circle as a system for detecting proportion is by trial and error, that is to say, one keeps on redrawing the analysis chart pattern starting from a different centre and with a different radius, or with the use of modern photocopiers one keeps either enlarging or reducing the frame pattern until it is possible to confirm or reject the existence of a proportional system.

Having discussed what we actually mean by the term 'squaring the circle', in the next chapter we set out to examine this principle in relation to fourteen early Muslim monuments covering a diverse geographical area.

1. The square element in the composition (squared circle).

3.5 Two-dimensional Architectural Design

The function of decoration is central to any analysis of Islamic art. It is a unifying factor which, for centuries, has linked buildings and objects from all over the Islamic world and across an enormous geographic span. This form of artistry, which is reflected in architecture and in the applied arts, occurs independently of material, scale, and technique, culminating as decorative principles applicable to all types of buildings and objects.

Symmetry is one of the basic principles of design in Islamic art. Whether employed in the development of pattern or in the composition of a façade, it relies on the recognition of like parts balancing each other on opposite sides of a fulcrum or axis. *Symmetria* in classical terminology meant the proportionality between the constituent elements of the whole. Since the concepts of *symmetria* are based on harmonic proportions, geometric pattern designs become the most expressive mode in the art of two-dimensional decoration.

To our knowledge, no record has survived to instruct us in the theory of designing Islamic patterns and although many have attempted to describe the principles of constructing this art form, none is as clear as the work of Issam El-Said and Ayşe Parman (1976). They have illustrated how the craftsmen at different times and places in the Muslim world applied the geometric principles to the practical problems of making geometric patterns.

At the centre of El-Said and Parman's work lies the concept of the 'repeat unit'. It is the systematic arrangement of the repeat unit which produces the overall design. The shape of the repeat unit of a design is determined by the use of a 'squared circle'. The basic or unit measure is taken as the radius of that circle. The initial divisions of the circumference of the circle (described by the unit radius) into 4, 6, or 5 equal sections (or multiples of these sections) determine the system of proportioning used to generate the repeat unit of the design.

The key to the construction of complex geometric ornament is through a grammar of mathematical principles. The mode of their composition entails a constant abnegation of free choice on the part of the artist in favour of the constraints of symmetry and laws of proportion.

The basic component is a simple shape, such as a square or triangle, used repeatedly. This shape may not be evident in the final design, but it is the determining factor. In Islamic designs the sequence of the repeat units and the patterns built on them invariably follow bilateral or radial symmetry.

In the first of these, the proportional system based on the square root of 2, the rectangle on the right in fig. 3.5.1, shows the relations. Its proportions are derived from the square, whose side provides the height. The side of another

square, placed diagonally within the first at 45° , provides its width. The sides of the resulting rectangle are in a proportion of 1 to $\sqrt{2}$.

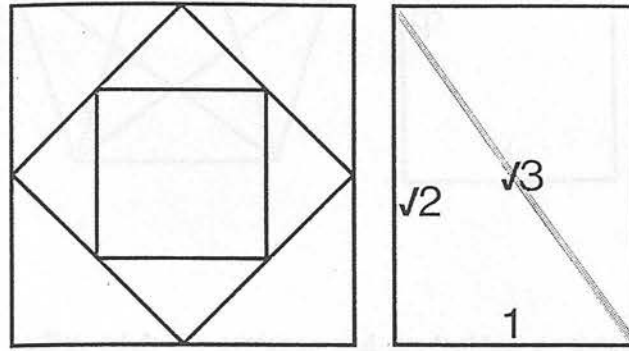


Fig. 3.5.1: the $\sqrt{2}$ proportional system and the $\sqrt{2}:1$ rectangle

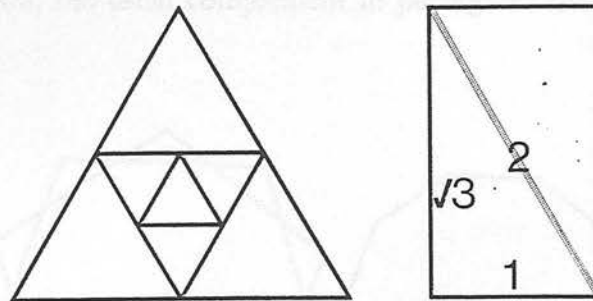


Fig. 3.5.2: the $\sqrt{3}$ proportional system and the $\sqrt{3}:1$ rectangle

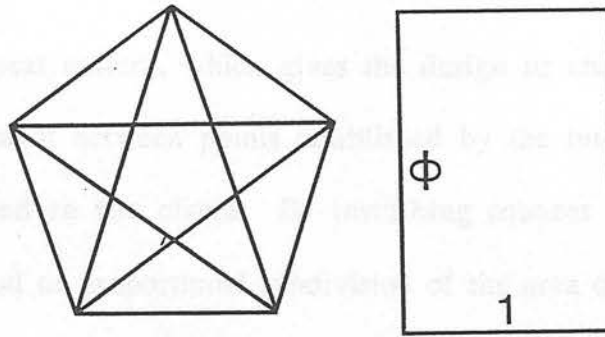


Fig. 3.5.3: the pentagon and the 'golden' rectangle

The second proportional system is based on the square root of 3 (see fig. 3.5.4). This third proportional system derives from the Golden Mean $(\sqrt{5}+1)/2$, with its obvious affinities with the pentagon. Two pentagons rotating at 36° give rise to the decagon, the usual complement to pentagonal design (see fig. 3.5.4).

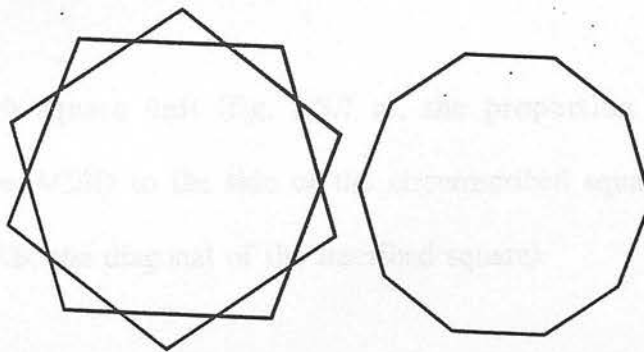


Fig. 3.5.4

Patterns incorporating the root two system of proportion

The repeat pattern, which gives the design its character, is determined by grid lines drawn between points established by the intersecting sides of the squares inscribed in the circle. By inscribing squares within the circle, a geometric method of proportional subdivision of the area of the repeat unit, and thereby of all the grid lines of the pattern, is achieved.

When a surface is to be decorated, one of its sides is divided equally into a number of parts corresponding to the number of repeat units required. The area is then filled with circles (fig. 3.5.5 a), the diameters of which are equal to the subdivisions of the side of the surface being decorated. The area is now equipartitioned into square repeat units by the point-joining method (fig. 3.5.5 b). This procedure is similar to the method of constructing a square as described in fig. 3.5.6.

In each square unit (fig. 3.5.7 a), the proportion of the side of the inscribed square ACBD to the side of the circumscribed square abcd is $AD/ad = 1/\sqrt{2}$ (i.e. $ad = AB$, the diagonal of the inscribed square).

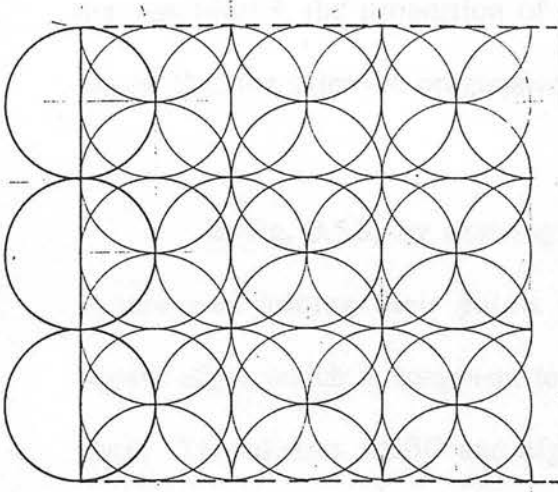


Fig. 3.5.5 a

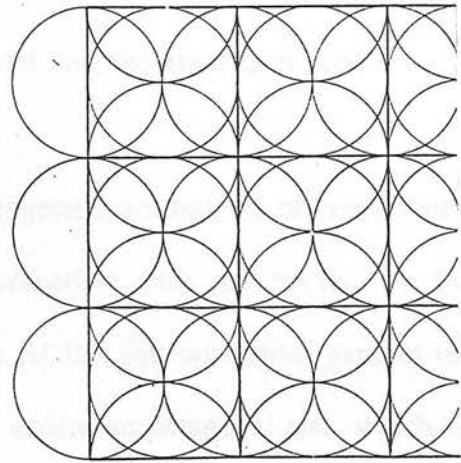
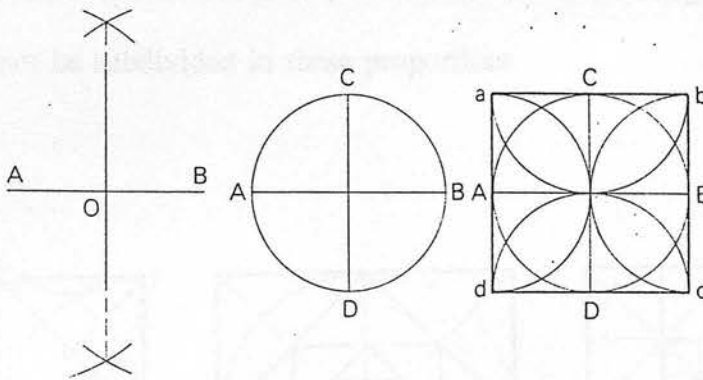


Fig. 3.5.5 b

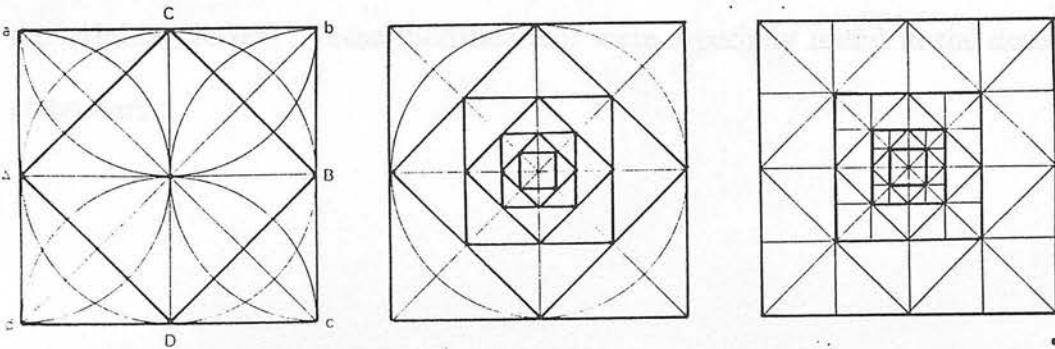


Figs. 3.5.6 a, b, & c

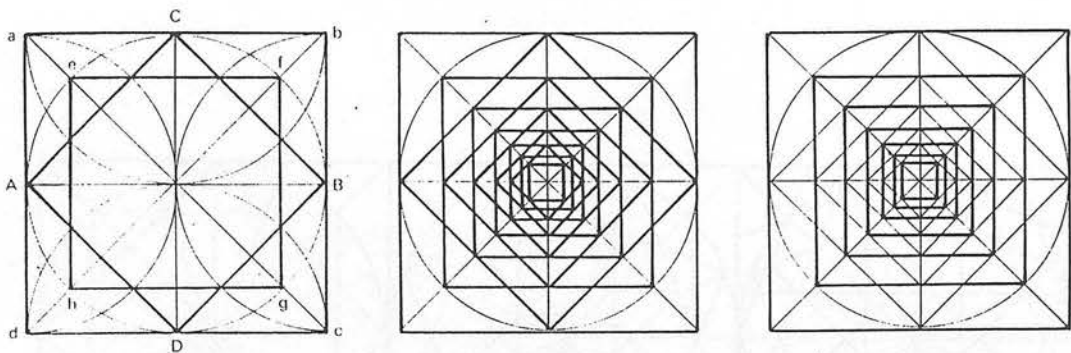
Thus, the sides of the series of concentric squares drawn in fig. 3.5.5 b are related by the proportion $1:\sqrt{2}$ and therefore the areas are progressively halved. In fig. 3.5.6 c, where only the squares of fig. 3.5.5 b with vertical and horizontal sides

are considered, the proportion of the sides of these series of squares is 1:2, which means that the sides are progressively halved and the areas quartered.

In fig. 3.5.8, by drawing the diagonals ac and bd of the circumscribed square and joining their points of intersection with the circles, we form the square efgh, which is congruent to square ACBD, but with sides parallel to square abcd. The squares ACBD and efgh thus create an octagonal star, which we refer to as the 'master grid' of the unit pattern based on this octagonal star. By drawing the concentric octagonal stars (fig. 3.5.8 b) using the point-joining method, we establish a geometric system of harmonious subdivision of the repeat unit. In this system of concentric squares, the consecutive parallel sides (fig. 3.5.8 c) are related in the proportion of $1:\sqrt{2}$ and the alternate parallel sides are related in the proportion of 1:2; therefore, all the sides and the diagonals of the square repeat unit can be subdivided in these proportions.



Figs. 3.5.7 a, b, & c



Figs. 3.5.8 a, b, & c

The designs shown in fig. 3.5.9 illustrate how a square repeat unit can be modified to a rectangular repeat unit. This allows for new repeat patterns and the incorporation of variations into a basic design. Alternatively, the basic square shape of the repeat unit is maintained, but the basic repeat pattern generated on a given set of grid lines is shifted, as illustrated in fig. 3.5.9, to create variations on the original design. These modifications were especially useful in the decorations of borders.

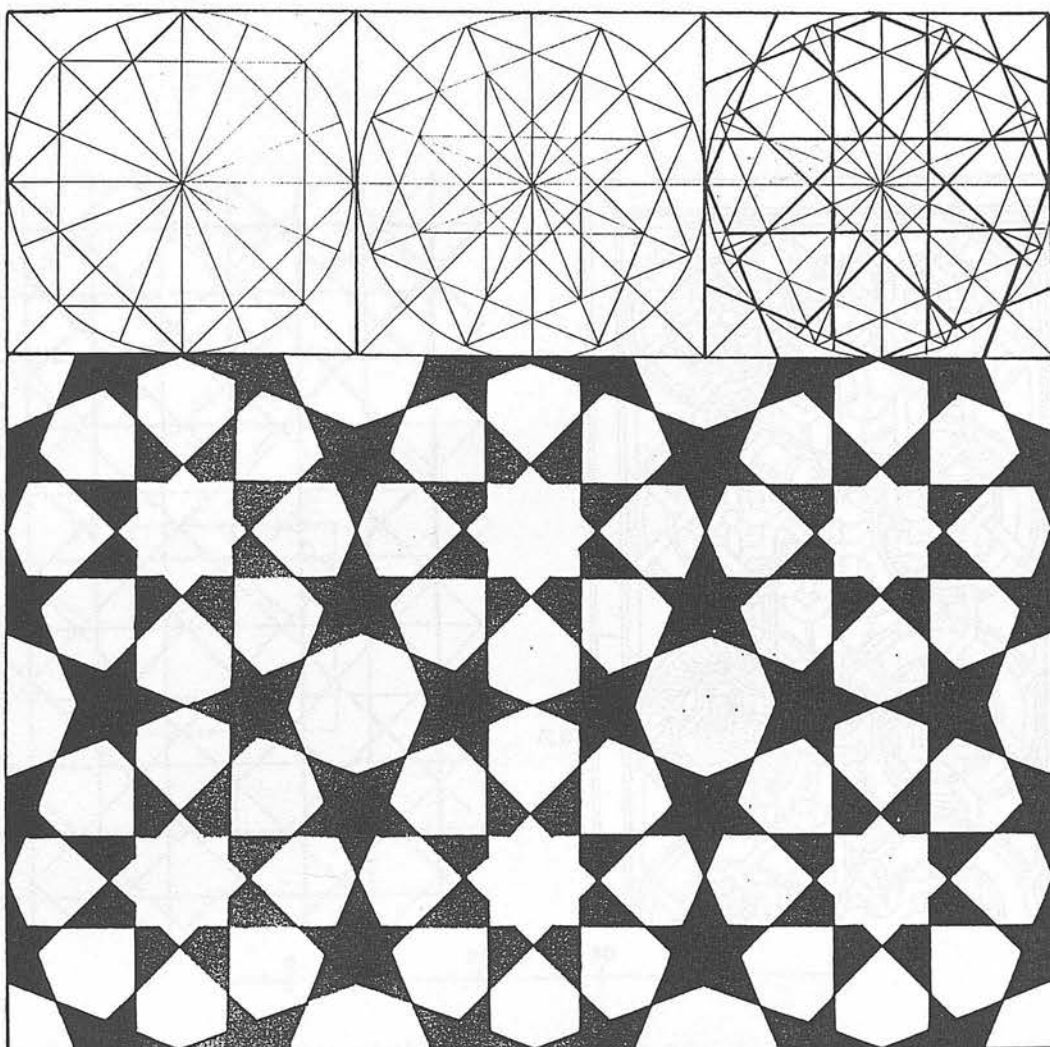


Fig. 3.5.9

Fig. 3.5.10 also reflects a design based on the root two system of proportion. In the Alcazaba of Balaguer (Lérida province of Spain), a reconstruction of stucco panel from the Ta'ifa period shows a familiar form of two overlapping octagons.¹

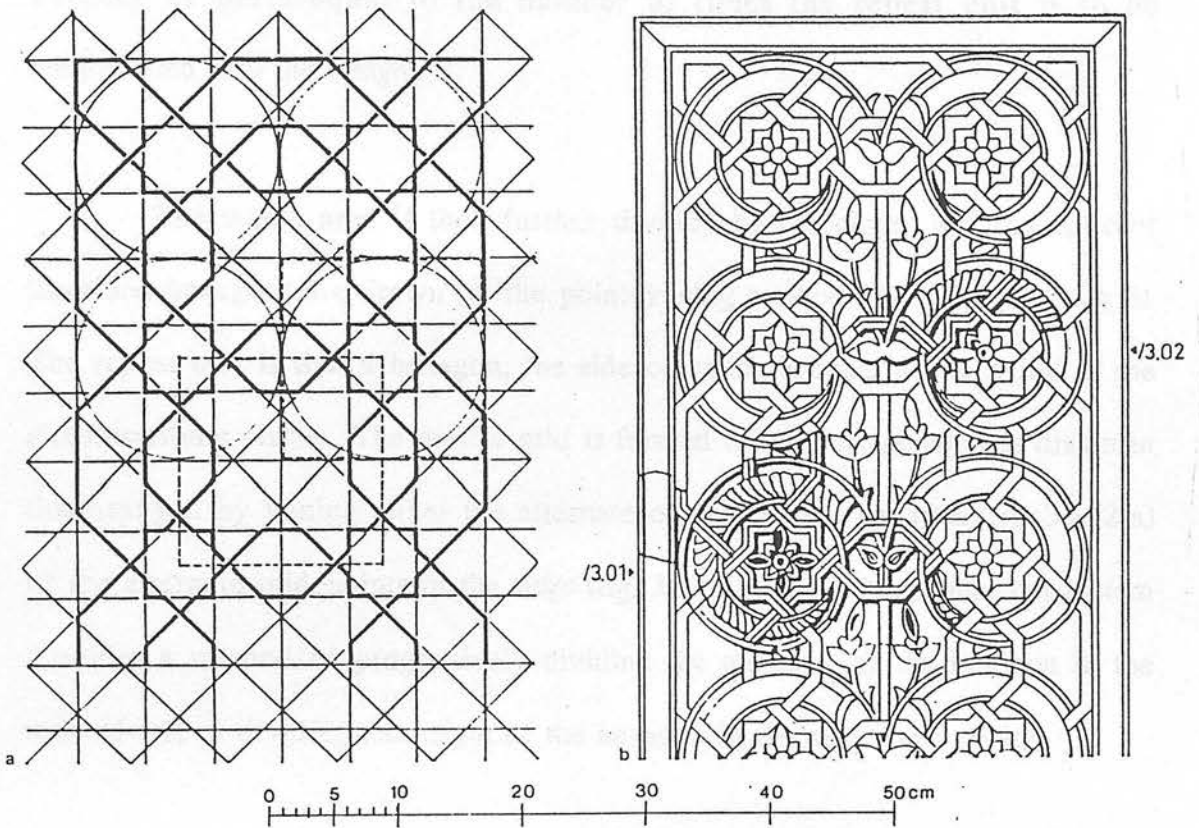


Fig. 3.5.10

1. C. Ewert, *Proceedings of the British Academy*, vol. LXXII (1980) p. 139 f.

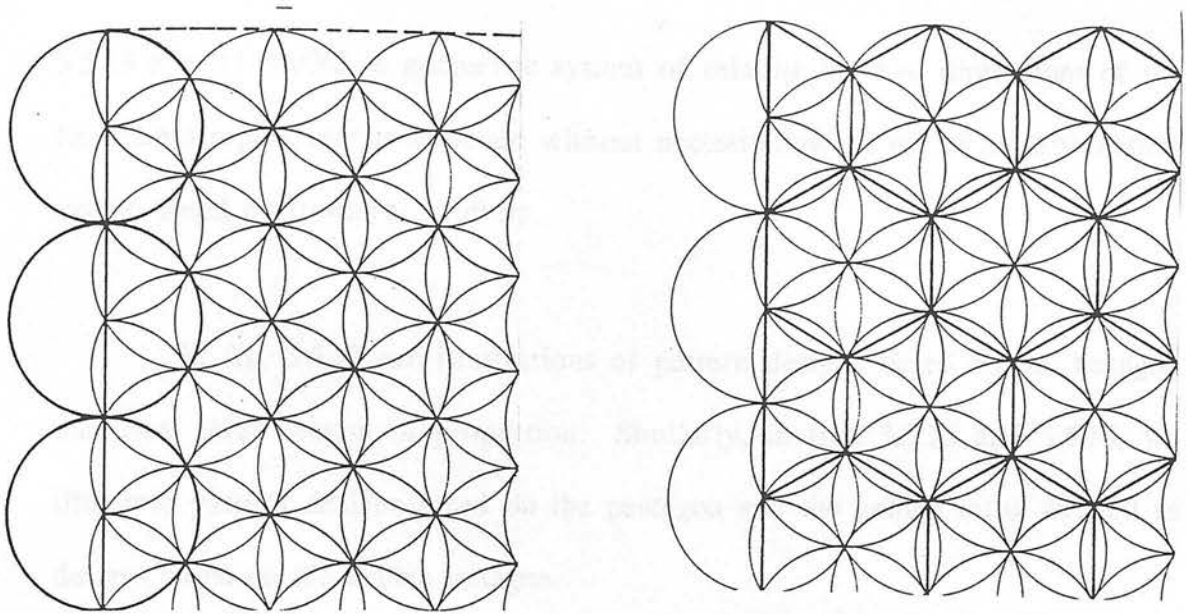
Patterns incorporating the root three system of proportion

The initial procedure for constructing the pattern is the same as described in those incorporating the root two system of proportion. Again, the side of the area to be decorated is subdivided by the use of the compasses into a number of parts equal to the number of times the repeat unit is to be incorporated into the design.

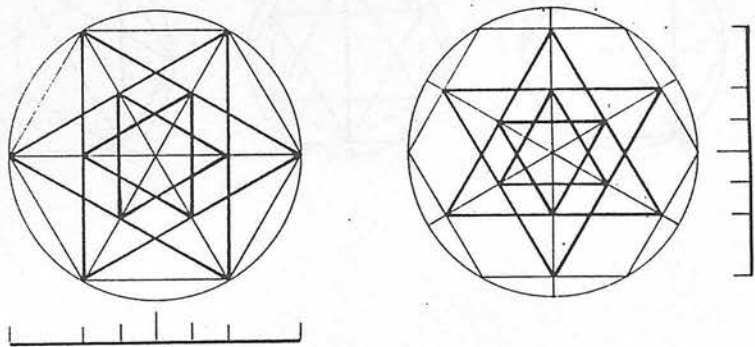
The whole area is then further divided by the circles and the adjacent inscribed hexagons are drawn by the point-joining method (see fig. 3.5.11 a & b). The repeat unit is thus a hexagon, the side of which is equal to the radius of the circumscribing circle. The master grid is formed by the hexagonal stars drawn in this hexagon by joining either the alternate corners of the hexagon (fig. 3.5.12 a) or the alternate mid-points of the sides (fig. 3.5.12 b). This hexagonal star system provides a method of progressively dividing the diameter of the hexagon in the ratio of 1:2, or dividing the height of the hexagon in the same ratio of 1:2.

Fig. 3.5.12 a & b

Since the ratio of the height AB of the hexagon (fig. 3.5.12 b) to the side of AB is the same as the ratio of the height of the hexagonal star (fig. 3.5.12 a) to the diameter of the hexagon (fig.



Figs. 3.5.11 a & b

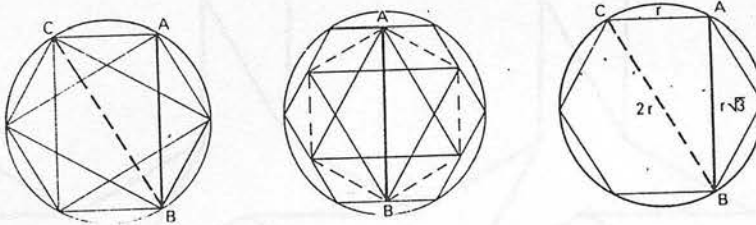


Figs. 3.5.12 a & b

Since the ratio of the height AB of the hexagon (fig. 3.5.13 b), i.e. the side of AB of the hexagonal star (fig. 3.5.13 a), to BC, the diameter of the decagon (fig.

3.5.13 a or c) is $\sqrt{3}:2$, a geometric system of relating the two dimensions of the hexagonal repeat unit is achieved without necessitating the use of an arithmetical system based on irrational numbers.

In fig. 3.5.13 are illustrations of pattern designs based on the hexagon and root three system of proportion. Similarly, in figs. 3.5.15 and 3.5.16, we illustrate pattern designs based on the pentagon and the golden ratio¹ as well as designs based on the double hexagon.



Figs. 3.5.13 a, b, & c

1. For detailed examination, see El-Said & Parman, *op. cit.*, 82-85, 98-113; K. Critchlow, *The Language of Islamic Pattern*.

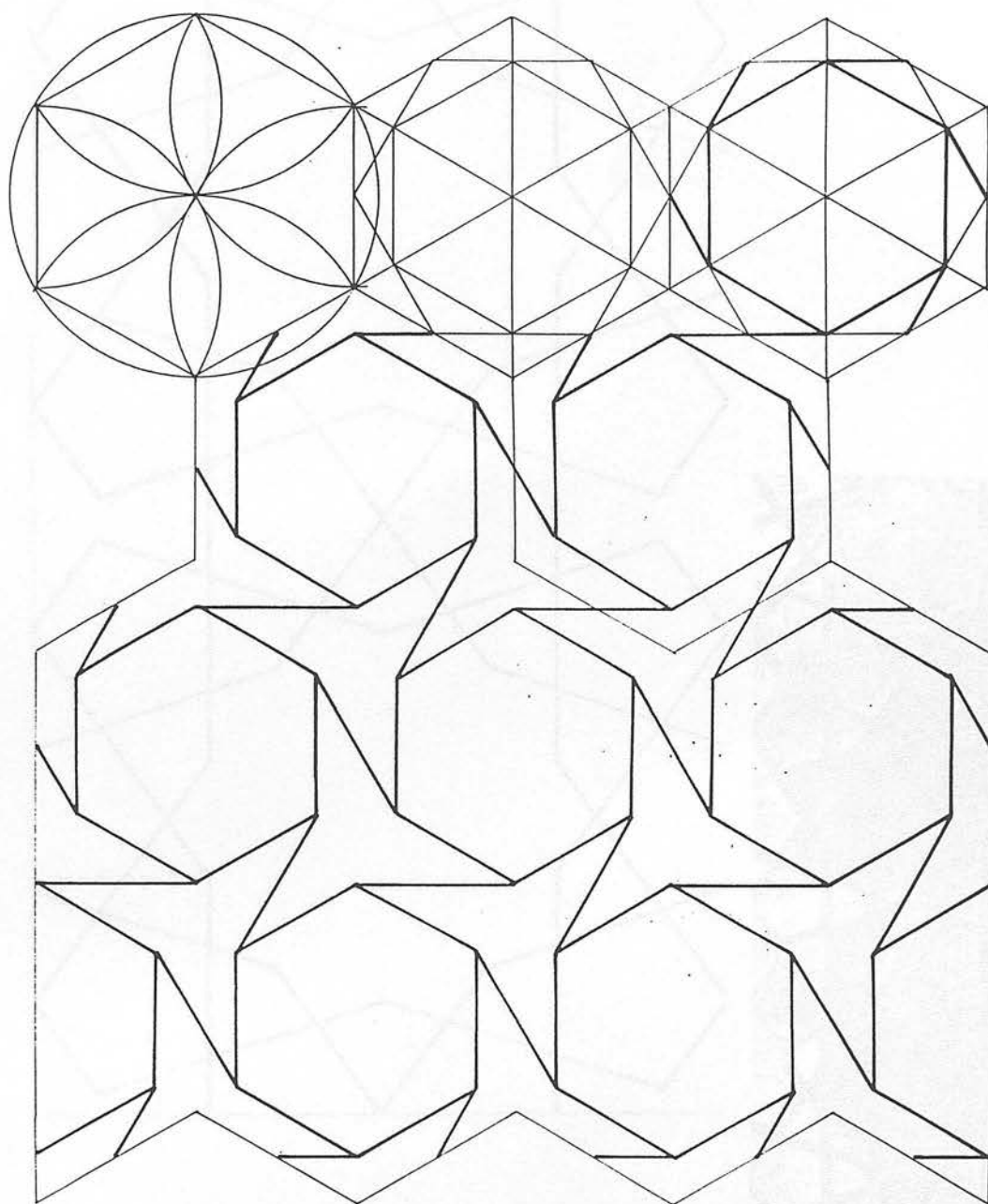


Fig. 3.5.14

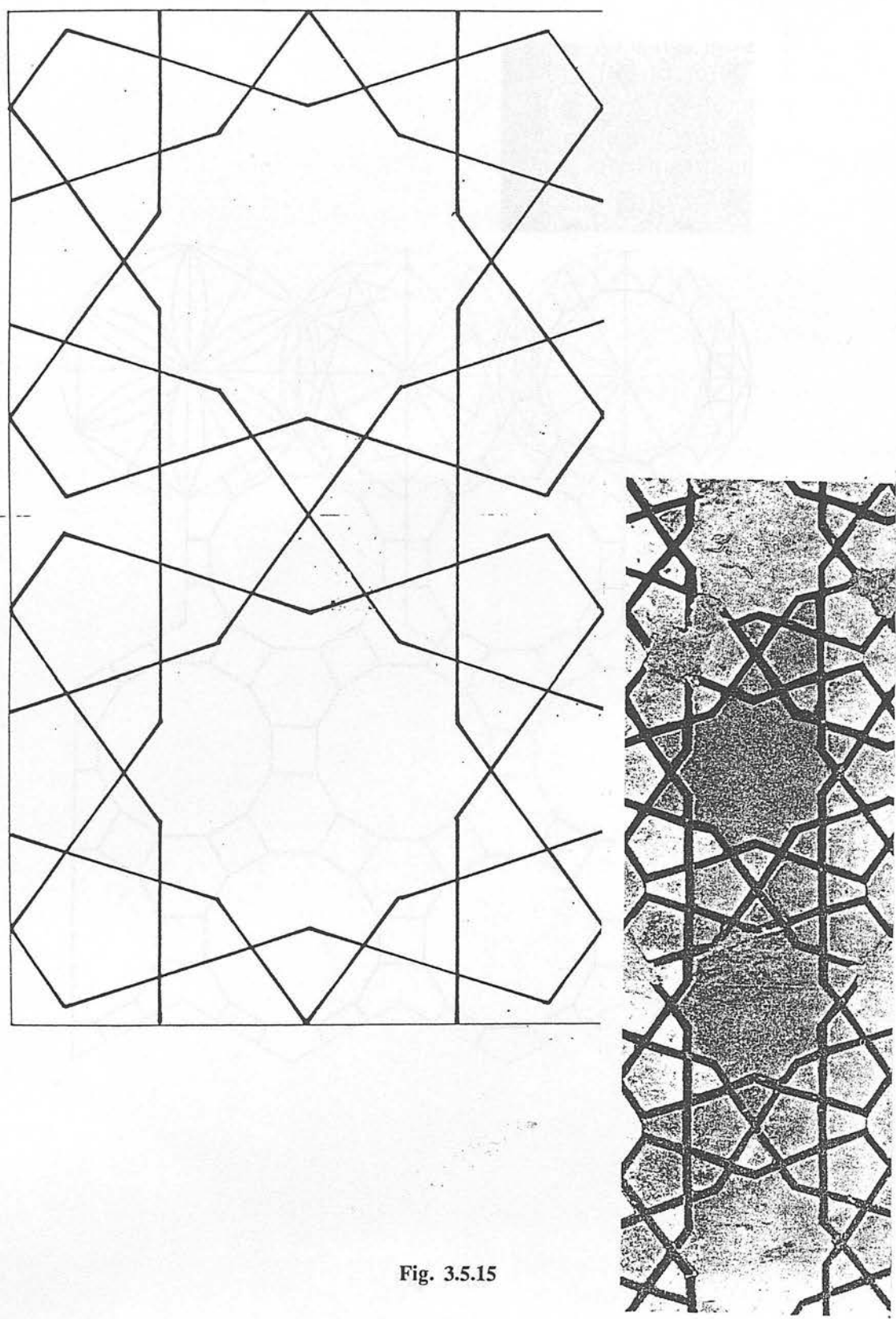


Fig. 3.5.15

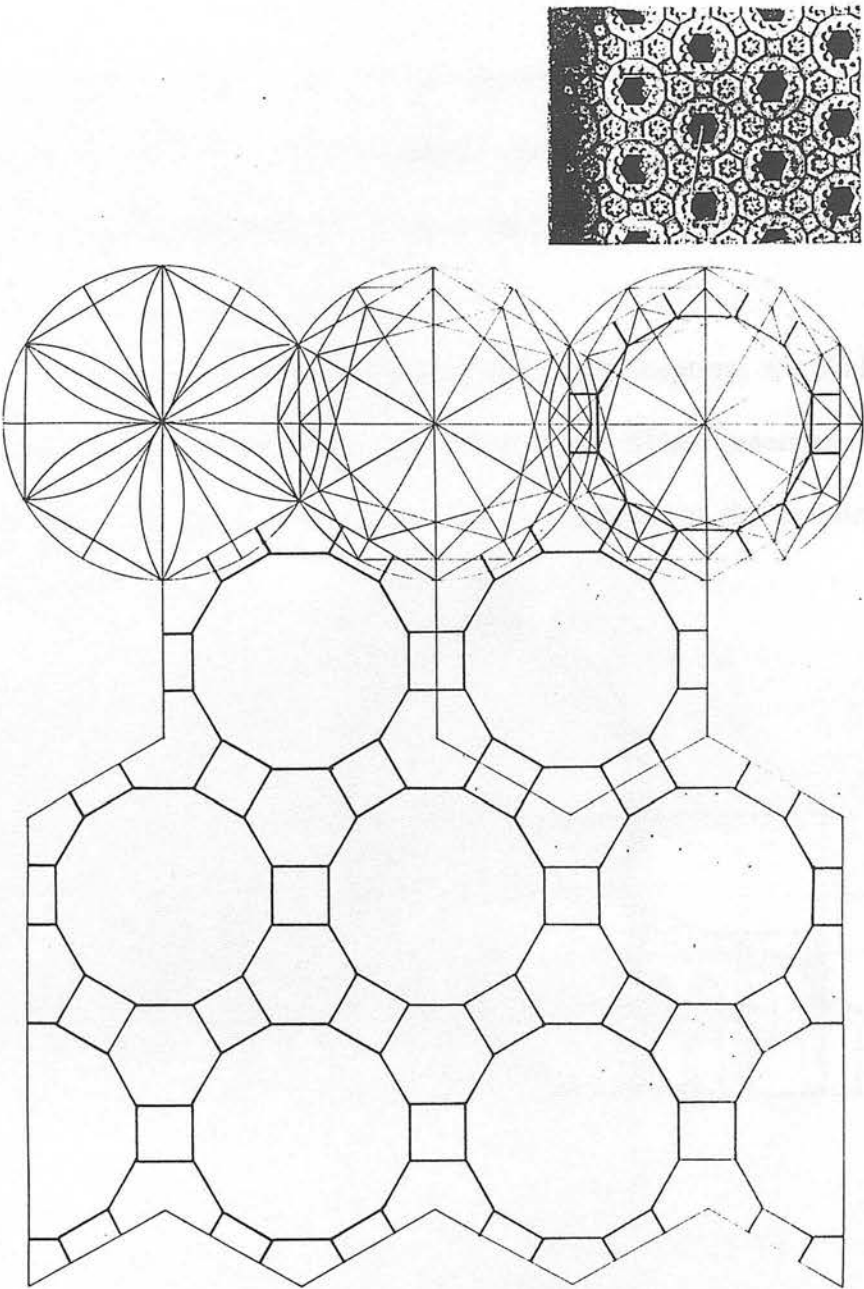


Fig. 3.5.16

3.6 The Plan and the Elevation

We are certain that the plan and the elevation were considered simultaneously by the Muslim builders. This we shall demonstrate in our analysis of the Great Mosque of Qayrawān¹ and our study of the *mihrāb* of al-Manṣūr.²

Here, as an introduction to the next chapters, we shall examine the probability that the octagonal geometric shape which generated the plan of the Dome of the Rock³ is also responsible for setting out the building's section and elevation (see fig. 3.6.1)

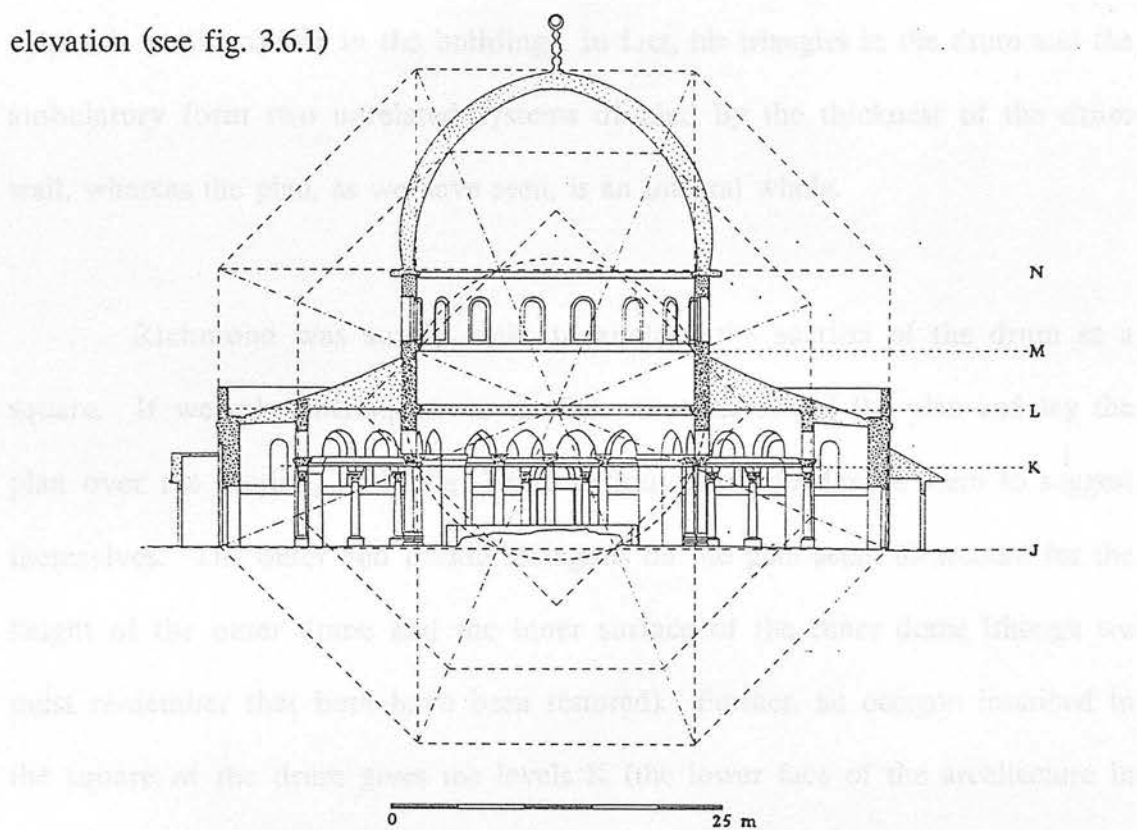


Fig. 3.6.1

1. See thesis, vol. I, chapter IV, section 6; and vol. II (drawings).
2. See thesis, vol. I, chapter IV, section 4.10.4; and vol. II (drawings).
3. See thesis, vol. I, chapter IV, section 1.

In analysing the geometry of the section, E.T. Richmond¹ came to the conclusion that the height and breadth of the drum were intended to be equal, and he marked them on the section by a square. He then analysed the drum and dome in terms of two large equilateral triangles and the inner ambulatory by smaller ones. But Richmond's analysis would thus produce irrational measurements involving $\sqrt{3}$ and consequently be out of harmony with those of the star diagram in fig. 3.6.1, which produces values related exclusively to $\sqrt{2}$. By splitting the length of the drum and dome in two, he lays geometrical emphasis on the mid point (the springing of the drum window arches), but no such emphasis is discernible in the building. In fact, his triangles in the drum and the ambulatory form two unrelated systems divided by the thickness of the drum wall, whereas the plan, as we have seen, is an integral whole.

Richmond was surely right to analyse the section of the drum as a square. If we take this square as the innermost feature of the plan and lay the plan over the section, as in fig. 3.6.1, various correspondences seem to suggest themselves. The outer and middle octagons on the plan seem to account for the height of the outer dome and the inner surface of the inner dome (though we must remember that both have been restored). Further, an octagon inscribed in the square of the drum gives the levels K (the lower face of the architecture in the middle octagon), L (the middle of the lowest cornice in the drum), and M (the lower edge of the middle cornice, which has been given a casing of thin

1. *The Dome of the Rock* (Oxford, 1924) p.9.

board). The cornice at the point where the drum meets the dome, being of wood, may well be a replacement for a cornice of another size.

Although some may wish to treat the following table confirming this theory of plan-elevation with caution, J. Wilkinson¹ is fairly convinced that the plan seems to fit the section quite well.

	K Lower face architrave middle octagon	L Middle of lowest cornice	M Lower edge of middle cornice	N Top edge of top cornice	Meeting of outer dome and abutment ring
A. True measurements: metres	6.01	10.17	14.57	20.45	35.36
B. Geometrical values when factor in 2nd column = 10.23 factors	6.0 $\frac{2x}{\sqrt{4+2\sqrt{2}}}$	10.23 $\frac{x(2+\sqrt{2})}{\sqrt{4+2\sqrt{2}}}$	14.47 $\frac{x(2+2\sqrt{2})}{\sqrt{4+2\sqrt{2}}}$	20.47 $x\sqrt{4+2\sqrt{2}}$	34.94* $\frac{x(6+4\sqrt{2})}{\sqrt{4+2\sqrt{2}}}$
C. Working figures: cubits of 46.24 cm. metric equivalent	13 6.01	22 10.17	31.5 14.57	44 20.35	77 35.60

Table 3.6.1

Dome of the Rock: section

Other examples in which it is clear that the plan and the elevation were considered simultaneously by Muslim builders are also mentioned.

In Chapter V, part 4 of this thesis, we refer to Bulatov's demonstration

1. "Architectural procedures in Byzantine Palestine", *Levant*, XIII (1981) p.168-171.

that in the Mausoleum of 'A'isha Bibi the elevation, with its main elements such as doorways, the height and position of porches, the height of the main enclosure, and the height of elements such as the dome, all appear to be controlled by the geometric proportion of the plan.

K.S. Kriukov¹ has analysed a number of surviving Central Asian buildings from the ninth century onwards in terms of geometric schemes and has found that their design conforms exactly to the actual buildings. He shows (see fig. 3.6.2) how the composition of the elevation is derived from the proportional setting-out of the plan, which is based on arcs drawn from the diagonals of squares to give ratios of $1:\sqrt{2}$.

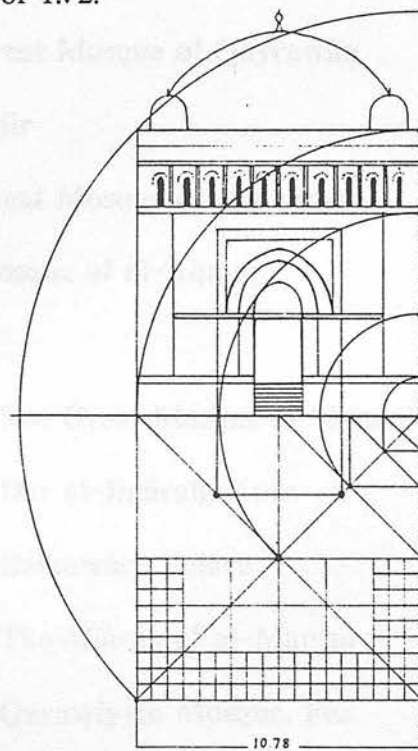


Fig. 3.6.2

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1. "The module in monuments of Central Asian architecture", *Architectural heritage*, XVII (1964), cited by G. Michell, *Architecture of the Islamic world*, p.132..

CHAPTER FOUR

GEOMETRIC ANALYSIS OF INDIVIDUAL BUILDINGS

4.1 The Dome of the Rock

4.2 Qubbat al-Bārūdiyyīn

4.3 Ajdābiyyah Palace

4.4 Ajdābiyyah Mosque

4.5 The Great Mosque of Córdoba

4.6 The Great Mosque of Qayrawān

4.7 Ukhayḍir

4.8 The Great Mosque of Damascus

4.9 The Mosque of al-Aqmar

4.10

4.10.1 The Great Mosque of Sāmarrā'

4.10.2 Dār al-Imārah, Kufa

4.10.3 Balkuwāra Palace

4.10.4 The *Mihrāb* of al-Manṣūr

4.10.5 Qarawīyyīn Mosque, Fez

4.1 THE DOME OF THE ROCK

In view of the numerous articles and detailed studies that have already appeared on the Dome of the Rock, one may perhaps wonder if the octagonal monument built by 'Abd al-Malik from c. AD 688 has already been more than adequately researched. However, the area in which one would be able to contribute to the existing body of work without too much repetition is in the geometric analysis of the building using the principle of the squared circle adopted throughout this thesis as a proportional gauge. This, as we shall see, will allow us to reproduce the exact layout of the Dome of the Rock with all its subdivisions in what is perhaps a much more efficient and simpler method than has been previously proposed.

Before discussing this in detail, we need to examine briefly Creswell's work, since he wrote extensively on the architectural origins of the Dome of the Rock, and to study the further details supplied in the superb analysis of Ecochard. It will be useful also to refer to the oldest and most original analysis of the Dome of the Rock as it was drawn by Choisy.

The Setting of the Plan: Mauss's Theory

As described by Creswell, Mauss's theory states that the plan of Qubbat al-Šakhra was based on the principle of two crossed squares inscribed in the exterior circle of the central rotunda (see fig. 4.1.1).

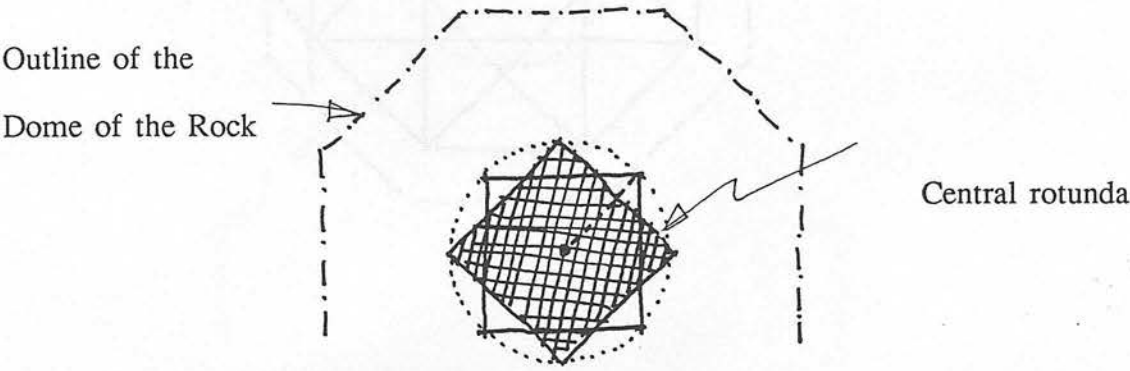


Fig. 4.1.1

The sides of these squares, when prolonged, determine, by their intersections, the

regular octagon which divides the two ambulatories (see fig. 4.1.2)

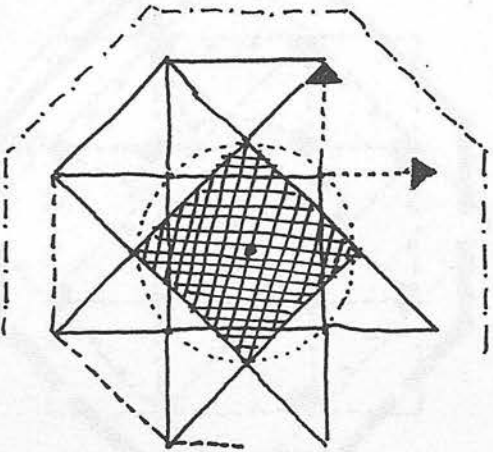


Fig. 4.1.2

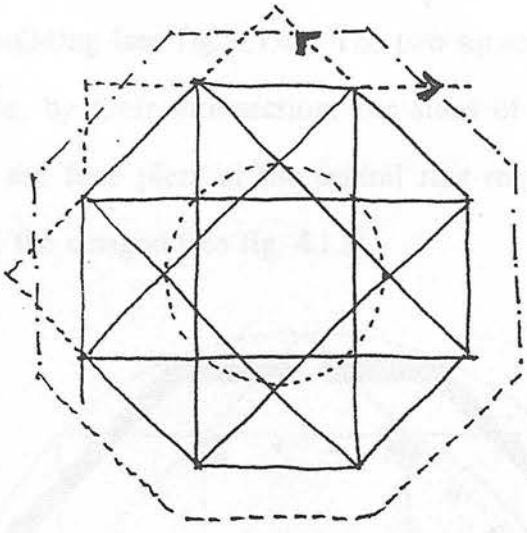


Fig. 4.1.3

The sides of this octagon, when prolonged, form two other squares, which may be circumscribed by another circle (see fig. 4.1.3).

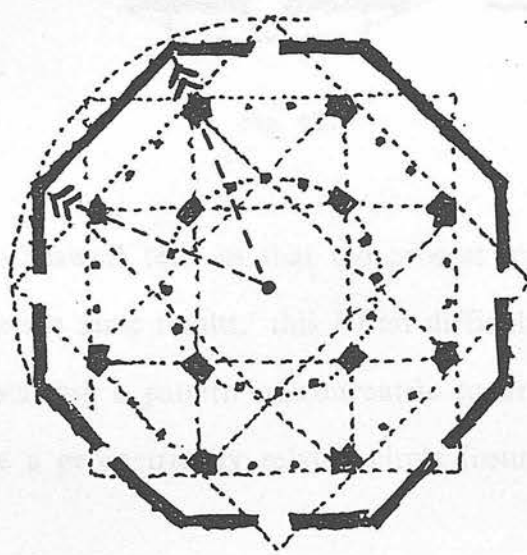


Fig. 4.1.4

An octagon inscribed in this circle, with sides parallel to the first, determines the exterior of the building (see fig. 4.1.4). The two squares, set crossways within the main circle, give, by their intersection, the sides of the intermediate octagon. The position of the four piers of the central ring may be established by joining up the corners of the octagon (see fig. 4.1.5).

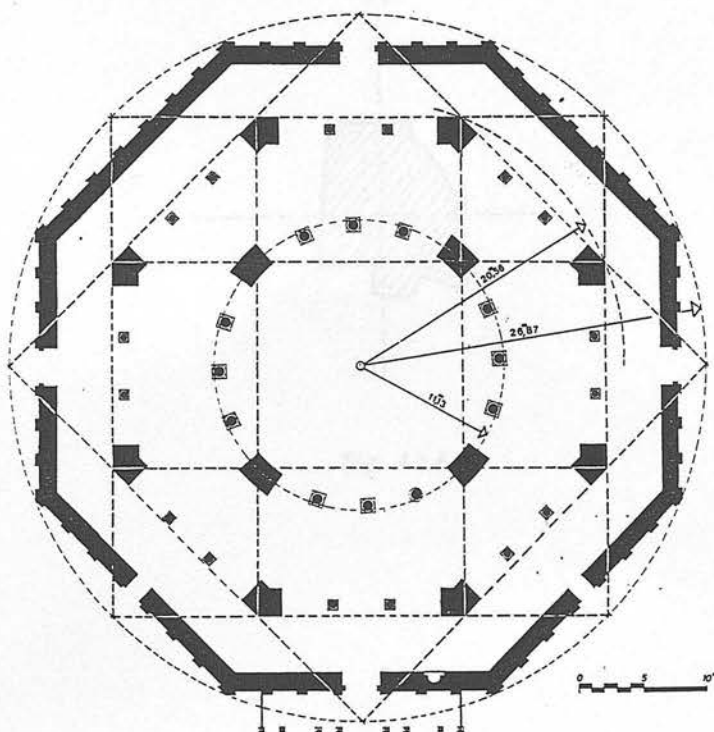


Fig. 4.1.5

Although Creswell tells us that the process in Mauss's theory can be reversed to produce the same results,¹ this I find difficult to accept, especially as we know that by starting a pattern of concentric squares inwardly it would be difficult to produce a geometrically related circle (rotunda) perfectly encircling

1. *Early Muslim architecture*, vol. I, p. 73.

al-Şakhra (the Rock). Therefore, one may disagree somewhat with Mauss's initial setting out and suggest that a circle was drawn, the centre of which was a central point on the Rock and the radius of which circumscribed the Rock. However, to what degree the circle would in fact circumscribe the Rock would be left to the judgment of the architect/geometer (see figs. 4.1.6 & 4.1.7).

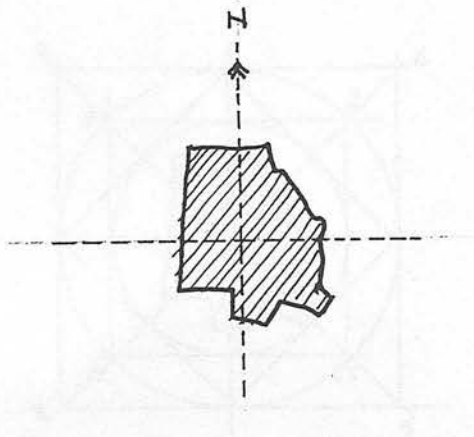
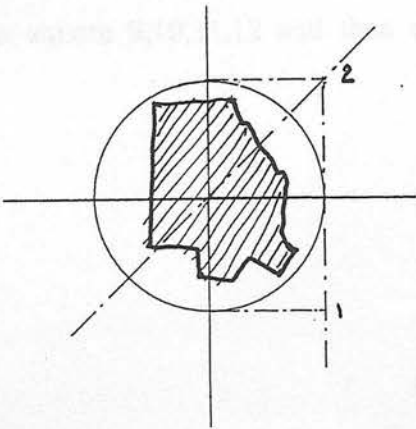


Fig. 4.1.6

Stage A



The Rock

Fig. 4.1.7

Having established the cardinal points, the next stage would have been to square circle 'D' by the square 1,2,3,4 (see fig. 4.1.8).

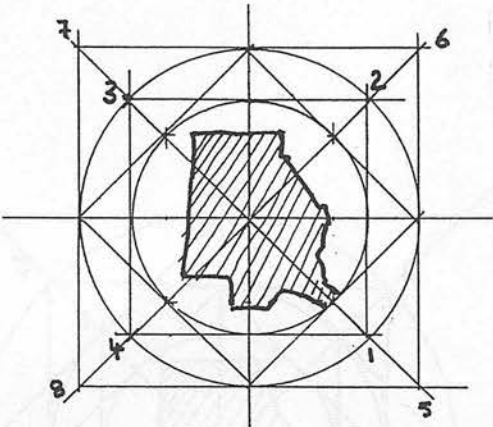


Fig. 4.1.8

We then circle square 1,2,3,4 and construct the outer square 5,6,7,8. This process is repeated to construct square 9,10,11,12 and then the diagonal square e,f,g,h is drawn (see fig. 4.1.9).

We then circle square 1,2,3,4 and construct the outer square 5,6,7,8. This process is repeated to construct square 9,10,11,12 and then the diagonal square e,f,g,h is drawn (see fig. 4.1.9).

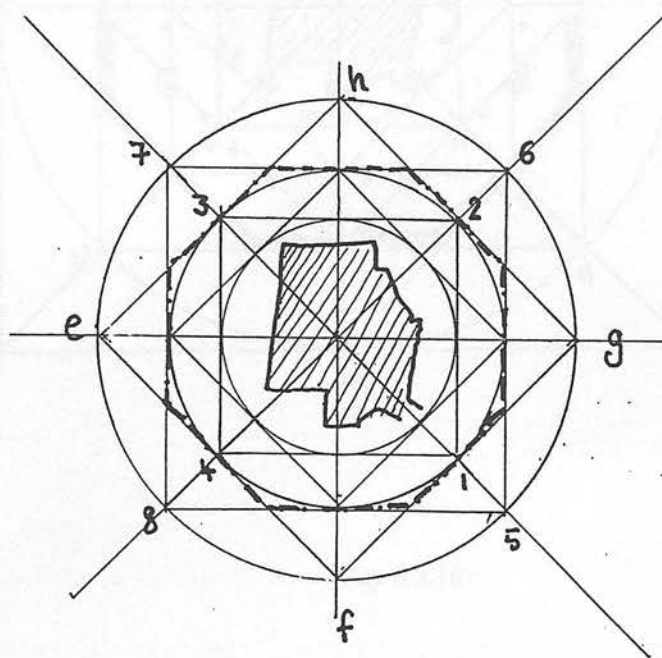


Fig. 4.1.9

We arrive at the basis for establishing the position of the internal octagonal arcade by circling square 9,10,11,12, and squaring it by square 13,14,15,16 and drawing its equivalent diagonal square i,j,k,l (see fig. 4.1.10).

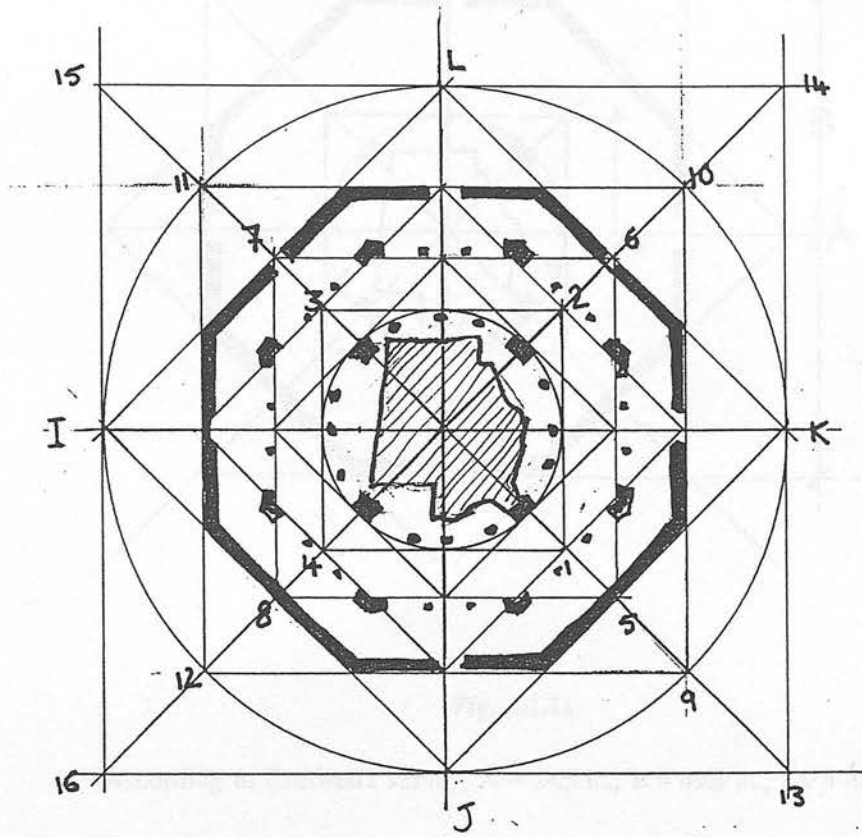


Fig. 4.1.10

The resultant octagon is the outline of the external wall of the Dome of the Rock.

My proposal can be tested mathematically by using the formula which states that in this geometric pattern of concentric squares, the consecutive sides relate in a $1 : \sqrt{2}$ proportion, whereas the alternate sides relate in a $1 : 2$ proportion (see fig. 4.1.11).

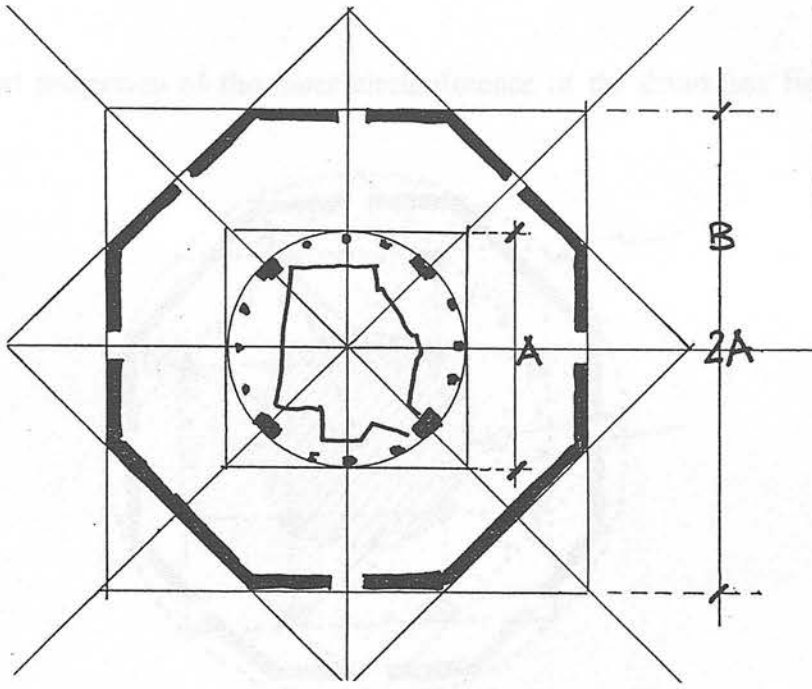


Fig. 4.1.11

(According to Creswell's survey, $A = 24.5$ m., $B = 49.6$ m., $2A = 49$ m.¹)

Alternative Analysis of the Dome of the Rock

Basing his analysis on Mauss's theory cited in Creswell's *Early Muslim Architecture* (p. 73 f.), Creswell's own calculations, and Richmond's description of the geometrical relationship between the elements of the structure, Doron Chen reveals the mathematical and harmonic system which, according to his analysis, is the key element in the design of the Dome of the Rock. Chen identifies the harmonic system underlying the plan of the Dome of the Rock as a ratio between two circles, one circumscribing the exterior of the building and the other being

1. *Early Muslim architecture*, vol. I, p. 71.

the horizontal projection of the inner circumference of the drum (see fig. 4.1.12).

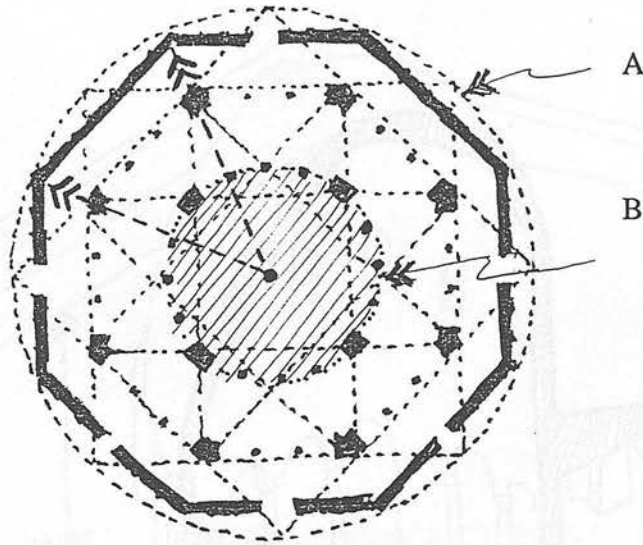


Fig. 4.1.12

The diameter of the outer circle, calculated according to the measurements taken by Creswell, is 53.75 m. (its radius R is 26.875 m.). The average length of the inner diameter of the drum, also calculated according to the measurements taken by Creswell, is:

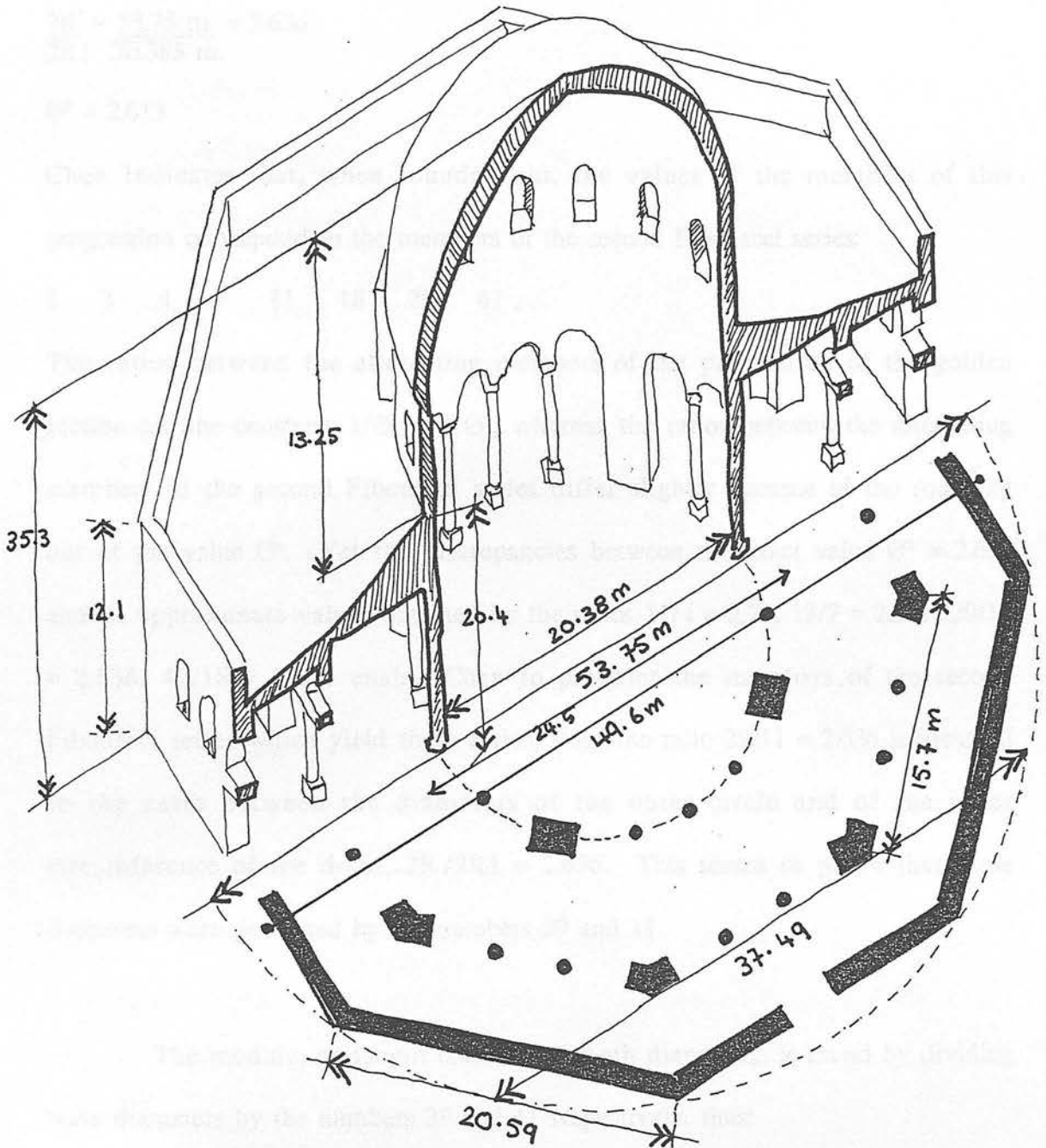
$$2R_1 = \frac{20.33 \text{ m.} + 20.44 \text{ m.}}{2} = 20.385 \text{ m.}$$

The ratio between these diameters is:

$$\frac{2R}{2R_1} = \frac{53.75}{20.385} = 2.636$$

Chen finds that the value 2.636 closely approximates to the square of the golden number, i.e. $\phi = 2.613$, taken from the progression of the golden sections:

$$\emptyset = 1.618, \emptyset^2 = 2.618, \emptyset^3 = 4.236, \emptyset^4 = 6.853 \dots \emptyset^n.$$



$$2R_1 = \frac{20.33\text{ m} + 20.44}{2} = 20.385$$

$$\frac{2R''}{2R'} = \frac{53.75\text{ m}}{20.385\text{ m}} = 2.636$$

$$\emptyset^2 = 2.613$$

Fig. 4.1.13

$$2R1 = \frac{20.33 \text{ m.} + 20.44 \text{ m.}}{2} = 20.385 \text{ m.}$$

$$\frac{2R''}{2R1} = \frac{53.75 \text{ m.}}{20.385 \text{ m.}} = 2.636$$

$$\phi^2 = 2.613$$

Chen indicates that, when rounded out, the values of the members of this progression correspond to the members of the second Fibonacci series:

1 3 4 7 11 18 29 47 ...

The ratios between the alternating members of the progression of the golden section are the constants $1/\phi^2$ and ϕ^2 , whereas the ratios between the alternating members of the second Fibonacci series differ slightly because of the rounding out of the value ϕ^n . Yet the discrepancies between the exact value $\phi^2 = 2.618$ and its approximate values obtained by the ratios $11/4 = 2.75$, $18/7 = 2.571$, $29/11 = 2.636$, $47/18 = 2.611$ enable Chen to pinpoint the members of the second Fibonacci series which yield these ratios, since the ratio $29/11 = 2.636$ is identical to the ratio between the diameters of the outer circle and of the inner circumference of the drum, $2R''/2R1 = 2.636$. This seems to prove that these diameters were generated by the numbers 29 and 11.

The module, or length common to both diameters, is found by dividing these diameters by the numbers 29 and 11 respectively, thus:

$$\frac{2R''}{29} = \frac{53.75 \text{ m.}}{29} = 1.853 \text{ m. and } \frac{2R1}{11} = \frac{20.385}{11} = 1.853 \text{ m.}$$

The cardinal dimensions of the plan of the Dome of the Rock on which Chen

bases the ancient foot of 0.3089 m.¹ (a unit of measurement which was found carved beneath a Byzantine inscription discovered near Bethlehem which has been dated to about the sixth century AD²) are:

1. Inner diameter of drum	66 ft.	20.38 m.
2. Diameter of outer circle	174 ft.	53.75 m.
3. Base of the square inscribed in outer circle	123 ft.	$\frac{53}{\sqrt{2}} = 37.49$ m.
4. Side of outer octagon	66.5 ft.	20.59 m.
5. Inner width of doorways	9 ft.	2.78 m.
6. Height of outer doorways	14 ft.	4.35 m.
7. Side of intermediate octagon	51 ft.	15.7 m.
8. Diameter of centre line of piers and columns which carry the drum	72 ft.	22.3 m.

Chen believes that, since in antiquity one of the major engineering problems was the construction of the dome, the inner diameter of the dome must have been the *initial parameter* of the diagram of the Dome of the Rock. This is interesting because we too arrive at this conclusion, except that we suggest it was because of other reasons, principally the marking of the first circle on the ground. This was determined by the most practical way of including the actual rock within a perfect circle (see Analysis).

1. *Palestine Exploration Quarterly* (1980) p. 44.

2. F.M. Abel, "Inscription grecque de l'Aqueduc de Jérusalem avec figure du pied byzantin", *Revue Biblique*, XXXV (1926) 284-295.

Chen goes on to explain that the harmonic diagram of the plan was most probably generated by means of the golden rectangle $1 + \sqrt{5}/2 : 1$ (see figs. 4.1.14 & 4.1.15).

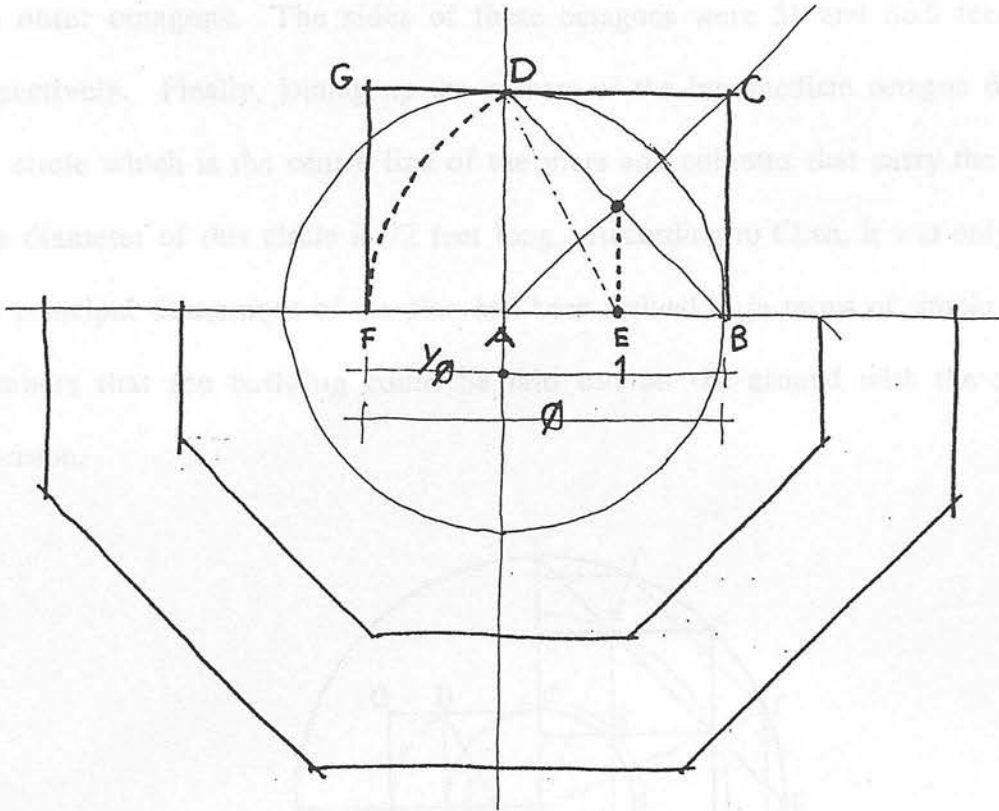


Fig. 4.1.14

The inner radius of the drum AB served as the base of the square A,B,C,D, while the length ED (which is the diagonal of the half of the square), projected on the side, generated the length FB. This length served as the base of the golden rectangle F,B,C,G. Again, the length FB served as the base of the new square F,B,H,I, which in turn generated the radius KB of the circle which circumscribes

the exterior of the building. Once the harmonic system was set, the inner diameter of the drum (20.1 m. or 11 modules) and the diameter of the outer circle (53.75 m. or 29 modules) were defined. The outer circle generated the square with the base of 37.5 m., which in turn engendered the intermediate and the outer octagons. The sides of these octagons were 51 and 66.5 feet long respectively. Finally, joining up the corners of the intermediate octagon defined the circle which is the centre line of the piers and columns that carry the drum. The diameter of this circle is 72 feet long. According to Chen, it was only after the principal dimensions of the plan had been arrived at in terms of simple whole numbers that the building could be laid out on the ground with the utmost precision.

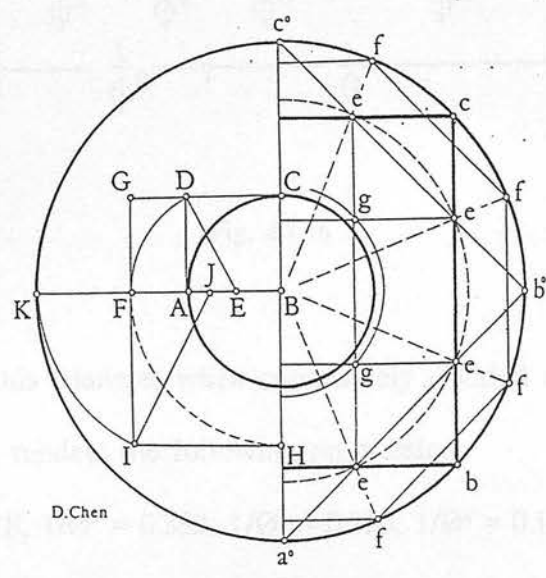


Fig. 4.1.15

Basing his analysis of the façade on Lecomte's detailed drawing, Chen employed the \emptyset triangle¹ in order to study the elevational elements of the outer octagon. The sides of the triangle (see fig. 4.1.16) are related to each other in the ratio 1 : 1/2.

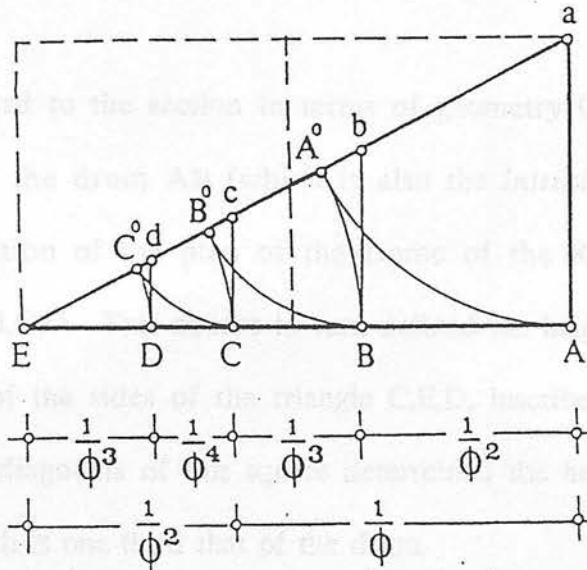


Fig. 4.1.16

The longer side of this triangle, when successively divided according to the rule of the golden section, renders the following progression:

$$1/\emptyset = 0.618, 1/\emptyset^2 = 0.382, 1/\emptyset^3 = 0.236, 1/\emptyset^4 = 0.146... 1/\emptyset^n.$$

By substituting the height of the octagon (12.04 m.) for the longer side of the \emptyset triangle, Chen managed to obtain the following results:

1. *Op. cit.*, p. 47.

Height of window sills $12.04/\varnothing^2 = 4.63$ m. – 4.65 m. actual measurement.¹

Height of arch spring point $12.04 \times (1/\varnothing^2 + 1/\varnothing^3) = 7.414$ m. – 7.45 actual measurement.

Height of roof gutter $12.04 \times (1/\varnothing^2 + 1/\varnothing^3 + 1/\varnothing^4) = 9.267$ m. – 9.3 m. actual measurement.

With regard to the section in terms of geometry (see fig. 4.1.17), the inner diameter of the drum AB (which is also the *initial parameter* of the harmonic composition of the plan of the Dome of the Rock) generated the vertical square A,B,C,D. This square in turn defined the height of the drum AC. The intersections of the sides of the triangle C,E,D, inscribed within the square A,B,C,D, and the diagonals of this square determined the height of the piers of the inner ring, which is one third that of the drum.

In this way, Chen reveals the unity of the horizontal and vertical projections of the building and, by the use of whole numbers, the commensurability of its principal dimensions.

Chen insists that for the direct analogy to the plan of the Rock we must turn to the Chapter of the Holy Sepulchre. The Byzantine Architect with the diameter of the gallery generated by Hagia's perfect number 48 and the circumference of the interior defined by the ratio of the golden section did indeed, according to Chen, serve as the model for the Dome of the Rock, both in

1. Creswell's measured survey.

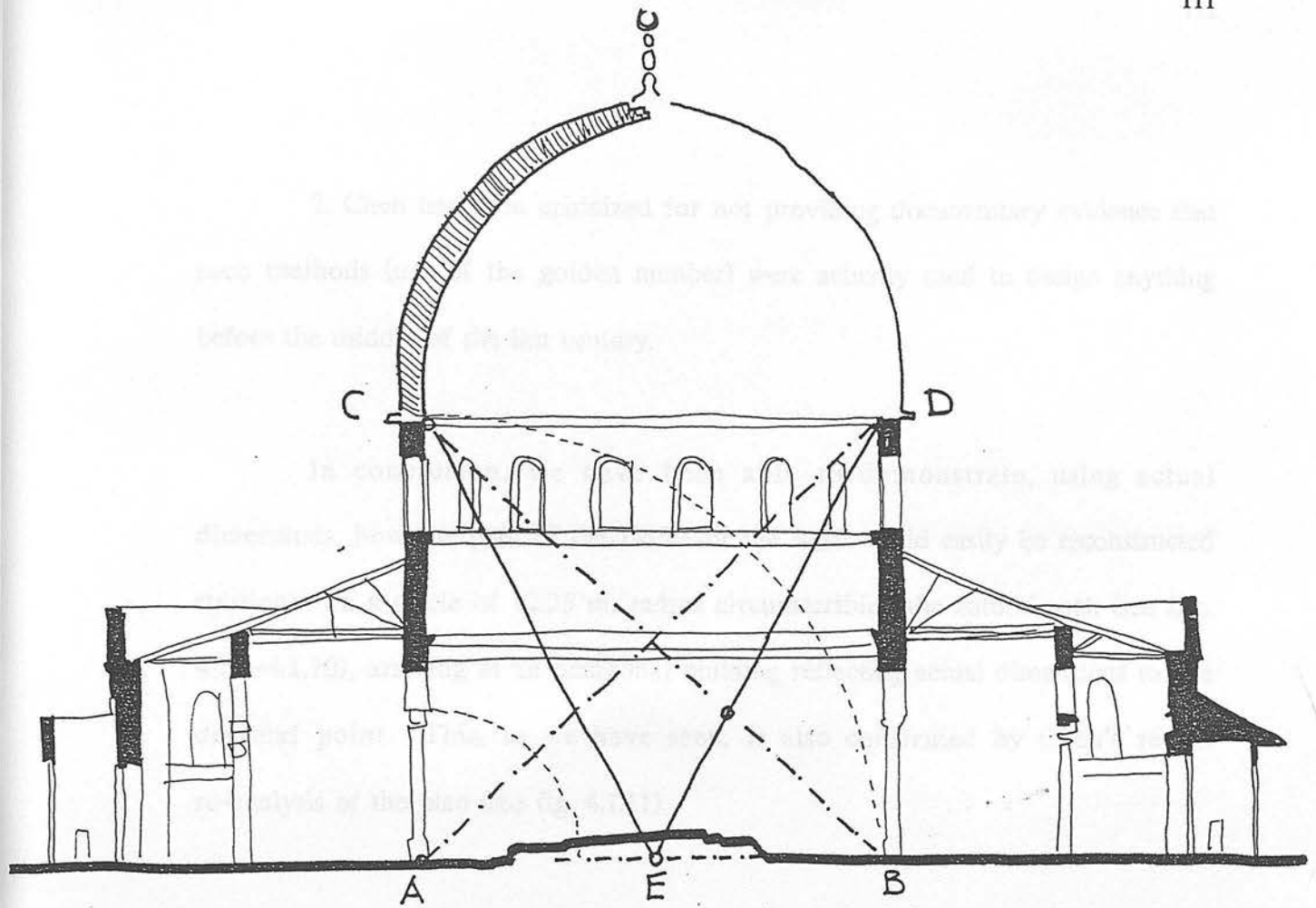


Fig. 4.1.17

Conclusions

1. Chen insists that for the closest analogy to the Dome of the Rock we must turn to the Church of the Holy Sepulchre. The Rotunda Anastasis with the diameter of its gallery generated by Euclid's perfect number six and the circumference of its interior defined by the ratio of the golden section did indeed, according to Chen, serve as the model for the Dome of the Rock, both in grandeur and in the planning technique employed.

2. Chen has been criticized for not providing documentary evidence that such methods (use of the golden number) were actually used to design anything before the middle of the last century.

In conclusion, we have been able to demonstrate, using actual dimensions, how the plan of the Dome of the Rock could easily be reconstructed starting with a circle of 12.25 m. radius circumscribing the natural rock (see figs. 4.1.6–4.1.10), arriving at an octagonal building reflecting actual dimensions to one decimal point. This, as we have seen, is also confirmed by Chen's recent re-analysis of the plan (see fig. 4.1.11).

Although we may have some reservations with regard to Mauss's method of arriving at the outer octagon (see fig. 4.1.4), we fully support his conclusion that the plan of the Dome of the Rock could not have been achieved by first establishing the outmost circle¹ relating to the setting out of the main walls. Rather, it was accomplished by starting from the inmost circle, which determined the centre line of the piers and columns which carry the drum.

1. See Creswell, *Early Muslim architecture*, i, 73.

4.2 QUBBAT AL-BĀRŪDIYYĪN

Qubbat al-Bārūdiyyīn in Marrakesh is one of the clearest examples of applied geometry in early Muslim architecture, showing how the two-dimensional pattern metamorphoses into three-dimensional art. Qubbat al-Bārūdiyyīn shelters a fountain for ablutions in an annex to the Friday mosque of 'Alī ibn Yūsuf constructed in c. A.D. 1106, and now entirely rebuilt. The structure is a rectangular pavilion with a two-tiered elevation: the lower storey houses the entrance arches, while the upper storey contains five windows on one side and three on the other. A central brick dome covers the structure, decorated in relief with interlaced arches and a seven-pointed star in a herring-bone pattern.

It is the inside of the dome, however, that interests us most particularly in relation to the geometry of the subdivisions of the internal dome. Independent of the exterior is a quite complex decorated vault. Eight interlacing arches support concave 'scoops' within the domes. In the eight upper arches and in the four seven-pointed star-shaped vaults in the corners of the octagon appears the *muqarnaṣ*, yet with a rather flat appearance and not as sophisticated as the lower designs.

Geometric Analysis

Drawing QUB/A5 gives us an overall view of the result of the analysis. In order to see how this geometric representation is arrived at, we will take the analysis stage by stage.

In drawing QUB/A1 we draw a circle from centre O with radius O-10 which is the limit of the eight interlacing arches supporting the central dome. We then square the circle 1,2,3,4 and draw the diagonals 1-3 and 2-4. Points 5, 6, 7, and 8 mark the intersection of circle O with the diagonals. By drawing the first alternate concentric square 9,10,11,12, an 'octagram'¹ is produced. The intersections resulting from superimposing square 9,10,11,12 upon square 5,6,7,8 (as shown in drawing QUB/A2) give points 13, 14, 15, 16, 17, 18, 19, and 20.

We connect the central point O with each of the above intersection points. By extending these connections to meet the sides of the generating square 1,2,3,4, we obtain points 21, 22, 23, 24, 25, 26, 27, and 28. It is these points on which we relate the inner square 1,2,3,4 (see drawing QUB/A2) to produce the eight-pointed star which is the basis for the eight interlacing arches supporting the dome (shown in green in drawing QUB/A3 and in QUB/A4 as 21, 22, 23, 24, 25, 26, 27, and 28).

1. One square superimposed upon another whose orientation differs by 45°, also known as *ad quadratum*.

By circling the generating square 1,2,3,4, using diagonal O-3 as the radius, the resulting circle when squared is 29,30,31,32. Sides 31-32 and 29-30 appear to correspond to the pavilion's external north and south facing walls.

Again, by circling the resulting square 29,30,31,32 using its diagonal O-31 as radius, the resulting circle when squared is 33,34,35,36. Here sides 33-36 and 35-34 appear to line up with the pavilion's external east and west walls (see drawing QUB/A3).

Having found the proportional criteria for the main shape of the pavilion, we find that the structural support also appears to adhere to the same generating geometry. In drawing QUB/A4 we see how the main beams supporting the dome are in line with the extensions of points 21, 22, 23, 24, 25, 26, 27, and 28 which meet the rectangle representing the overall shape of the pavilion at 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, and 52.

This analysis can be taken one stage further by circling the resulting square 33,34,35,36 using diagonal O-35 as the radius. The resulting circle when squared at 53,54,55,56 and a hexagon drawn within it, side a-b can represent the width of the pavilion.

Arithmetical check

As indicated in drawing QUB/A4-A5 (see vol.II: drawings), side 2-3 of the square 1,2,3,4 = 3.72 m.¹ Side 6-7 of square 5,6,7,8 = 2.61 m.² According to our proportional principle, alternate sides in the squared circle diagram relate in a 1:2 system of proportion. Consecutive sides, however, relate in a 1:√2 proportion.

$$3.72/2.61 = 1.425$$

$$\sqrt{2} = 1.414$$

Thus, the proportional system appears to apply with a margin of error of one decimal point.

1. J.D. Hoag, *Islamic architecture* (New York, 1977) p.94-98.

2. *Ibid.*

4.3 AJDĀBIYYAH PALACE

Ajdābiyyah lies near the eastern end of the Gulf of Sirte, 150 km. south of Benghazi and 18 km. from the coast. The region consists of low sand-gravel hills, with salt flats towards the sea. The site, however, does possess one essential for settled existence: water. Ajdābiyyah has a number of wells, several of which are sweet, and for this reason Ajdābiyyah became the meeting-point of two important routes: the Sirtic route between the populated areas of Cyrenaica and Tripolitania (itself part of the longer coast road linking Egypt and the Maghrib), and the desert route which led south through the oases of Jalu and Kufrah towards Chad and the Sudan.

The earliest recorded occupation of Ajdābiyyah is in the Roman period,¹ and in the Islamic period its main occupation took place under the Fāṭimids from shortly before c. 964 until 1051-2 when it was sacked by the Banū Hilāl and Banū Salīm.

Writing in the mid-eleventh century, al-Bakrī describes Ajdābiyyah in the following words:

1. Blake, "Ajdābiyyah", p.107.

A great city situated in a desert of hard stone and possessing several rock-cut well[s] yielding good water. There is also a sweet water spring. This city contains a mosque¹ [with an] octagonal minaret²...baths, caravanserais...much-frequented bazaars...a sea-port...and a total of three castles.³

Al-Idrīsī, writing before 1154, adds that Ajdābiyyah was surrounded by a wall. In his day only two forts remained.⁴ Blake, Hutt, and Whitehouse⁵ note that several of the features mentioned by al-Bakrī are visible today: part of the mediaeval city survives as an area of low mounds to the south-east of the modern town; within this area stand the remains of a large mosque, plausibly identified as the mosque of Abu'l-Qāsim; 1050 m. from the mosque, and undoubtedly outside the walled city, stands a fortress-palace, perhaps one of the three castles of al-Bakrī and almost certainly one of the two forts described by al-Idrīsī.⁶

The Fortress Palace

According to Blake, Hutt, and Whitehouse, the principal monument of mediaeval Ajdābiyyah is the fortress palace, a rectangular building with the major axis aligned approximately northeast to southwest. It was entered from the

1. For the mosque, see plan and vol.II (Geometric Analysis) of the present work.

2. The minaret is not apparent in the reconstructed plan.

3. Blake, *loc. cit.*

4. *Ibid.*

5. David Whitehouse, "Interim Report", *Libya Antiqua* (1980) p. 13.

6. Blake, *loc. cit.*

northeast, and at the southwest end stood a suite of impressive apartments. According to Whitehouse,¹ Pacho recognised the building as a "Château sarrasin",² and Hamilton placed it "not later than the ninth century of the Hejira".³ Although Ferri repeated the Islamic origin,⁴ and notwithstanding an Arabic inscription built into one of the imposts of room 9 the palace could be described frequently as a Byzantine structure. The palace was built of large ashlar blocks bonded with lime mortar. The outer walls, which are more than 1 m. thick, rest on solid foundations and are faced with ashlar masonry outside a rubble core. The walls of the main apartments, at least, were faced with white plaster, and, as in the mosque, the floors were paved with limestone slabs.

The plan of the palace, here reproduced after Abdussaid,⁵ reveals a rectangular building with external dimensions (salient excepted) of 33.5 by 25.5 m. The angles were reinforced with circular towers, and halfway along each side stood a rectangular salient. In the centre was a square courtyard, 14 m. across, with ranges of rooms on all four sides. At the northeast end the salient forms a monumental entrance giving access to a cross-hall 8.2 m. long. At the opposite end of the palace stood the suite of three vaulted rooms approached through a second cross-hall entered from the courtyard. The longer sides of the building

1. *Op. cit.*, p.15.

2. Abdussaid, "Early Islamic Monuments at Ajdabiyah", *Libya Antiqua*, vol. I (1964) p. 115-119.

3. *Ibid.*

4. *Ibid.*

5. *Ibid.*

contained ranges of rooms, presumably used for accommodation.

The entrance (room 1) consisted of a passage 2.7 m. long, on either side of which was a recess with three semi-circular niches, the whole structure contained within a salient 5.3 m. wide and 4.2 m. deep. The cross-hall, which served as a vestibule (room 2) occupied the centre of the north-east side of the palace. In its original form it had an opening at each end, while another opening, 2 m. wide, *may* have given access directly to the courtyard.

The ranges of rooms on the long sides of the palace require little comment. The salients on the two sides presumably were defensible bastions, and part of a staircase leading to an upper storey survives in the north-west range. No comparable feature survives in the south-east range.

The main apartments comprise four rooms (7-10) of which only the principal chamber (9) retains any distinctive architectural features...The suite was self-contained. It was entered through room 7, an ante-room 14 m. long with three openings from the courtyard. Rooms 8 and 10 each measured 6.4 m. long and in room 10 the base of a semicircular niche survived. Room 9, in the centre, included the southwest salient and was 9.6 m. long. Pachó's engraving shows that rooms 8-10 had barrel-vaults, although today the only indication of vaults are the fragments in room 9 and the seat of the springing on the southeast side of room 10. In room 9 the salient was covered by a semi-dome supported on squinches

and closing the end of the vault.¹

In the conclusions to their study, Blake, Hutt, and Whitehouse note that the palaces of this period seem to have a number of features in common. Firstly they are rectangular with circular towers projecting at the corners and a number of other salients either consisting of entire rooms or parts of rooms. The monumental portal is also salient continuing an earlier tradition and is provided with niches on the interior which presumably formed seats for the guards. All the palace entrances examined have had a bent entrance of some sort. The main reception room facing the entrance is supplied with an ante-room transversal to the main axis of the building, which ante-room would appear to have been barrel-vaulted, and it is possible that there may have been a dome at the T junction so formed, analogous to those in North African mosques. The audience or 'throne room' section of this reception area is also salient. Many of these features can be paralleled either in earlier Islamic constructions, particularly the desert palaces of Syria and Jordan, or in buildings of the Roman-Byzantine-Hellenistic tradition which was still current in North Africa.² A.M. Lézine, however, discussed the apparently Iraqi element in the plan: namely the palatial suite, corresponding to Hira and Khirbet al-Mafjar.³ Many of the forms would appear to have a Mesopotamian origin, either transmitted via Egypt or brought directly, although the Syrian influence coming through Umayyad Spain

1. "Ajdābīyah", p.109f.

2. D. Whitehouse, *op. cit.*, p.19 f.

3. *Recherches d'archéologie islamique* (Paris, 1965) p.30.

and the western Maghrib is also very important.¹

Geometric Analysis

When examining the palace's plan, two of the most striking features of the building, besides the overall rectangular shape,² are the four circular corner towers and the central square courtyard.

On close examination we discover that the centres of each of these circular towers line up with the outer face of the external walls. This must be the first indication of a possible hidden geometric principle underlying the structure (AJD/A1).

An examination of the square-shaped central courtyard is our next step towards deciphering the plan. First one must try to establish its centre. This might be achieved by drawing the diagonals, except for the fact, evident on the plan, that the surveyed internal corners of the courtyard have since crumbled away, making it difficult to determine an accurate centre point. Therefore one must try to establish a more rigid framework on the basis of which right angles can be fixed, which in turn can serve as a basis for the springing of the diagonal lines.

1. "Ajdābīyah", p.118f.

2. The shape is in fact nearer to a square than to a rectangle. Its length in relation to its width is less than 1 1/4.

Going back to our previous observation, by connecting the centre points of the circular towers, using a set square,¹ we establish rectangle ABMG (AJD/A1). V is the centre point² between A and B. With centre V and radius VB, we draw a circle, crossing the vertical axis at X (AJD/A2) and U. With centre X and radius XV, we draw a second, interlocking circle cutting the vertical axis at Y and V. If we now draw a 45° line from B towards X, cutting AG at D, and similarly from A, cutting BM at C, then connect CD, ABCD now circumscribes circle no.2 and is therefore a square.

ABCD is a unit square with diagonals BD and AC. If sheet AJD/A2.1 is drawn on transparent material and we lay this over sheet AJD/AO, maintaining A, B, M, and G as the centres of the four circular corner towers, we may observe the following:

1. X is the centre of the internal courtyard;
2. DC lines up with a major dividing internal wall proceeding to the throne room; and
3. the courtyard appears square with surrounding rooms of equal widths.

The next step is to carry on with the 1/2 interlocking circle concept and draw from Y circle no.3 with radius YX (P/AJD/A3), cutting the vertical axis at

1. The set square is set to a vertical line taken through the centre of the plan from V.
 2. The method of establishing the centre point of a given straight line has been discussed in earlier sections. See chapter II on systems of proportion.

X and Z. In order to obtain a master circle for detailed analysis, we return to the middle circle with centre X and draw a larger circle with radius XZ, cutting the vertical axis at U and Z and the at K and L. We then square the master circle and draw the main diagonals.

The next step is to draw two overlapping hexagons, the first with corners at ZNPUIH, the second at MKQJLG.

Analytical work from AJD/A4 to AJD/A6 and A7 not included in the thesis) did not achieve the final objective, which was to attain the geometrically line MG, that is to say finding the geometry which determines the connecting of the centres of the top circular towers, although, as in AJD/A4, M-G is the top side of one of the hexagons drawn within the major circle which appears to pass close to the external projection of the throne room at Z. Upon reaching this stage and not being able to continue any further, the solution was to undertake a complete re-examination of approach in analysing the structure of Ajdābiyyah Palace. In this enterprise, the proportional method used by Gothic builders (as analysed by Professor Eric Fernie), which basically hinges on turning the unit square once at a 45° angle, was of great help. AJD/B1 is based on the original second unit circle with centre X and radius XV and with master square 1,2,3,4. We now square unit circle no.2 as in AJD/B2 and rotate the unit square no.2 by 45° , resulting in an overlapping unit square .5,6,7,8 (see drawing AJD/B3). If this square is pulled upwards to meet square 1,2,3,4, we obtain a new 45° square

.13,.14,.15,.16. We note that corners .14 and .6 are on the same line and, most importantly, we also note that two sides of the square .13,.14,.15,.16 are tangent to the sub-master circle with centre at X. The withdrawn square .13,.14,.15,.16 can also be rotated 45°, giving us square .9,.10,.11,.12. Thus, .11,.12 should now explain our dilemma relating to M-G shown on AJD/A2,A3, whereas AJD/B2 explains the degree by which tangent A is raised.

If we now lay the palace's plan on the resultant pattern – which is basically ten pointed stars pulled apart – we begin to appreciate this enquiry into the palace's plan. AJD/B6 explains the next step in understanding the palace's internal geometry, which is basically five equal longitudinal compartments achieved by dividing square .9,.10,.11,.12 into five equal strips.

If we connect the centre points of .9-.10 with .11, the centre point of .10-.11 with .12, the centre point of .11-.12 with .9, and the centre point of .12-.9 with .10, the intersecting points divide the plan into five equal divisions. AJD/B8 is the reconstructed final plan based on the geometric grid of two ten-pointed stars pulled apart.

If we compare the reconstructed plan based on geometric principles (P/AJD/AO) with the site measurements published by Blake, Hutt, and

Whitehouse, i.e. external dimensions of 33.5 x 25.5 m.,¹ we find a discrepancy of just 0.5 m. in that the geometric reconstruction shows a longitudinal measurement of 33 m. with a width between two tower centres of 25.5 m. This discrepancy is not enough to disprove our theory, since if we divided 25 m. by 0.5, the possible error is only 2 m. in every 100 m. and the courtyard is out by 0.3 m., i.e. 14.3 m. as against the site measurement of 14 m. It seems quite probable that these figures could be improved upon by more accurate site surveys and further detailed analysis.

1. "Ajḍabīyah", p.110.

4.4 AJDĀBIYYAH MOSQUE

The Jāmi‘ Saḥnūn is a ruined Fatimid mosque. By the nineteenth century, only the minaret and parts of the sanctuary were still standing. Jean-Raymond Pacho, who visited Ajdābiyyah in 1824 sketched the remains and subsequently published an engraving of the sketch. Although the engraver misunderstood the perspective of the site, Pacho’s plate gives us useful information about the *miḥrāb* and the sanctuary façade.¹ Even in the nineteenth century, the ruins commanded respect, mainly because tradition associated them with the religious reformer Sahnun, and long after Pacho’s visit a small mosque was built on the site, incorporating the Fatimid *miḥrāb*. A cemetery surrounded the mosque and this is still the town’s burial ground.

The Libyan Department of Antiquities began to excavate the mosque in 1954, by which time all trace of the nineteenth-century building had vanished. The Department cleared the courtyard and the surrounding arcades. In 1971, at the Department’s invitation, Blake’s team resumed work at the mosque. The old excavations were examined, the demolished remains of the nineteenth-century mosque were excavated, and a series of trenches were cut in the sanctuary. By

1. *Interim Report on Excavation at Ajdabiya*, plan on p.18.

the end of the season the team were able to reconstruct the plan of the original mosque with some confidence. With the help of the geometric analysis of the plan shown above in this study, the team would probably be able to consolidate the plan even further.

The Jāmi' Saḥnūn had maximum dimensions of 47 metres by 31.¹ It was roughly rectangular. Blake believed that, as in the Great Mosque at Susa, the builder had little regard for right angles or parallel lines.² It was built with a combination of stone and mud-brick: stone for the minaret, piers, jambs and other weight-bearing elements, and mud-brick for the walls. On the simplified plan published here, original masonry is shown in solid black and mud-brick is hatched; all secondary features are omitted.

Looking at the mosque's plan, the lay-out is simple. The mosque had a

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1. On examination of the plan, it appears that the 47 x 31 m. dimension has been averaged out due to the apparent misalignment of the walls.
 2. This may not be necessarily true, since for early builders the setting of right angles was an easy task. Misaligned walls in such buildings are usually so for a good reason. In the case of the Fāṭimid mosque at Ajdābiyyah, the *qiblah* wall appears to be the most obvious deviation from the right angle. This is almost a repeat of the Qairawan Great Mosque *qiblah* wall and the Zaytuna mosque's *qiblah* and northern *ṣaḥn* entrance wall. The reasons for such deviations usually lie in attempts to conform to an existing town fabric, although adjusting the *qiblah* wall slightly to achieve correct orientation is far less dramatic than for the full length of the mosque to adopt the required angle. The other possible reason for such a layout is that in some cases it is practical to build on an existing foundation which in itself may not have the correct orientation towards Mecca. It is difficult to accept that a builder who can achieve a design containing proportional relationships, tying the length to the width of a building, which in turn relates to the proportion of the inner courtyard, would have little regard for right angles (see AJD/A1-3).

courtyard surrounded on *all four sides* by a single arcade. There were three entrances to the *sahn*: one on the axis and two in the sides. On the left of the axial entrance stood the minaret. Although we excavated only part of the sanctuary, we found sufficient to indicate the original plan. The sanctuary was four bays deep and the facade showed that it was nine bays wide. The spacing of the piers and columns indicated that it had a T-shaped plan: that is, with a broad axial 'nave' and broad 'transepts' immediately in front of the *qiblah* wall. At the intersection of the stem and the arms of the T was a square compartment, presumably covered by a dome. This arrangement, widely used in the Maghrib, is best illustrated by the local prototype, the Great Mosque at Qairawan, as rebuilt by Ziyadat Allah in 836. As can be seen from the general analysis, both mosques were generated by interlocking two circles..

The minaret has been demolished almost completely – apparently between c. 1934 and 1954 – and today only the square base, which is 3.5 metres across, survives.¹

In the *sahn*, the sanctuary façade consisted of a series of piers, decorated on the outer face with semicircular niches.

Turning to the sanctuary itself, the columns of the nave towards the ruined *mihrāb* had three elements: two circular shafts resting directly on the floor

1. D. Whitehouse, "Interim Report", *Libya antiqua* (1980) p. 15.

and a third column standing on a rectangular base (not original). Thus, in the restored plan of the original mosque groups of two (not three) columns are shown supporting the nave. In front of the *mihrāb*, where the nave and transepts meet, stood two composite piers, each with a quatrefoil plan.

The outer walls were supported by massive buttresses. Blake's team excavated two angle buttresses, one on the *qiblah* wall and one on the façade of the mosque, and a third buttress was visible on the surface. Two of the buttresses were steeply battered. With its heavily-buttressed walls, the building recalls the external appearance of the Great Mosque at Qairawan, which was buttressed at frequent intervals.

Geometric Analysis

In M/AJD/A1 we immediately notice that the central courtyard is not a perfect square. This is, of course, substantiated by the site measurements (20 x 17.2 m.) published by Blake, Hutt, and Whitehouse, i.e. 20 m. by 17.2 m.

The excavation, although incomplete, showed that the Fāṭimid mosque, known today as Jāmi' Saḥnūn, was a major building. Its *ṣaḥn*, which measured 20 x 17.2 m., if constructed in accordance with a proportional system, ought to relate to the building's width of 31 m. This proved to be the key to the geometric analysis of the Fāṭimid mosque because (bearing in mind that the

centre of the courtyard is normally the centre of its geometry) the next logical step was to assume that its dimensions were either 17.2 x 17.2 m. or 20 x 20 m. M/AJD/A2 shows a courtyard with dimensions of 20 x 20 m. and from this we may observe the following.

Having established the vertical axis Y-Y1, unit circle I has a radius X-6 measuring 11 m. In squaring this unit circle, the intersection of the main diagonals of the unit circle gives us the unit square .1, .2, .3, .4. Side .2-.3 lines up with the outer wall of the colonnade (i.e. opposite to the courtyard's side); so does .1-.2 and .1-.4. Side .3-.4 appears to line up with the outer face of the prayer hall (improvised).¹ If we take square .1, .2, .3, .4, rotate it 45°, and carry it towards the *qiblah* wall by half a unit square, the corners of this rotated square (.9, .10, .11, .12) will touch the two side walls of the mosque (outer face)² and the outer west-side wall before the misalignment.³

The same result can be arrived at by drawing another interlocking circle from centre .7 with the same radius as unit circle I. This circle will reach the *mihrāb* and by squaring unit circle II, there is produced the square .8, .6, .15, .16, of which side 15-16 appears to line up with the outer face of the *mihrāb*, but

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1. Making a deviation from the resultant geometric grid.
 2. With more time this can be worked out mathematically.
 3. The plan at present appears to be marked by two pilasters (M/AJD/A2.1).

purely geometrically. Before the misalignment,¹ it would have certainly lined up with the outer walls of the prayer hall.

In conclusion, it may be asserted that the Fāṭimid mosque at Ajdābiyyah is based on the 2:3 proportion, i.e. a full circle and a half, as in the proportions of the Ka'ba in the period of Ibn Zubayr.²

Arithmetical check

We have proved geometrically (see drawing M/AJD/A2 in vol. II) that the overall building demonstrates a 2:3 proportion. Checking this against Hutt and Whitehouse's 'measurments' of 47 x 31 m., the following results are produced:

$$31 \text{ m. wide}/2 = 15.5$$

$$15.5 \times 3 = 46.5 \text{ m.} - 1/2 \text{ m. margin.}$$

However, if we take geometric lines as the basis for the calculations, using the scale provided in drawing M/AJD/A2,

$$31 \text{ m. wide}/2 = 15.5$$

$$15.5 \times 3 \text{ produces an overall length of } 46.5 \text{ m.,}$$

which is spot on if we measure line 14-15 as the average length of the building.

The other check which can be carried out is in the geometric

1. The correction of angle to achieve orientation with Mecca.

2. Not including the *Hijr*.

relationship between the width of the building and the width of the *ṣaḥn*. According to our geometrical analysis, which is based on the squared circle, this ought to reflect the $1:\sqrt{2}$ proportion.

Whitehouse's measured external width of the building is 31 m. and this is also confirmed by our geometric line 13-14 (see drawing M/AJD/A2 in vol. II). According to our calculations based on the geometric lines 1-2 and 3-4, the average external width of the *ṣaḥn* is 21.5 m., including the piers complete with nibs (750 mm. on each side).

$$\sqrt{2} = 1.414$$

$$31\text{m.}/2.5 \text{ m.} = 1.44,$$

with a margin of one decimal point if we consider the average published internal width:

$$18.6 \text{ m.} + 1.5 \text{ pier thickness} = 20.1 \text{ m.}$$

$$31 \text{ m.}/20.1 \text{ m.} = 1.5, \text{ with a margin of } 100 \text{ mm.}$$

We may thus conclude that Ajdābiyyah Mosque in Libya was based on a proportional system which included 2:3 and $1:\sqrt{2}$ systems of proportion.

4.5 THE GREAT MOSQUE OF CÓRDOBA

Introduction

The layout of the mosque at Córdoba is a unique one. It is not a square nor is it a rectangle, which can easily be explained. The building has many elements belonging to various different periods and, as a whole, was subjected from the late 8th to the late 10th century to a total of five extensions, one merging with another with no apparent break line.

The key which began the process of deciphering the language of the building was the tracing of the wall which appeared in M. Gomez Moreno's plan published in R.A. Jairazbhoy's *Outline of Islamic Architecture*.¹ The question was, was this enormous divided mosque structure pre-planned (which is of course impossible because of the numerous later extensions), or could the architectural concept have been allowed to grow and multiply for two hundred years or thereabouts, so that the later structure would still relate to the original principle? What follows is an attempt to unravel this intriguing question. Of all the published plans of the Great Mosque, Gomez Moreno's survey appears to be the

1. (1972) p.76.

most accurate to date. Although I have not seen any previously published geometric analysis of the building, our analysis, as we shall see, suggests very strongly that Gomez Moreno's measurements were in fact accurate.

The Great Mosque of Córdoba

At the height of its fame, in the 10th century A.D., Córdoba was 'twenty four miles long by six miles wide and had over 250,000 buildings, including 3,000 mosques, palaces and baths.'¹ Today these architectural monuments are nothing but ruins, with one exception – the Great Mosque.

Period A: 'Abd al-Raḥmān I (r. A.D. 756–788)

The building was begun in A.D. 785 by the Emir 'Abd al-Raḥmān I and was frequently enlarged and embellished over the years by his successors. 'Abd al-Raḥmān I was a member of the Umayyad dynasty, who, after the overthrow of his house by the 'Abbāsids, settled in Córdoba in A.D. 756 and established an independent emirate that acknowledged the supremacy of Baghdad in matters of religion. Towards the end of his reign, 'Abd al-Raḥmān I realized the incongruity of the Muslims praying in a low-ceilinged improvised shelter, and began negotiating with the Christians with a view to purchasing for a price of 100,000 *dīnārs* the site of the church of St. Vincent which, on the advice of the

1. Conti, *Architecture as Environment*, p.37.

Caliph 'Umar ibn al-Khaṭṭāb, they had shared for over fifty years.¹ The Christians rebuilt their new church on the outskirts of the city, while the emir demolished the old church and built on its site the Great Mosque of Córdoba in A.D. 784.

The plan of the mosque consisted of a walled *ṣaḥn* (courtyard) preceding an eleven-aisled, twelve-bay sanctuary (the prayer hall). Creswell states that the mosque originally consisted of a sanctuary 73.5 m. wide and 36.8 m. deep, preceded by a *ṣaḥn* 73.21 m. wide and 60.07 deep.² This means that the *ṣaḥn* was deeper than the sanctuary space, which makes the overall shape of the mosque that of a rectangle. This is quite clear in the drawn plan appearing on p.301 of Creswell's work. Creswell seems to arrive at this conclusion even though his larger plan, shown on p.292 of the same book, shows clearly that the actually surviving evidence of 'Abd al-Raḥmān's period (shaded in black) only extends northward for approximately 10 m. from the T-shaped piers by the entrance.

Hoag, in describing the Great Mosque of Córdoba, does not mention the *ṣaḥn*'s dimensions, nor does he describe its geometric shape, but he notes that the *ṣaḥn* had no *riwāqs* (colonnades), and communicated directly with the prayer hall

1. See Jairazbhoy, *op. cit.*, p.75.

2. A *Short Account*, p.300.

through doors set between heavy T-shaped piers.¹ What is interesting is that the plan which accompanies Hoag's text on the original mosque building appears almost perfectly square in shape.

Geometric analysis

Accepting that the original mosque had 12 bays, as is agreed by all scholars, we establish a vertical central axis (Y1-Y2), as shown on analysis sheet COR/A1, passing along the centre of the middle aisle, cutting the *qibla* wall-line, and again cutting the sanctuary entrance at V. Using a compass, we now draw a circle from centre V and with a radius VF (F is to the external face of the wall). We immediately see that the circle touches the mosque's east wall at E, its west wall at F, and the *qibla* wall-line at X. But on the north side, the circle touches the traces of a wall appearing on Gomez Moreno's plan.² The reality of this wall seems to have been accepted by Hoag though he gives no reason for his view. Although it is not mentioned in the text relating to the mosque's first period, the plan on p.77 of his study clearly shows a wall (with the same degree of misalignment) situated almost 12 bays northward from the sanctuary's façade. This appears as an actual wall on Hoag's plan, not a speculative one. If this plan

1. *Islamic Architecture*, p.77.

2. See Jairazbhoy, *Outline of Islamic Architecture*, p.87. I have decided to use Gomez Moreno's plan for my analysis as it appears to be the most accurate representation of the mosque's development available to date. The dotted line in that plan refers to foundation walls discovered during excavations in the courtyard in the 1930s. Unfortunately, this information has not been incorporated into later published plans of the mosque.

(indicated by Gomez Moreno on the basis of excavation and accepted by Hoag) is acceptable, the result would be that we would have a square-shaped mosque with corners at A, B, C, and D, and with diagonals B-D and A-C crossing exactly at V. The question presents itself, does this area actually represent the design of the original mosque of 'Abd al-Raḥmān I, or is Creswell right in regarding the original mosque as a rectangle? Evidently not, if we consider the recent finding by Félix Hernández of an earlier minaret built by Hishām I c. the 790s; its north face was about 36 metres from the present sanctuary façade, whereas the present courtyard is 58.5 m. deep. He also found the remains of a wall parallel to the northern side of the sanctuary façade under Hishām I which had its south side aligned to the old minaret. At a time which is still uncertain, the *ṣaḥn* was made deeper by about 24 m.¹

In a section headed "Islam and Muslim Art", Alexandre Papadopoulos states that

Abd-al-Rahman I in 786-788 erected a mosque that K.A.C. Creswell, H. Terrasse, and J. Sauvaget thought was an oblong about 328 feet long and 246 feet wide. However according to present-day knowledge, the original mosque seems to have been an almost perfect square of about 258 feet, thereby following the Medina model.²

Unfortunately, Papadopoulos does not give the source for this 'present-day

1. E. Lambert, "Histoire de la Grande Mosquée", p.177; and see analysis sheet COR/A1.

2. *Islam and Muslim Art*, p.252.

knowledge'; presumably it is the archaeological evidence embodied in the present courtyard. My own geometrical evidence supports this. Thus, contrary to Creswell's view, 'Abd al-Raḥmān's original mosque was a truly square building. The mathematical analysis of its layout given in this section would, at worst, offer corroborative evidence for such a view; and, at best, would clinch the suggestion incorporated in Gomez Moreno's plan.

Period B: 'Abd al-Raḥmān II (r. A.D. 822-852)

'Abd al-Raḥmān II had the wisdom to preserve the general character of the mosque's plan with aisles perpendicular to the *qibla* and with an interior dominated by superposed columns and double-tier arches. According to most authorities, 'Abd al-Raḥmān II was aware of the pressures caused by population expansion and amplified the mosque by carrying the *qibla* wall another 8 aisles southward.¹ Although all scholars talk of 8 bays² extension towards the south, Creswell, under the heading "History of the Mosque", states, basing himself on the observations of Ibn Adharī, that 'Abd al-Raḥmān II's extension of the mosque was 50 cubits deep and 150 cubits wide. Creswell gives the depth of the original sanctuary (prayer hall) as 36.8 m.³ Thus Creswell gives the correct depth of the extension but draws the wrong conclusion from that information, in that he

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1. Cf. Jairazbhoy, *Outline of Islamic Architecture*, p.77; Hoag, *Islamic Architecture*, p.81. Hoag writes, 'Between 832 and 848 the prayer hall of the Great Mosque received eight more bays in the direction of the qibla.'
 2. However, no scholar explains why 'Abd al-Raḥmān II extended the mosque only by 8 bays.
 3. *A Short Account*, p.300.

assumes that the extension was 8 bays in depth. If a cubit be taken as approximately equivalent to 750 mm., 'Abd al-Raḥmān II's extension of 50 cubits must therefore have equalled 37.5 m.,¹ which is clearly quite close to the original mosque's prayer hall. This proves that, according to Ibn Adharī, the new extension must have been at least 12 bays deep, matching the depth of the original mosque. Geometrically this makes sense for the following reasons.

Geometric Analysis

A-B-C-D marks the original square mosque with its centre at V (see analysis sheet COR/A 1). Y1-Y2 is the vertical axis cutting C-D at X.

With the same radius as previously noted, i.e. VF, if we draw a circle with its centre at X, we find that it touches the west wall at C, the east wall at D, and on the north it touches the prayer hall's entrance wall at V. On the south, however, the circle goes beyond 'Abd al-Raḥmān II's *qibla* wall (as reconstructed by Creswell) by a certain depth, which leads me to believe that 'Abd al-Raḥmān II's extension was never by 8 bays only, but in fact by 12 bays, and thus constituted an exact duplication of 'Abd al-Raḥmān I's prayer hall. If this is true, the whole history of the Córdoba mosque will need to be rewritten.

When we come to period C of the mosque's development, we see the

1. 50 cubits x 750 mm. = 37.5m.

significance of the area by which the second circle (radius X-C and centre at X) intrudes into the space of al-Ḥakam II's extension. This space seems to be marked, as I have indicated on analysis sheet COR/A2, by two heavy piers enclosing an area 1 aisle wide and 3 bays deep.

Now we have a square of E-F-G-H, with centre at X, with horizontal axis D-C, and a vertical axis Y1-Y2, and with diagonals F-H and E-G crossing at X. This overlaps the earlier square A-B-C-D.

The question arises, where are the physical remains of the original *qibla* wall of 'Abd al-Raḥmān II on line H-G on my drawing (COR/A2)? It would not be strange if nothing whatever were left, since it would be the obvious course of action to remove the old *qibla* wall entirely when extending the mosque southwards. Nevertheless, I hope to prove that something *does* remain. The answer to our question lies in the next period of the development of the mosque during the reign of al-Ḥakam II. According to Jairazbhoy, 'Great compound piers [were] left to mark the old *qibla* wall';¹ yet this assumes that the original *qibla* wall was either an open arcade carried on these piers or a solid wall punctuated by them. Neither type of *qibla* wall is known in early Islamic architecture. Thus some other explanation must be found for this imposing sequence of piers. Another and quite different explanation for their presence will be suggested below.

1. *Outline of Islamic Architecture*, p.81.

Period C: 'Abd al-Raḥmān III (r. A.D. 912-961)

'Abd al-Raḥmān III assumed the title of 'caliph' in A.D. 929 under the name of al-Nāṣir li-Dīn Allāh, in opposition to the recent founding of the Shī'ite caliphate of the Fāṭimids at Mahdiya. 'Abd al-Raḥmān III's principal foundation was the residential and administrative centre of Madīnat al-Zahra begun in A.D. 936 and to which the court moved in A.D. 945. Nevertheless, he was also active in extending the Great Mosque of Córdoba. It was in A.D. 951 that he extended the *ṣaḥn* to the north¹ and built a new minaret for the Great Mosque. The minaret projected into the *ṣaḥn* and avoided the *miḥrāb* (prayer niche) axis as it did at Madīnat al-Zahra. Its base covered an area of 91 square feet and its height was 111 feet. He was probably also responsible for 3 *riwāqs* in the *ṣaḥn*; these have since disappeared. He appears to have left the sanctuary untouched.

Period D: al-Ḥakam II (r. A.D. 961-976)

As we have seen, 'Abd al-Raḥmān III confined himself to rebuilding the minaret in A.D. 961 and to extending the *ṣaḥn*. It was his successor, al-Ḥakam II, who applied to the construction all the resources of Córdoba art. We are told that al-Ḥakam went in person, along with his jurists and architects, to the

1. Hoag says to the south ('In 962, a year after he came to the throne, Al-Hakam II began the final southward extension of the Great Mosque. He added twelve bays and a double qibla wall.' *Islamic Architecture*, p.84).

mosque to draw out the plan¹ and set down the details of the third enlargement of the prayer hall.²

According to ‘Abd al-‘Azīz al-Dawlatī, ‘On the appointment of his *khilāfa* (trusteeship), al-Ḥakam II began enlarging the prayer hall by two-thirds [the size of] the old hall’ (بادر الحكم الثاني عند توليه الخلافة بتوسيع بيت الصلاة بمقدار ثلثي مساحة القاعة القديمة).³ By ‘the old hall’, I take it that he means the area including the extensions of periods I and II. If we divide the old hall into three and then compare it with the plan, the extension must have started from the twin heavy piers (3 bays in front of ‘Abd al-Raḥmān II’s suggested *qibla* wall).⁴ If so, this supports my geometric theory of three intersecting circles encompassing perfectly the three periods of the mosque’s extensions A, B, and D (C is the extension of ‘Abd al-Raḥmān III).

There is no trace of ‘Abd al-Raḥmān II’s *qibla* wall. What is there now and what everyone refers to, is an arcade resting on massive piers and running across the prayer hall from east to west (line N-O on analysis sheet).

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1. Jairazbhoy, *Outline of Islamic Architecture*, p.80.
 2. To me this suggests the same method, discussed in an earlier chapter, of setting out the building on site using rope stretching and chalk marking.
 3. *Masjid Qurṭuba*, p.27.
 4. Most scholars appear to agree that ‘Abd al-Raḥmān II extended the mosque by 8 bays. See e.g. M. al-Tilimsānī, *Naf’ al-Ṭīb fī Ghuṣn al-Andalus al-Raṭīb* (Cairo, 1949); R. Arié, *L’Espagne Musulmane aux Temps des Nasrides (1232-1492)* (Paris, 1973); R. Castejon, *La Mezquita Al Jama de Córdoba* (Léon, 1973).

The question is, why should the arcade be on the line of the old 'Abd al-Raḥmān II *qibla* wall? It is an assumption which is yet to be proved. The only support for this theory is Creswell's statement that 'if we walk northwards from the *qibla* wall until we have passed twelve bays, we come to an arcade, resting on massive piers and running right across. This evidently marks what was the limit of the mosque before al-Ḥakam's addition.'¹ This conclusion is difficult to accept, as it is merely an assumption. Creswell attempts to prove his theory conclusively by saying, 'and further confirmation is supplied by the fact that the masonry of the western wall breaks bond immediately to the south of the point where this arcade strikes it.'

I am of the opinion that the traces of 'Abd al-Raḥmān II's *qibla* wall are still there to be seen. They consist of 2 heavy piers located 3 bays south of the arcade O-N, and I believe that they were to commemorate the old *qibla* wall.²

The only way al-Ḥakam could have achieved a 12-bay extension - i.e. an extension on the same scale as those of his predecessors - as well as a double *qibla* wall, and yet not reach the river, would have been by taking down 'Abd al-Raḥmān II's *qibla* wall (G-H)³ and rebuilding it northward 3 bays into 'Abd al-Raḥmān II's prayer hall. His actual extension was of the same size as those of

1. *A Short Account*, p.297.

2. See Jairazbhoy, *Outline of Islamic Architecture*, p.81.

3. Creswell mentions a similar exercise by al-Ḥakam in taking down structures in that area: 'in doing this [meaning the extension] he demolished the covered passage of Abd Allah and replaced it by a new one at the extremity of the west wall.' (*A Short Account*, p.293.)

his predecessors, but might have appeared smaller than it was because the double *qibla* wall took up so much space: indeed it swallowed up the equivalent of 3 full bays. Thus he could have been accused of building 'only' a 9-bay extension. To recuperate the 'lost' 3 bays, he took a sizable chunk out of 'Abd al-Raḥmān II's extension and staked out the space thus claimed with an elaborate new arcade (N-O).

Al-Ḥakam built his northern arcade resting on massive piers lining up with the rest of the aisles, thereby achieving a prayer hall 12 bays deep, a structure as impressive as that of 'Abd al-Raḥmān I. Al-Ḥakam left great compound piers to mark the old *qibla* wall and, as a mark of respect, he went on to build the most spectacular enclosure of multi-leafed arches. Jairazbhoy describes this as follows:

In the central aisle at the start of the new extension is the so-called Chapel of Villaviciosa. It is a vaulted bay whose walls are open screens of complex arch forms carried on marble columns.¹

Geometric Analysis: Period D

We now have, as shown in COR/A4, three interlocking circles with a diameter the exact width of the mosque and encompassing perfectly the three

1. *Outline of Islamic Architecture*, p.81.

mosque extensions A, B, and D.¹ These proportions are reminiscent of those of the Great Mosque of Sāmarrā' (al-Malwiyā). There we shall see, as I will demonstrate in the section 13 of this chapter, a similar proportional system.

On analysis sheet COR/A2.2 I have tried to demonstrate how, if we were to take the sightings of al-Adharī as a basis for argument, E-F-G-H (the size of the prayer hall before al-Ḥakam) is divided into three parts. Al-Ḥakam's extension equals $\frac{2}{3}$ of this area. If, on the other hand, we were to propose that the prayer hall before al-Ḥakam's extension was E-F-N-O and divided that into three, al-Ḥakam's extension does *not* measure $\frac{2}{3}$ of this area.

Period E: al-Manṣūr (r. A.D. 987-990)

Twenty-five years after al-Ḥakam's extension, the mosque could no longer accommodate the masses of worshippers, especially with the new arrival of Berber settlers.² Al-Manṣūr began to enlarge the mosque for the fourth time. His dilemma was deciding in which direction to make the extension. On the west, Qaṣr al-Khilāfa (the Palace of Trusteeship) was attached to the mosque's wall,³ southward the river was only a short distance from the *qibla* wall, and on the north side 'Abd al-Raḥmān II had built the new minaret. The only direction left for him therefore was the east side.

1. Stage C was to the north only.

2. See al-Dawlatī, *Masjid Qurṭuba*, p.28.

3. *Ibid.*

Al-Manṣūr decided to extend the Great Mosque of Córdoba eastward along its full length for another 8 aisles,¹ so that now there were 19 parallel aisles that looped away for more than 30 bays in a southerly direction. Finally, in the enlarged *ṣaḥn* which had trees planted in it since the ninth century, al-Manṣūr built a *jābiya* (underground cistern). Later we will see how this configuration of design relates to the overall geometry of the mosque.

The question that kept recurring when I examined the mosque's plan was: why was al-Manṣūr's extension to the east by 8 aisles and not by 9, or 7? Another question was, if the previous three extensions were one geometric progression, how would al-Manṣūr carry this through?

Geometric analysis: Period E

Having established an overlapping repeated square unit D-C-L-M circumscribing a circle with centre at Y, diameter H-G, and vertical axis Y1-Y2 (see drawing COR/A4) and having decided to extend the mosque eastward, al-Manṣūr must have sat down with his architect/geometer² and pondered on how to extend within a geometric formula in order (a) to establish a practical system of building on the basis of which clear instructions could be given to workers;

1. See J. Allan, *A Short Account*, p.293.

2. When 'Abd al-Raḥmān III built Madīnat al-Zahra, his chief architect and geometer was Maslama ibn 'Abd Allāh. See *ibid.*, p.78.

and (b) to arrive at a plan which would relate mathematically to the four preceding extensions, i.e. from the north corners of 'Abd al-Raḥmān's extension to the extension of al-Ḥakam (L-M) on the south.

The solution was simple. Al-Manṣūr's architect had two fixed limits to his extension. On the north he had 'Abd al-Raḥmān III's new external wall,¹ while on the south he had al-Ḥakam's *qibla* wall which was restricted by the river. The only limit he had to decide on geometrically was the east edge. All he had to do was to return to the original unit square, 'the original mosque of 'Abd al-Raḥmān I', divide the area vertically into three equal parts (see drawing COR/A5), and extend these vertical bands along the entire mosque, then simply go eastward by exactly two parts, i.e. two thirds of the old prayer hall.

Seeking to correlate the past stages of the mosque's building, it is interesting to discover that 'Abd al-Raḥmān III extended the *ṣaḥn* to the north by a third, and al-Ḥakam also extended to the south by two thirds. If we were to draw a 'grid' based on the division of the unit square (i.e. the original mosque of 'Abd al-Raḥmān I) into thirds, the result would be a large pattern of small squares horizontally and vertically, in which the entire mosque, including al-Manṣūr's final extension, occupies a total of exactly 35 units (see drawing COR/A7).

1. Obviously there was no need to extend the *ṣaḥn* further north.

When the grid of squares is superimposed on the mosque plan (COR/A7), it yields 7×5 equal squares. The *jabiya* is on the sixth line of squares, south of the *qibla* and the minaret is on the seventh line, occupying exactly one-ninth of its square.

Arithmetical check

According to measurements published by RIBA,¹ the overall width of the building is 130 m., taken from external face to external face of the walls. The width of the inner building, which, as we have already explained, is related by many historians to Period A² of the building's history, is 78.39 m., taken from external face to external face of the walls.

Towards the end of our geometric analysis of the Great Mosque of Córdoba, we concluded that the entire building appears to form a great rectangle made up of five equal units across its width and by seven of the same units along its length. The area relating to Period A is made up of three units by seven units (see drawing COR/A7 in vol. II). The width of Period A building is $78.39/3$, i.e. 26.13 m. Could therefore 26.13 m. square be the area of a unit measure on which the entire building was based?

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1. "The mosque-cathedral of Cordova", *Transactions of the Royal Institute of British Architects*, vol. VIII (1882-3) fig. 127.
 2. 'Abd al-Rahmān I (AD 756-788).

$26.13 \times 5 \text{ units} = 130.65 \text{ m.}$

published width 130 m.

a margin of 650 mm.

$26.13 \times 7 \text{ units} = 182.91$

published length = 182

a margin of 910 mm.

In a recently published article by Martin H Mills, entitled "The Pre-Islamic Provenance of the Great Mosque of Cordova"¹ the author bravely questions the true origin of the Great Mosque of Córdoba, stating that it is time to re-examine the concept of an Islamic origin of this building, one which has been accepted by historians for over a thousand years, including al-Rāzī in the fourth/tenth century, al-Idrīsī, and Ibn Adhārī. Mills invites his readers to evaluate with him the evidence that could support a more ancient provenance and, although I had already concluded my chapter on the Great Mosque of Córdoba years prior to reading this article, I am glad to be able to consider Mills's work. For many years during my research I have pondered the question of the orientation of the Great Mosque of Córdoba, constantly coming up against the question, why does this mosque not face Mecca? The common answer is that just as the Syrians back in Syria turned southwards to face Mecca, the founder of the Great Mosque of Córdoba likewise remained facing south, even when in

1. *Al-Masāq*, IV (1991) 1-16.

Spain, to emphasize the sentiment that the Umayyad dynasty never in reality left its homeland. Consequently, the founder's descendants maintained the custom of praying towards the south, even though they had numerous chances to change the *qiblah* during the many extensions that were made to the Great Mosque.

Although the subject of orientation itself does not directly affect our analysis of the building's geometric characteristics, we nevertheless find that the plan's rigid geometric principles, even during the process of adding extensions, may confirm the fact that the Muslims were perhaps building within already marked-out boundaries.

First, however, it is necessary to evaluate Mills's evidence. What we will consider here is the possibility that the Great Mosque of Córdoba is the result of a process of renovation and extending of an existing pre-Islamic building, rather than a freshly created structure. What we need also to appreciate is that the quintessence of the Spanish Islamic style, whose architectural vocabulary spread within Spain and North Africa, may owe its appearance and substance to an earlier culture.

With regard to the question of the *qiblah* and the mosque's orientation,

Mecca from Córdoba is 10 degrees and 14 minutes south of east,¹ yet the mosque actually faces 60 degrees south of east. We may accept Mills's evidence, derived from Hawkins and David King, that the Ka'ba also aligns 60 degrees south of east, so that the Great Mosque of Córdoba and the Ka'ba are axially parallel. Mills attempted to learn more of the mosque's construction history by using carbon-14 analysis. With the help of the Spanish Government, he collected wood samples from the mosque and submitted five of them for carbon-14 dating analysis. The results were detailed and quite astonishing, for some of the datings produced had a mid-point date of AD 681 plus or minus 60 years. One can only comment on their accuracy by examining the full results – a task which is however beyond the scope of this research. However, their implications are important, for, if they are correct, we may be looking at a building first constructed long before any Muslim had arrived in Spain.

Working from the building's interesting orientation, Mills puts forward a hypothetical reconstruction of its development through time as follows. During the Phoenicians' presence in Andalusia, from the twelfth to the seventh centuries BC, they may have built a combined religious and commercial centre. The building may have functioned simultaneously as a temple, an audience hall, and a warehouse. In addition, its tower may have served as an astronomical observatory. Later, the building could have been used by the Carthaginians from

1. Mills confirmed this orientation in October 1990. Readings were taken on the roof-tops of the mosque. He aligned the pinnacles of the two mosque cupolas on an axis with the *mīhrāb*.

the sixth to the second centuries BC. The Romans would have found it advantageous to retain the building and continue using it as a warehouse from the second century BC to the fifth century AD. Continuing with Mills's hypothetical historic reconstruction, the Visigoths, from the fifth to early eighth centuries AD, may also have continued to use the building, perhaps even renovating it. This would correspond in time to Mills's wood sample taken from a roof beam. The Visigoths' presumed need for a watch-tower and a lighthouse may have encouraged them to rebuild a destroyed tower, but this time with a central masonry wall spire for reinforcement. The accepted account of this tower's being the second minaret built by 'Abd al-Raḥmān II may be somewhat adrift from reality, since his work may only have involved repairs and cosmetic work. The fact that this tower had replaced an earlier minaret built by Hishām I has, according to Mills, never been proved, although foundations for some structure still exist in the courtyard. In the Visigothic period, the north-west area of the building, the reputed Stage I of the mosque, may have served as a functioning Church of St. Vincent.

Mills's sample no. 4, which was taken from a wood ceiling panel and dates from AD 785, may have been the replacement for an existing panel. Taking this further, according to Mills, interpreting Stage I of the mosque as a reworking rather than as a new construction, allows for the otherwise improbable assertion by al-Rāzī that Stage I of the mosque was built in one year. If the foundations and exterior walls were already in place, the repairs and replacements

of the interior columns and arches would account for the presence today of reused Byzantine, Visigothic, and Roman capitals in the prayer hall. Stages II and III, from AD 822 to 961 respectively, probably completed the reworking of the remainder of the hypostyle hall as far south as the present *mihrāb*, which, in Mills's opinion, may originally have been a Phoenician altar.

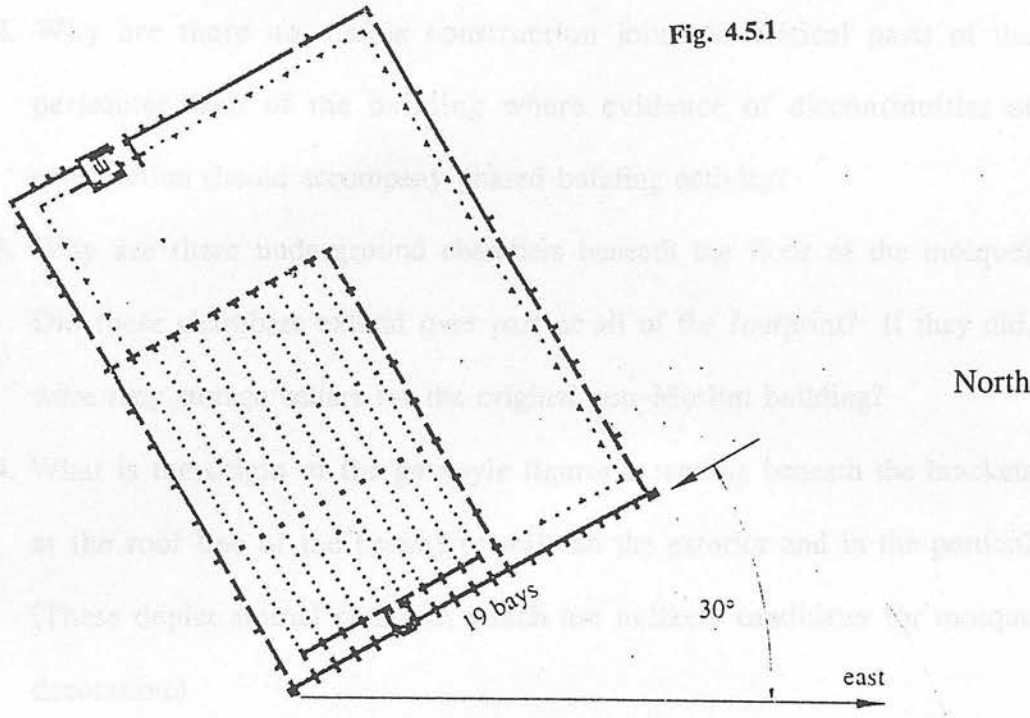
If this is correct, i.e. that phases I, II, and III of the Great Mosque of Córdoba were in fact merely internal work built into an already existing structure, there will no longer be a conflict between the extent of the third 'external' enlargement deduced by our geometric analysis¹ and the more accepted level of work carried out by al-Ḥakam II (961-876),² especially if no external walls or roofs were constructed. However, our conclusions are still valid as far as the sequence of the internal enlargements are concerned and these, as we have seen, reflect a pure geometric system of three interlocking circles with evidence of 1:2 and 1:√2 proportions.

Whether or not Mills's theory proves correct, this does not affect the credibility of our results for the simple reason that, whoever constructed this building, its dimensions adhere to a strict geometric framework, one which must have been charted by the Muslim occupiers even if their work was contained

1. See thesis, vol. II (drawings), p. 18-22, drawing no. COR/A4.

2. See thesis, vol. I, Chapter IV (geometric analysis), section 5 on the Great Mosque of Córdoba.

within an existing structure.¹ However, Mills's conclusion appears to go a long way in overturning Creswell's findings, for if the Great Mosque of Córdoba² has always been one building, one needs to re-examine Creswell's report 'and further confirmation is supplied by the fact that the masonry of the western wall breaks bond immediately to the south of the point where this arcade strikes it.'³



Mills accepts that Stage IV of the mosque's expansion was a Muslim expansion (apart from the eastern wall) to the previous phases I, II, and III, and

1. Each enlargement was by a certain amount relating closely to the previous size and proportions of the prayer hall. See thesis, vol. I, Chapter IV, section 5.
 2. By 'mosque' here, we refer to phases I, II, and III.
 3. *A Short Account*, p. 297.

adds, 'This Stage IV construction seems to be generally inferior in that the voussoirs of the interior arches are of stone with painted stucco rather than alternating red brick and limestone.' At the end of his article, Mills leaves us with the following questions that he believes still require answering.

1. How did the style of double-tiered, lobated interlaced arches and intertwined cupola arches come about with such skill and without apparent precedent?
2. Why are there no visible construction joints at critical parts of the perimeter wall of the building where evidence of discontinuities of construction should accompany phased building activity?
3. Why are there underground chambers beneath the floor of the mosque? Did these chambers extend over part or all of the footprint? If they did, were they storage cellars for the original non-Muslim building?
4. What is the origin of the gargoyle figures extending beneath the brackets at the roof line of the perimeter wall on the exterior and in the portico? (These depict animal creatures which are unlikely candidates for mosque decoration.)
5. Besides the fortress-like character of the construction, suitable for a secular construction, why would the Umayyads have built Stage II towards the south, between 961 and 976, thereby creating an elongated oblong form atypical of mosque configuration?
6. The mosque in the palace-city of Madīnat al-Zāhirah, built shortly after the Great Mosque of Córdoba in 936, some five miles to the north-west

of Córdoba, is closely oriented toward Mecca. Why should this be so since, if reason had been strong enough, it could have been constructed parallel to Mecca like the Great Mosque of Córdoba?. Why do the roofs of the Great Mosque of Córdoba appear not to correlate with alleged stages of construction as one would expect?

Having examined Mills's proposal and pondered over his questions, we may conclude that the building evidently had a complex history involving extensive destruction and rebuilding over centuries. It therefore requires closer attention and a more efficient programme of excavations if archaeologists can make use of our geometrical analysis of the mosque and the probing questions of Mills, in order to present a picture closer to the true history of this magnificent building.

I agree with Mills that the Great Mosque of Córdoba's key position in the history of Islamic architecture will, when thoroughly re-evaluated, lead to the revision of seemingly settled concepts. As we have seen in our geometric analysis of the Great Mosque of Damascus,¹ we were able with confidence to challenge the more accepted view of the building.

1. See thesis, vol. I, § 4.8 and vol. II, drawing DAMA/A3.

4.6 THE GREAT MOSQUE OF QAYRAWĀN

‘Uqbah ibn Nāfi’ was appointed over the Maghrib by Mu‘āwiyah in c. A.D. 670. At Qayrawān, in the vicinity of a deep well,¹ ‘Uqbah marked out the ground² of his mosque, an extremely modest structure of which very little is known.

In examining the plan of the present structure in detail, one is able to observe certain elements which can be identified with the original mosque. This helps us to reconstruct ‘Uqbah’s original building, which seems to have comprised:

1. The Main Qiblah Axis

The *mihrāb*’s present location along the *qiblah* wall has remained virtually unaltered for centuries. But while ‘Uqbah presumably built no *mihrāb*, it would have been sensible for the *imām* to take up his place on the central axis of the *qiblah* wall whether or not that axis was marked in some special way. It seems reasonable to assume that this spot is that marked by the

1. D. Hill & L. Golvin, *Islamic Architecture in North Africa* (London, 1976) p.91.

2. K.A.C. Creswell, *Early Muslim Architecture*, vol.II (Oxford, 1940) p.208.

present *mihrāb* if one assumes that the mosque was extended symmetrically. Therefore, one can establish a fixed vertical axis which we shall call Y-Y1, passing through the middle of the *mihrāb*'s position down through the middle of the central aisle and that of the end portico, through to the centre of the courtyard, and finally lining up with the western wall of the minaret (see analysis drawing QAI/A1, vol.II – plan after Creswell).

Now that a vertical central axis Y-Y1 is established, we next proceed to fix the horizontal axis X-X1. Assuming that 'Uqbah's original mosque was square, on the Medina and Kufa models, the length of the original mosque should equal its width. Once that is established, the mid-point along the length should be the position of the horizontal axis.

2. Width of the Original Mosque

There have been a number of theories regarding the width of the original sanctuary. Sébag and Lézine deduced that the original *qiblah* wall must have been 50 m. long, to one side of the present mosque (see analysis drawing QAI/A2), with the *qiblah* window to the present *maqṣūra* being the original

mīhrāb built by ‘Uqba.¹

However, when the above theory is examined geometrically, it proves to be inconclusive, particularly as the cistern on which they based their theory of the *maqṣūra*-window alignment falls outside the proposed 50 m. sq. mosque which they believed was constructed on the Medina model, also approximately 50 m. (100 cubits) sq. It could also be argued that for the well to fall outside the boundaries of the original mosque is not necessarily a miscalculation, especially as many historic structures have been set out in relation to a significant point outside the immediate boundaries. However, in the case of Qayrawān, this can hardly be practical when, as in any mosque, water is most essential for the required ablutions. Taking Kufa (A.D. 638) as a raw model (200 cubits, or approx. 100 m. sq. – see fig. 4.6.1), it is therefore not inconceivable for a mosque constructed in A.D. 670 to measure 70 m. in width. The dimension of 70 m. is suggested so as to *include* the wall rather than exclude it; and a square mosque is more likely than a rectangular mosque at this period.

Having assumed the original mosque width to be 70 m., we can now proceed to establish the horizontal axis X-X1. Its location ought to be the mid-point of the mosque's length which, as assumed above, equals its width of 70.2 m. (see analysis drawing QAI/A3 in vol.II).

1. J. Allan, *Short Account*, p.329.

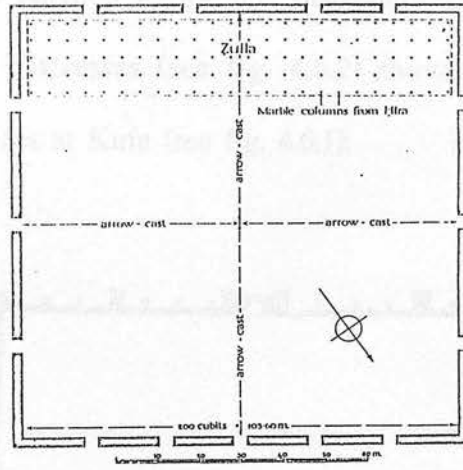


Fig. 4.6.1

3. Depth of the Original Sanctuary

There appears to be no historical record of the depth of 'Uqbah's original prayer hall, which means that one is left either to deduce certain facts or to theorize on the basis of a detailed analytical study of the present plan, or both as in the case of Qayrawān.

One problem concerns the juxtaposition of the two existing side entrances in relation to the prayer hall. These are Bāb Lallā Riḥānah on the east side fourth from the north, and the corresponding entrance on the west side known as Bāb al-Sulṭān. Although in time these entrances became elaborate,

consisting of domed porches, Creswell's¹ suggested reconstruction of Qayrawān's original scheme of buttresses (see fig. 4.6.2) shows them as simple doorways reminiscent of the doors at Kufa (see fig. 4.6.1).

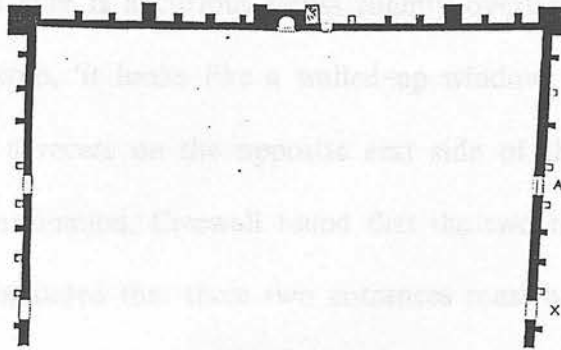


Fig. 4.6.2

These must at one time have led directly to the mosque courtyard, as it is impractical in Islam for a prayer hall to be entered directly from the street. It is essential to arrive first at a courtyard or a covered arcade, from which the faithful are directed to their ablutions or to the formal prayer hall entrance in order to arrive facing the *mihṛāb*.

Having deduced that Bāb Lallā Riḥanah and Bāb al-Sulṭān must have led directly to the courtyard, the original prayer hall must therefore have terminated beyond those entrances, as in Kufa (see fig. 4.6.1).

1. *Early Muslim Architecture*, vol.II, p.216.

Creswell's observations with regard to the third door from the north on both east and west sides of the mosque add credibility to our theory about Bāb Lallā Rihānah and Bāb al-Sulṭān. Creswell finds that to the south of the third door on the west side is a curious recess slightly overlapped by a buttress. 'At first sight,' he wrote, 'it looks like a walled-up window. I found however that my plan showed a recess on the opposite east side of the mosque at the same spot.'¹ On re-examination, Creswell found that the two recesses were walled-up entrances. He concluded that these two entrances must have exactly flanked the old façade of the sanctuary, as at Damascus and Cordoba. Creswell believes that the portico was added when Ibrāhīm II ordered fresh doorways to be made to flank the new façade,² and to lead the faithful directly on to the courtyard for visual contact with the water source for ablutions.

4. The Total Geometric Composition

Analysis drawing QAI/A10, which we will be discussing later in more detail, illustrates how, in establishing a comprehensive geometric picture which corresponds to most elements of the building, one can confirm previous assumptions, such as:

- a) the central vertical axis Y-Y1;

1. *Ibid.*, p.216.

2. *Ibid.*, p.218.

- b) the location of horizontal axis X-X1; and
- c) the progressively increasing depth of the sanctuary.

In addition, one may also observe other major elements which appear to adhere to the same geometric system. These are:

- a) the width of the central aisle, which appears to correspond to the diameter of both domes, i.e. above the *mīhrāb* and above the front portico; and
- b) the width of the existing covered courtyard arcade which appears to correspond to the width and length of the minaret.

5. A Second Opinion

It is interesting to cite the following statement, which supports the conclusion already arrived at independently:

Although we have learnt very little about Uqbah's original mosque, recent investigations can give us an idea of the structural value of this first edifice. We think it is possible to assert that it was as wide as the present mosque but the plan was more or less square like that at Kufa.¹

1. D. Hill & L. Golvin, *Islamic Architecture in North Africa*, p.92.

Reconstructing 'Uqbah's Original Plan

The present sanctuary façade, in relation to the vertical axis Y-Y1, appears to halt at a distance, measured from the centre of the *mihrāb*, equal to half the width of the present mosque (see analysis drawing QAI/A3).

Historically it is appropriate enough to take the present sanctuary façade as a point of departure. It runs unbroken from the east side of the mosque to the west side, despite the abutment of the present arcades. This may prove that originally there were no side *riwāqs*, similar to the arrangements found at Cordoba.

The point at which the vertical Y-Y1 axis crosses the present sanctuary façade becomes the centre of our master circle A. The centre we shall call 50. Approximately along the line of the present sanctuary façade is our horizontal X-X1 axis, i.e. half-way across 'Uqbah's presumed original mosque. We now draw the master circle from centre 50 with radius 50-51, i.e. to the centre of the semi-circular *mihrāb*. Squaring master circle A, we obtain square 1,2,3,4. We draw the diagonals 1,3 and 2,4 and continue with further concentric squares in the usual manner (see analysis drawing QAI/A4). We find that the south side of concentric alternate square 9,10,11,12 lies beyond the line of the south entrances. Allowing both Bāb Lallā Rihānah and Bāb al-Sultān to lead directly to the proposed original courtyard indicated by area 1,2,42,43 (see analysis drawing QAI/A4), area 43,42,3,4 reflects the proposed extent of 'Uqbah's original prayer

hall. By that reckoning it would account for one quarter of the original mosque; the remaining three quarters would be courtyard.

The above geometric interpretation enables more than ever direct comparison to be made between the layout of the original Qayrawān mosque and the original mosque at Kufa.

Evidence for 'Uqbah's Original Mosque and the Search for the Missing *Ṣahn* Wall

The discussion so far has suggested that assuming that the present *qiblah* wall is identical with that of 'Uqbah's mosque and that the central axis (however that was emphasized) has not changed, and finally that the first mosque was square, the northern wall of 'Uqbah's mosque lies somewhere under the present courtyard. Geometry suggests that it should be sought on the line 1-2.

Two particular aspects made a field trip to the Great Mosque at Qayrawān even more essential. First, there was the possibility of finding some physical evidence which might be considered relevant to the original mosque of 'Uqba; and second, there was the likelihood of finding an explanation for the obliqueness of the western wall.

It is difficult to resist expressing the astonishing experience of actually seeing the Great Mosque in reality. "Uqbah's Mosque", as it is known to the

locals, was impressive, dignified, and grandiose, projecting an exceptional aura and an intense impression of history.

Armed with a geometric chart of the present layout (see analysis drawing QAI/A10), one enters the mosque's compounds from a covered entrance midway along the western wall.

According to the geometric analysis chart drawn for this site, the missing *ṣahn* wall lies transversely beneath the present courtyard between points 1 and 2, as shown in analysis drawing QAI/A4. Therefore, there would be no need at this stage to progress further into the mosque. The search for the missing wall begins by seeking a break-line within the arcade wall itself internally and externally.

Plate no.5 shows the area immediately to the north of entrance E where there is no apparent evidence of a joint or break-line in the brickwork. In plate no.6, which illustrates the arcade's front elevation, there is again no sign of any change in the structure except for some recent repointing. Plate no.7, on the other hand, shows a vertical band on the wall area between the two buttresses north of entrance E, when the brick joints appear to stop and restart. However, when this is examined in detail, it appears to be approximately 3.2 m. away from the proposed missing wall.

The next task was to examine the arcade wall on the opposite side of

the courtyard. The excavation of a trench 1 x 2 m. wide and 2.5 m. deep within the sanctuary end of the eastern covered arcade seemed promising. When it was examined carefully, however, although it appeared to reveal interesting foundation deposits, its location in relation to the proposed line of the missing wall was out by 10 m. to the south.

In the knowledge that all existing walls have been rebuilt several times, one may entertain the hope that some indication or sign might have been left by the builders to celebrate the original outlines which 'Uqbah himself had marked out on the ground.¹ This is not unusual in Islam. There was an example of it in Córdoba where traces of 'Abd al-Rahmān II's *qiblah* wall are commemorated by two heavy piers and bays south of the arcade.² We have also seen an example of this in Qayrawān's original entrances B and L. Although these entrances might not have been in daily use since, consequent upon the expansion of the prayer hall, they led directly into the prayer hall, they are commemorated with impressive and decorative domed porches.

The final hope, however, rested in examining in great detail the surface of the courtyard itself. In the middle of the present courtyard and among the apparently haphazard and scattered water holes, one cannot help but notice the exquisite piece of decorative marble work surrounding water hole no.1 (see

1. *Ibid.*, p.208.

2. See chapter 3/8.11.

drawing QAI/A9). This particular water source (see plate no.10) lies exactly on the *qiblah* axis and at a certain distance from the proposed missing *ṣaḥn* wall. The extremities of the marble pattern form a large square measuring 6.61 m. square. Its north side appears to line up with the proposed missing wall of 'Uqbah's Mosque. Having carried out a measured survey of the marble decoration, cross-checking the distance between the centre of the hole to the edge of the pattern with the proposed missing *ṣaḥn* wall, one obtains almost equal readings within 95 mm. tolerance.

Superimposing the geometric analysis drawing for the total complex, including the proposed line of 'Uqbah's missing *ṣaḥn* wall,¹ onto Creswell's original survey plan,² the distance between the central water hole within the marble square and the proposed line of the missing wall measures (using Creswell's scale of 1: 300) 47 mm. The distance between the central water hole and the edge of the marble pattern, according to the measurement survey carried out on site (see drawing QAI/A11), is 3,305 mm. The difference is a gap of 1,445 mm. In the knowledge that there are no original walls remaining from 'Uqbah's period for comparison, taking an average thickness based on the earliest surviving wall in this mosque³ will result in the following:

-
1. The proposed original northern wall of 'Uqbah's Mosque is referred to here as 'missing' only in the sense that its exact location does not appear to be well documented.
 2. *Early Muslim Architecture*, vol.II, fig.180.
 3. *Ibid.*

wall thickness of 1.5 m. + wall thickness of 1.2 m. = 2.7 m.

$2.7/2 = 1.35$ m. average wall thickness.

Therefore, if one assumes that the line shown in the geometric analysis indicates the outer face of the missing assumed wall of an assumed average thickness of 1.35 m., the distance from the central water-hole to the outer face of the missing assumed wall ought to be 3.455 m.

The square marble pattern is in line with the inner face of the assumed wall with a tolerance of $1,445 - 1,350 = 95$ mm. (see drawing QAI/A12).

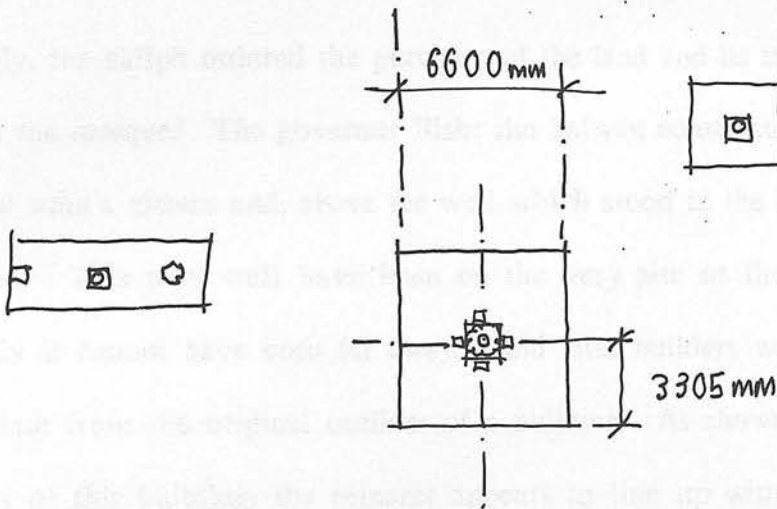


Fig. 4.6.3

Ḥassān ibn al-Nu'mān, c. A.D. 703

Ḥassān ibn al-Nu'mān is believed to have pulled down 'Uqbah's Mosque in its derelict state with the exception of the *mihrab*¹ and in A.D. 703 to have inaugurated a restored mosque. On this basis it seems likely that Ḥassān's reconstructed mosque was similar to 'Uqbah's original edifice.

Hishām's Mosque, c. A.D. 724

Bishr ibn Safwān wrote to the new Caliph Hishām stating that the mosque at Qayrawān was no longer large enough to hold the assembly of the faithful and that immediately to the north of the mosque was a vast garden. In his reply, the caliph ordered the purchase of the land and its inclusion within the area of the mosque.² The governor Bishr ibn Safwān constructed in the centre of the new *ṣaḥn* a cistern and, above the well which stood in the included garden, a minaret.³ This may well have been on the very site of the present minaret; certainly it cannot have been far away. And later builders were often reluctant to deviate from the original outlines of a building. As shown in the geometric analysis of this building, the minaret appears to line up with the centre of the *mihrab*. Further observations relating to the purpose of the minaret will be

1. *Ibid.*, p.209.

2. *Ibid.*, p.211.

3. *Ibid.*

discussed in more detail towards the end of this section.

With regard to the extent of Bishr's prayer hall, there appears to be no historic record. However, one can perhaps deduce that for the governor to request urgent extension must have meant that the mosque was not large enough to accommodate the much enlarged congregations of the two *'īd* prayers (see drawing QAI/A5). Normally these extra worshippers – who tripled the normal numbers of worshippers – would be accommodated in the courtyard; hence the usual proportions of one-third *zulla* and two-thirds *sahn*.

One can only assume that Bishr's extension ended roughly where the *zulla* ends now, especially as the minaret's location in relation to the well upon which it was built may well have remained a fixed reference.

Bishr ibn Safwān's Extension

In examining the present plan in detail, one could deduce that having obtained permission to purchase the land to the north of the old mosque, Bishr proceeded to mark out the ground for his extension. Squaring yet another circle and dividing yet another square, with the help of his architects/geometers, Bishr achieved a layout which became the foundation for all later reconstructions.

Analysis drawing QAI/A6 illustrates the likely steps Bishr's geometers

would have taken, marking out the ground for the great extension.

First, draw circle 'B' overlapping the original master circle 'A' from centre '53' with radius 53,50, which has the same radius as original master circle 'A'.

Second, square circle 'B' in the usual manner (see chapter 2.1 on systems of proportion).

Third, draw in the standard alternate and consecutive concentric squares initiated by the main diagonals 47,40 and 46,41 crossing at the centre of the second master circle 'B'.

Bishr and his architects/geometers had thus, despite all the constraints, achieved a mosque plan of perfect proportion, i.e. with one third prayer hall and two thirds courtyard. All this assumes, of course, that the dimensions of Bishr's *zulla* were identical with those we see in today's *zulla*.

Arithmetical Check

As a result of our geometric analysis (see drawing QAI/A6 in vol. II), we concluded that the overall plan of the Great Mosque conforms to the geometry of two interlocking circles, demonstrating an overall proportion of 2:3.

According to Creswell, side 3-4 (the mosque's width), as shown in

drawing QAI/A6, measures 70.26 m.¹ If we then add the thickness of the outer walls including their piers (but excluding the corner salients),

$$70.26 + 1.15 + 1.03 = 72.44 \text{ m.}$$

$$72.44/2 = 36.22 \text{ m. (unit measure).}$$

Side 47-3 (the mosque's original length),² as given by Creswell,³ measures 109.63 m. If our hypothesis regarding the mosque's geometry is correct, then the 2:3 proportion would be proved by multiplying the unit measure 36.22 by 3 to give the overall length, thus:

$$36.22 \times 3 = 108.66 \text{ m.}$$

Compared with the published dimension of 109.63 m., this gives a margin of 970 mm., which is less than 1 %.

However, we seem to obtain slightly better results using our own measurements taken with the help of the Department of Antiquities in Qayrawān. This makes half the width, i.e. from the mosque's central axis to the external face of the northern walls containing Bāb Lallā Riḥānah, 37 m. This forms the unit measure. The length of the mosque from the *qiblah* wall (internal face) to the face of the minaret facing the *qiblah* is 111.89 m.

$$37 \text{ m.} \times 3 = 111 \text{ m.}$$

$$111.89 \text{ (own measurements)} - 111 \text{ (geometrically)} = 890 \text{ mm. margin.}$$

1. *Early Muslim architecture*, fig. 180 (Qairawān).
2. That is to say, leaving out the minaret which is proud of the mosque's western wall. See the following section on the location of Qayrawān's minaret.
3. *Loc. cit.*

Either of the above results appears to be good enough, since they leave less than a 1 % margin. This encourages us to take seriously the geometric schemes proposed for this building and not to consider them as mere lines on a drawing.

The Minaret

Location

Bishr and his architects/geometers must have discussed in detail the location of their proposed minaret and they must have agreed that it ought to be central along the *qiblah* axis, lending this grand extended composition the correct balance horizontally and vertically. This practice is in line with the great architecture of earlier periods such as that underlying the ziggurats of Mesopotamia and the temples of ancient Egypt¹ where great emphasis was placed on the central axes to celebrate entrances and inner chambers.

Having agreed to locate the minaret in a generally central position and not on either of the north corners, the next decision for Bishr and his architects was whether the minaret should be centred absolutely exactly on the *qiblah* axis or positioned slightly on one side of the central axis. If the latter option were chosen, then there would also be the question of which side, right or left, it

1. Banister Fletcher, *A History of Architecture*, p.54.

should be positioned.

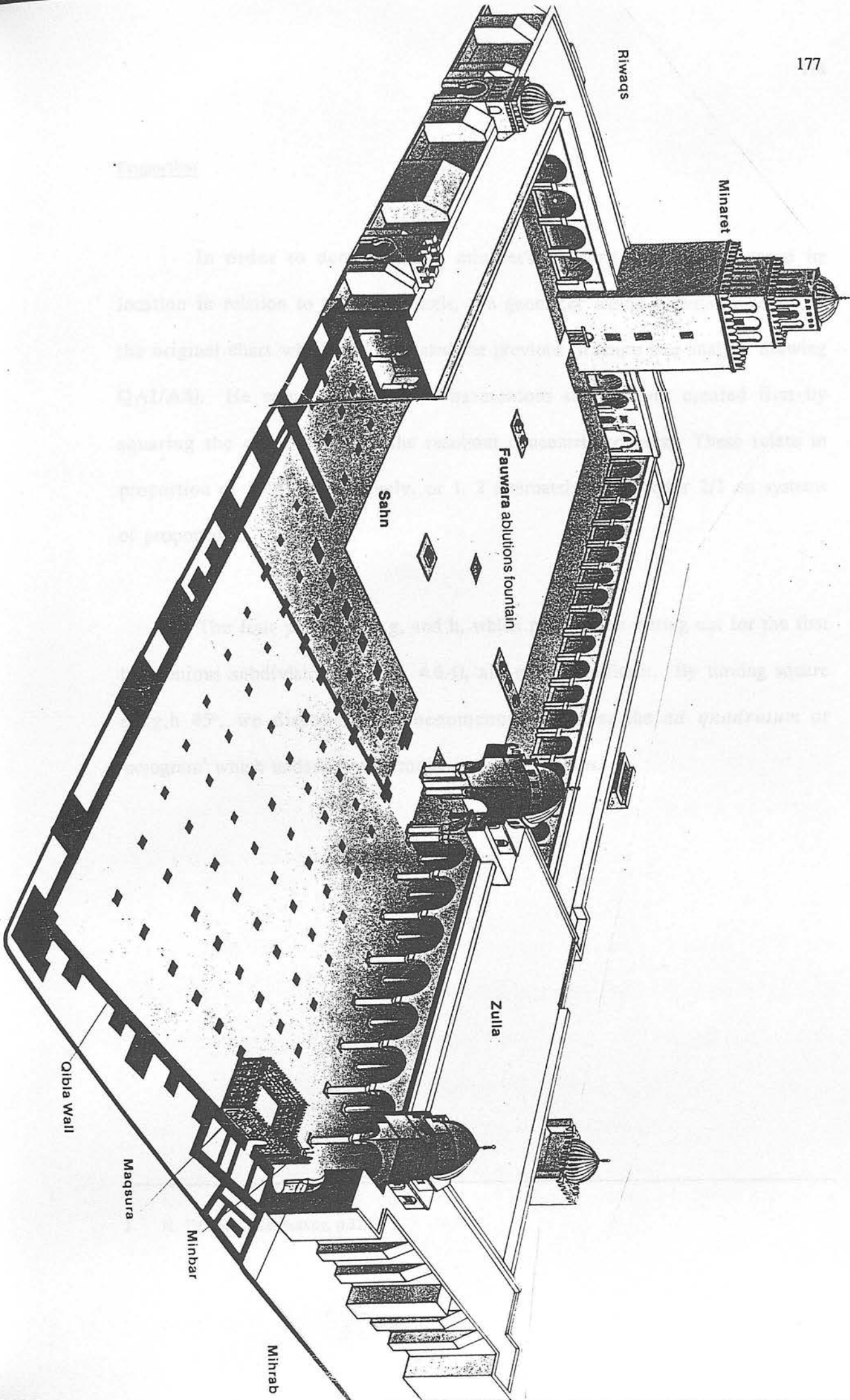
The solution to the problem is inherent in the basic understanding of the sequential order of precedence in the obligations of congregational prayer. In reverse order, these are:

3rd- The call for prayer 'component' (minaret).

2nd- The delivery of the sermon 'component' (*minbār*).¹

1st- The *imām*'s leading in prayer 'component' (*mīhrāb*).

1. The *minbār* (pulpit) must be positioned to the right-hand side of the *mīhrāb* (facing Mecca).



Proportion

In order to determine the minaret's proportions, having agreed its location in relation to the *qiblah* axis, the geometer would invariably return to the original chart which had regulated the previous structure (see analysis drawing QAI/A4). He would examine the harmonious subdivisions created first by squaring the circle, then by the resultant concentric squares. These relate in proportion of 1: $\sqrt{2}$ consecutively, or 1: 2 alternately (see chapter 2/1 on systems of proportion).

The four points e, f, g, and h, which produce the setting out for the first harmonious subdivision (see fig. 4.6.4), are quite significant. By turning square e,f,g,h 45°, we discover the phenomenon known as the *ad quadratum* or 'octogram' which underlies very many ancient structures.¹

1. N. Pennick, *Geomancy*, p.125.

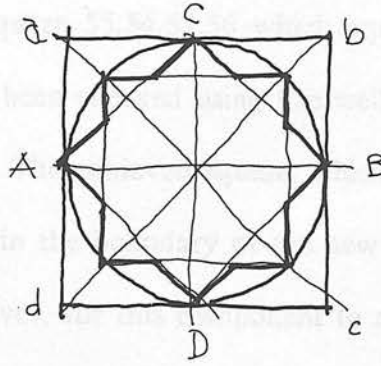


Fig. 4.6.4

Only through its applications in the decorative arts of Islam¹ did this figure become well known to the Western world.²

The geometer finds the width of this subdivision interesting as it may be sufficient for the size of a minaret. He then projects this band all around, including the area of the new extension (see QAI/A7).

Grid drawing QAI/A13 indicates the intersection of side 1,54 with connecting line between 59 and 60 which resulted from the squared circle B cutting the main diagonals of square 46,47,41,40. This results in a triangle 54,57,56 by extending the bisecting-diagonal 57,58 to meet side 46,47.

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1. See Creswell, *Early Muslim Architecture*, vol.II, pl.86A (122 and 129).
 2. One hopes that this thesis may go some way towards redressing this balance and proving its application also in Muslim architecture.

We now have square 55,54,57,56 which equals the area of the present minaret's base. This has been checked using Creswell's original measured survey drawing (copy attached). The achieved square, which will act as the base for the minaret, is naturally within the boundary of the new extension (squared circle B) (see fig. QAI/A7). However, for this component to act as a true 'tower' of great height celebrating the *qiblah* axis, it needs to stand freely or almost freely (see fig. 4.6.5).

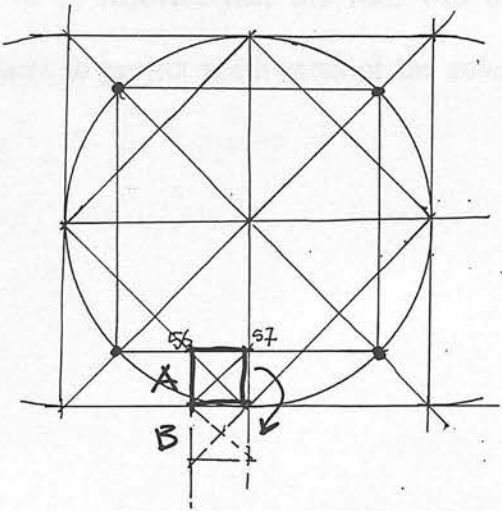


Fig. 4.6.5

The solution, as we see in the detailed geometric analysis, must have been to 'flip over' this square portion B in order to make it fall outside the mosque's external boundary. But the process whereby this was done was by no means casual (see drawing A7).

This disposition of the minaret was taken a stage further in Sāmarrā',¹ where the minaret is central on the *qiblah* axis and now stands freely away from the mosque's *ṣaḥn* wall (see § 3/16).

At present, Qayrawān's minaret does not project proud of its wall, as suggested in fig. 4.6.5, except that if one examines pl.22,² one can see from a distance that the wall differs in shade to the materials of which the minaret is constructed. This suggests that the minaret and wall were built in different periods, thus allowing us to theorize that the wall was originally built further south, enabling the minaret to project northwards of the *ṣaḥn* exterior wall.

1. See pl.63a, b in Creswell, *Early Muslim Architecture*, vol.II.

2. Photograph taken in April 1991 after recent restoration by the Dept. of Archaeology when layers of plaster and paint had been removed (cf. old photographs in Creswell, *Early Muslim Architecture*, vol.II, pl.46b).

Geometric Analysis of the Proportions of the Minaret

The Plan

Adopting Creswell's base plan¹ for the minaret (10.67 m. sq.) as a pure geometric shape, we draw in the square's diagonals in order to establish its centre. From this centre we draw a circle touching the four sides (see drawing QAI/A14.1, 14.2, and 14.3). In order to establish the vertical and horizontal axes, we draw an arc from corner no.4 to the centre of the circle cutting side 4,3 at A and side 4,1 at D, and we draw another arc from corner no.2 cutting side 2,1 at B and side 2,3 at C. Then we connect A with B and C with D. We then draw in six concentric squares in the usual manner.

Having drawn the above diagram on transparent material, we superimpose the drawing on Creswell's second plan showing level two of the minaret and we immediately notice that the first concentric square (5,6,7,8) corresponds exactly with the base plan of the middle section of the minaret at Qayrawān.

Superimposing the same diagram on Creswell's third plan of the top section of the minaret, we discover that the four sides of the square plan correspond directly with the second concentric square (9,10,11,12).

1. *Early Muslim Architecture*, vol.II, fig. 568.

Arithmetical cross check

As discussed in Chapter Two (on systems of proportion), the consecutive sides in the series of concentric squares relate in a $1:\sqrt{2}$ proportion, whereas the alternate squares relate in a $1:2$ proportion. Therefore, the measured width of the main minaret, i.e. 10.67 m., ought to relate to the width of the middle section in a $1:\sqrt{2}$ proportion..

$$10.67/7.63 = 1.398 \text{ m.}$$

$$\sqrt{2}/1 = 1.414.$$

The base of the top section of the minaret ought to relate to the base of the main trunk according to our geometric formula in a $1:2$ proportion.

$$10.67/2 = 5.335$$

$$\text{measured width} = 5.48.$$

There is thus a tolerance of 145 mm., which is a satisfactory reading considering the age of the minaret. Thus the widths of the three stages of the minaret relate to each other in a relationship of $1:\sqrt{2}$ (first stage to second stage) and $1:2$ (first stage to third stage).

The Elevation

The elevation of the minaret is best decoded when placed at a certain level within the concentric squares diagram.

After circling a square 1,2,3,4 that represents the base of the minaret at Qayrawān three times concentrically (see QAI/A15), you arrive at a square (-9,-10,-11,-12). Now extend sides 1,4 and 2,3, which represent the width of the minaret's base. Also extend sides 6,7 and 5,8 of square 5,6,7,8 which represents the base to the minaret's middle section. Also extend sides 10,11 and 9,12 of square 9,10,11,12 which represents the base of the minaret's top section. Thus the squares which define the ground plans of the minaret's three storeys all line up with the elevation itself.

Now draw arc no.2 from corner -11 and radius -11,B (see drawing QAI/A16). When arc no.1 meets the extended side 1,4, this gives the height of the main trunk of the minaret. When arc no.3 with radius -11,C meets extended side 5,8, this gives the height of the minaret's middle section. When arc no.4 with radius -11,D meets again extended side 5,8, this gives the height of the minaret's top section. When the same arc no.4 is allowed to carry on, meeting the vertical Y-Y1 axis, this gives the pinnacle of the dome. Arc no.3 can also give the pinnacle of the dome where it intersects the extension of side 2-3 at 5.

If one circles square -9,-10,-11,-12, one arrives at square -13,-14,-15,-16 which cuts the minaret's courtyard elevation at a level which appears to correspond to the height of the *riwāq* wall, not including the parapet which may be a later addition. And, even more to the point, this is also equal to

the depth of the *riwāq*, for which Creswell gives the measurement 6.6 m. (see his fig. 180).

If we examine plates 28¹ and 29,² the above proposals, confirmed by Creswell's dimensions (see his fig. 568) are quite credible. The point at which side -15,-16 cuts the minaret is also the point where arc no.1 from centre -11 to centre A meets the minaret's main body (see QAI/A16).

Yazīd ibn Ḥātim's Mosque

Yazīd apparently demolished Bishr's mosque with the exception of the *mihṛāb* and rebuilt it in c. A.D. 772.³

Again we do not appear to have any references which would help us reconstruct the exact form of Yazīd's mosque, but, assuming it was built on the same foundation as the previous mosque, the new format would be exactly the same.

Ziyādat Allāh I's Mosque, A.D. 836

Ziyādat Allāh demolished the entire structure of Yazīd's mosque and

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1. A. Papadopoulos, *Islam and Muslim Art*, pl.114.
 2. J.W. Allan, *A Short Account of Early Muslim Architecture*, p.322, pl.203.
 3. Creswell, *Early Muslim Architecture*, vol.II, p.211.

even ordered the *mihrāb* to be removed. This decision was later reversed by adopting a unique solution, whereby the original *mihrāb* was contained behind the new construction.¹ This at least gratified those who objected to his original plan of destroying 'Uqba's *mihrāb*.

The new mosque was constructed c. A.D. 836 and down to this day the Mosque of Qayrawān is believed to have remained just as Ziyādat Allāh left it with the exception, of course, of later additions supplied by Abū Ibrāhīm and Ibrāhīm II.

The Work of Abū Ibrāhīm, A.D. 862-3

Abū Ibrāhīm constructed the dome above the *mihrāb* and decorated the new *mihrāb* with marble panels.² The only link with our geometric analysis exercise is the diameter of the dome above the *mihrāb*, for we shall see later that this diameter relates proportionally to the overall size of the mosque.

The Work of Ibrāhīm II, A.D. 875

Ibrāhīm prolonged the naves of the sanctuary and constructed a portico

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1. *Ibid.*, p.213. On visiting the mosque today, one is told that the *mihrāb* visible through the marble fretwork is the original *mihrāb* which Ziyādat Allāh agreed to preserve.
 2. *Ibid.*

at the end of the nave. The dome is known as Qubbat Bāb al-Bahū¹ (see drawing QAI/A8). Creswell also attributes the additional line of columns and arches flanking the inner face of the central aisle to Ibrāhīm II. We find this is compatible with our own analysis (see analysis drawings QAI/A10 & A13), where sides 29,32 and 30,31 of the smaller concentric square 29,30,31,32 are in line with the outer face of the inner central aisle columns. (See also the observations made at the end of this section.)

The Ṣaḥn and Riwāqs

It would appear that the arcades bordering the *ṣaḥn* are part of the restoration work carried out by Abū Ḥafṣ after Qayrawān had been neglected for over two centuries due to the Hilālī incursions in A.D. 1054–5.²

Geometric analysis shows that this proposal is compatible with the way that the mosque had developed (see QAI/A17). That is to say, the *riwāqs* were the last additions which, we believe, resulted in including the minaret structure within the mosque compound and marked the end of its projecting proud of the courtyard's northern wall.

1. *Ibid.*

2. J. Allan, *A Short Account of Early Muslim Architecture*, p.323.

General Observations

1. In geometric analysis drawings QAI/A4-A10, side 4,3 of the unit square which squares circle 'A' appears to give the best geometric results in the total analysis, when in line with the inner face of the *qiblah* wall. Subsequently, side 1,4 of the same square is not in line with the eastern wall's inner face, but it is in line with the outer face of the eastern external buttresses.

We have adopted the inner face of the *qiblah* wall as a reference and have observed how this has led to a satisfactory total analytical diagram for the entire complex. If we add to this the fact that the above discrepancy is not applicable to the entire length of the eastern wall (see QAI/A4-A10), we may conclude that side 1,4 of the unit square is in the correct position in relation to the mosque's eastern wall.

2. The Great Mosque at Qayrawān is one of the few examples of early Muslim architecture examined in this thesis where, in terms of its geometric structure, a unifying master circle encompassing the entire layout is not in evidence. Instead, what appears to have generated the present mosque's plan is a pair of two interlocking circles which, when circumscribed by straight edges,

form the current layout with the minaret attached externally.¹

3. In an effort to locate the courtyard's central decorative marble piece and cross-checking this with Creswell's measured survey plan,² one discovers that the extreme edges of the square marble pattern in the courtyard line up with the outer face of the inner row of columns which flank the central aisle. These dimensions also appear to correspond with the area of the central unit's square 29,30,31,32 in the geometric analysis drawings QAI/A13 and QAI/A9.

4. In geometric analysis drawing QAI/A12 the unit square which squares circle B is developed to show the usual concentric squares. One observes that all the water-holes which may appear haphazardly arranged within the courtyard must in fact have been predetermined, as each water-hole relates to a particular concentric square within the overall pattern. Clearly it is worth studying the courtyard floor in these early mosques.

5. Having concluded through geometric analysis that the minaret of the Great Mosque at Qayrawān must at one time have projected out of the mosque's original northern wall opposite the *qiblah*, it was interesting to discover that

1. Externally, that is, in terms of the geometric layout.

2. *The Minaret in the Maghrib*, p.91.

recent UNESCO investigations¹ suggest that the ante-*qiblah* wall at Qayrawān once stood some 17 m. inside the present walls. Although we would welcome the UNESCO team's specific archaeologically-based conclusions, geometrically we can still confirm that the original wall once stood some 10 m. inside the present wall (i.e. width of the minaret).

6. Finally, with regard to the obliqueness of the western wall and its angle in relation to the *qiblah* wall, this feature was also examined on site and perhaps plates 18 and 19 may help illustrate the conclusions.

First, three measurements were taken across the street which separates the mosque from the city fabric. One measurement was taken outside entrance F, the second outside entrance D, and the third at entrance B (see drawing QAI/A18 and plates 20 and 21). These were 5.79 m., 5.8 m., 5.79 m. and 6.09 m., proving that the Great Mosque's western wall is parallel to the façade of the dwellings opposite.

Second, an attempt was made to compare the red mortar behind the plaster on the dwellings opposite and the red mortar of the mosque wall which appeared to be of a similar texture and colour. The accompanying guide could not give a date for these dwellings.

1. *Ibid.*

In conclusion, one presumes that either 'Uqba or those who came after him respected an existing fabric. This is not unusual in Islamic architectural planning (see below § 3.20 on al-Aqmar Mosque).

Finally, it is gratifying to be able to decode the plan of the Great Mosque of Qayrawān and to find that the 2:3 proportion, elevated through the principle of the square and the circle and confirmed by actual dimensions, is at the heart of its geometry.

The Great Mosque of Qayrawān is a rectangular building, 112.85 m. long north to south and 160.05 m. wide east to west, with a great courtyard in the centre of each side. The main building of the palace is 112.85 m. long north to south and 160.05 m. wide east to west. The wall of the inner enclosure on the north side is provided with round towers and its main entrance is on the west side, with the main entrance of the palace building.

The palace building is a rectangular building, 112.85 m. long north to south and 160.05 m. wide east to west. The main building of the palace is 112.85 m. long north to south and 160.05 m. wide east to west. The wall of the inner enclosure on the north side is provided with round towers and its main entrance is on the west side, with the main entrance of the palace building.

1. *Journal of Islamic Architecture*, 1980, p. 11.
 2. *Journal of Islamic Architecture*, 1980, p. 11.
 3. *Journal of Islamic Architecture*, 1980, p. 11.

4.7 UKHAYDIR

The origins of this palace are still unclear, for there is neither a relevant Arabic text nor an inscription *in situ* to help establish the identity of its builder. Jairazbhoy,¹ basing his conclusion on architectural observations, asserts that there can be no doubt that it was built around the year A.D. 750 and that there are indications that it might have been constructed even earlier than the round city of Baghdad. Creswell, on the other hand, dates it to around 780.²

Ukhayḍir consists of a main enclosure measuring about 174.74 m. from north to south and 169.05 m. from east to west,³ with a great gateway in the centre of each side. The main building of the palace itself measures 112.85 m. from north to south and 81.83 m. overall from east to west. The wall of the inner enclosure on the three faces is provided with round towers and its main entrance forms one with the northern entrance of the outer enclosure.

The palace contains a mosque, a main court (the Court of Honour), audience halls, and four *bayts* (self-contained units consisting of a courtyard off

1. *Outline of Islamic Architecture*, p.55.

2. *Early Muslim Architecture*, vol.II, p.61.

3. *Ibid.*, p.52.

which open *iwāns* and flanking rooms on two sides).¹ The entrance hall leads into a square room with a fluted dome followed by a vaulted wall with arched recesses. South of this room is the Court of Honour, decorated with blind apses, patterned in geometric brick designs. This leads into the vaulted official reception room of the palace.

In the eastern part of the palace yard is an annex.² The building itself stands free, but if its north façade is prolonged, one end touches a pilaster of the main enclosure wall and the other end touches a tower of the palace wall. In both cases, it merely touches it, for it is not bonded in. This led Creswell to conclude that the annex is a later addition,³ but due to its close resemblance to the rest of the palace, he accepted that it could not be very much later. The geometric analysis UKHA/A4 appears to support this, as it shows that the proportion of the annex is almost a minute replica of that of the main complex.

The Site

Ukhayḍir is located in the desert of Wadi 'Ubayd about 100 miles (120 km.) south-west of Baghdad. The name in Arabic (أخيضر) means 'the Green One'. The site lies on a sheet of water just below the surface of the sand which helps maintain a patch of vegetation.

1. Michell, *Architecture*, key to monuments, p.251.

2. Possibly for the use of retainers, or horses.

3. *Early Muslim Architecture*, vol.II, p.80.

Observations

- 1) The orientation of the main axis of Ukhayḍir is towards the cardinal points.
- 2) It seems probable that the outer walls were intended to form a perfect square from which the proportions of the other parts were derived (1:√2 ratio).¹ Creswell states that the palace consists of a fortified *rectangular* enclosure measuring 174.74 x 169.05 m., but he also records the description of one of six English travellers,² Carmichael, who said that 'the envelope or surrounding wall, is an exact square, whose sides face the cardinal points of the compass, each measuring 700 feet being 250 of my paces.'
- 3) Repeating the plan units in both north and south sides of the court of Ukhayḍir was perhaps intended to take the most favourable advantage of the seasons by providing both summer and winter quarters for the household.³

This strict symmetry of planning was not so rigidly followed in later Muslim palaces, for example the Palace of Balkuwāra in Sāmarrā'.

1. See analysis.
 2. *Early Muslim Architecture*, vol.II, p.51, n.11.
 3. *Ibid.*, p.71.

Geometric Analysis

Despite the references quoted which describe the building as a square structure, Creswell describes Ukhaydir as a rectangular enclosure measuring 174.74×169.05 m., i.e. a 5.69 m. divergence from a perfect square. It is hard to decide whether the 5.69 m. difference developed as a difference in wall thickness between the eastern and northern outer walls, or whether it was the combination of a small error in the crumbling structure with the method of measurement actually employed.

Using the same geometric principles employed in analyzing the structures chosen in the rest of the study, where concentric squares with consecutive parallel sides are related in the proportion of $1:\sqrt{2}$ and the concentric squares with alternate sides are related in the proportion of $1:2$,¹ we observe, as indicated in geometric analysis drawing no. UKHA/A3, that JK is twice GH and GH is twice the length of EF. EF is twice the length of CD and CD is twice AB.

AB = 9.75 m. Therefore, based on the site measurements of Creswell (see analysis drawing UKHA/A6),

$$CD = 19.5 \text{ m.}$$

$$EF = 39 \text{ m.}$$

$$GH = 78 \text{ m.}$$

1. See chapter 2 on systems of proportion.

$$JK = 156 \text{ m.}$$

This can be checked using the scale shown on Creswell's plan (UKHA/A1). Therefore the building was clearly intended to be square.

A further check is to add the thickness of the external walls¹ to see if the total dimension matches Creswell's total measured dimension.

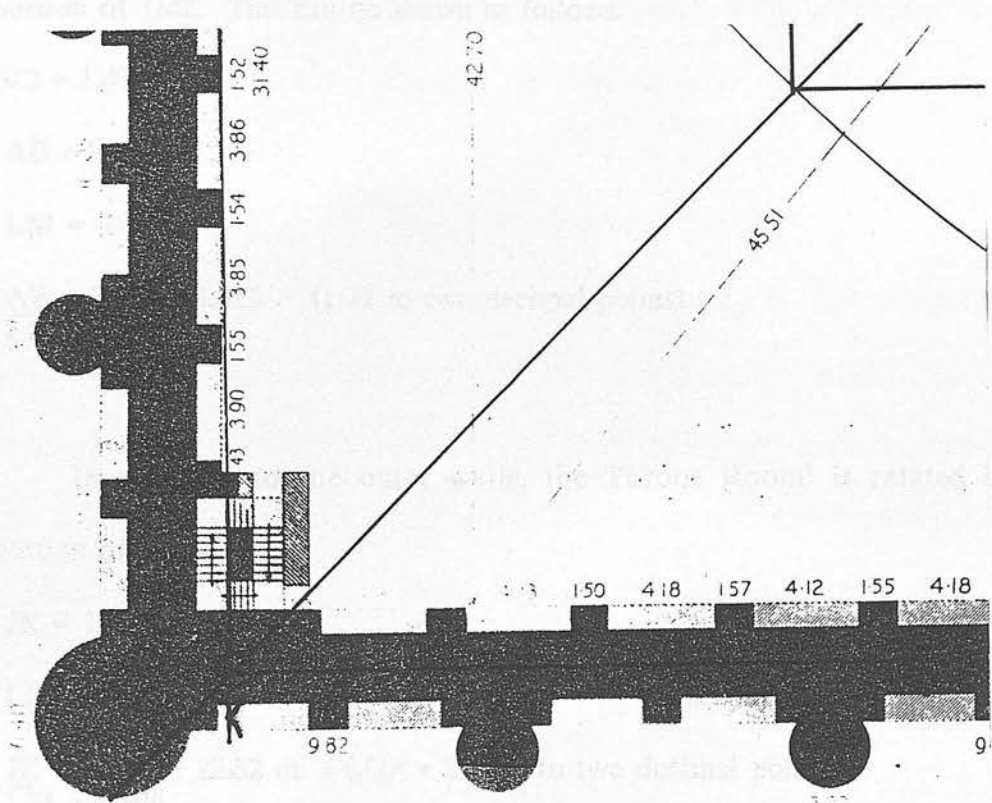


Fig. 4.7.1

$$6.4 \times 2 = 12.8 \text{ m. to include both corners.}$$

$$156 \text{ m.} + 12.8 \text{ m.} = 168.8 \text{ m. total length geometrically.}$$

1. This is because the geometric analysis takes into consideration the external walls from the inside face.

Creswell noted a total external wall length of 169 m. Therefore, the geometric system of the analysis proves that the external wall relates geometrically to the inner structures in the proportion of 1:2.

The external walls also relate to parts of the inner building in the proportion of $1:\sqrt{2}$. This can be shown as follows.

$$\sqrt{2} = 1.414$$

$$AB = 9.75$$

$$LM = 6.896$$

$$\frac{AB}{LM} = \frac{9.75}{6.896} = 1.413 \quad (1:\sqrt{2} \text{ to two decimal points})$$

In relation to the outer walls, the Throne Room¹ is related in the proportion of $1:(\sqrt{2})^8$.

$$JK = 156 \text{ m.}$$

$$LM = 6.896$$

$$\frac{JK}{LM} = \frac{156}{6.896} = 22.62 \text{ m.} = (\sqrt{2})^8 = 22.627 \text{ to two decimal points}$$

1. In Creswell's plan it is indicated as room no. 30.

General observations

1) Analysis drawing number UKHA/A5 illustrates the main complex of the palace (.1, .2, .7, .8), which is arrived at by drawing an arc from .8 starting from .6 and cutting .8, .1 at A. This method appears to have been reversed to give C,D. That is to say, an arc is drawn from .2 starting from .4 cutting .7, .2 at C. D, A, B, C (and *not* .8, .1, .2, .7 as one would expect) is a unique rectangle reflecting a plan of a unique palace, as if the *amīr* had instructed his architect/geometer to produce a plan no one could emulate. A thin corridor of space is left empty between the inner and outer walls.

2) With regard to the annexe, we see in geometric analysis UKHA/A4 that square 41,9,42,43, which contains the annexe is an exact replica of square 17,18,19,20 reflecting the heart of the main palace. In order to fix the annexe's location, its square was shifted forward to the new location a,b,c,d (see drawing UKHA/A7 & A8). Once this is achieved, a series of concentric squares were established for square a,b,c,d where square 49,50,51,52 represents the main body of the annexe and square 53,54,55,56 appears to correspond to the annexe's central hall (see drawing UKHA/A4 & A8).

To summarise the geometrical calculations given above, the width of the throne room (Room 30 in Creswell's plan) can be shown to be the basis of a whole series of subsequent larger dimensions culminating in the length of the outer enclosure wall.

Among the examples considered for analysis in this chapter, few, with the exception of the Dome of the Rock, have been geometrically analysed by others to a level which is sufficient for detailed comparison. However, in the case of al-Ukhayḍir, we find two completely different analytical approaches: one by El-Said and Parman¹ and the other by Alvaro Soler and Juan Zozaya.²

El-Said and Parman's analysis of al-Ukhayḍir is unfortunately quite elementary (see fig. 4.7.2), perhaps because the work in which the analysis appears is mainly concerned with two-dimensional decorative geometric patterns and not with analysing architectural plans. Nevertheless, we list below our general comments on their efforts to decipher the plan of al-Ukhayḍir.

1. The analysis appears to have been carried out only on the central section of the palace, disregarding the most important element, i.e. the outer main square enclosure, without which, as shown in our analysis,³ the inner buildings could not have been generated.
2. The main 'circle' used by El-Said and Parman to analyse al-Ukhayḍir, if squared as part of the process described in their geometric concepts, would relate to nothing either in or outside the compound of the palace.

1. *Geometric Concepts in Islamic Art*, p. 126.

2. "Castillos Omeyas de Planta Cuadrada su Relación Funcional", in *Congreso de Arqueología Medieval Española III*, 27 March-1 April 1989 (Universidad de Oviedo).

3. Thesis, vol. I, chapter IV, section 7; and vol. II (drawings - al-Ukhayḍir).

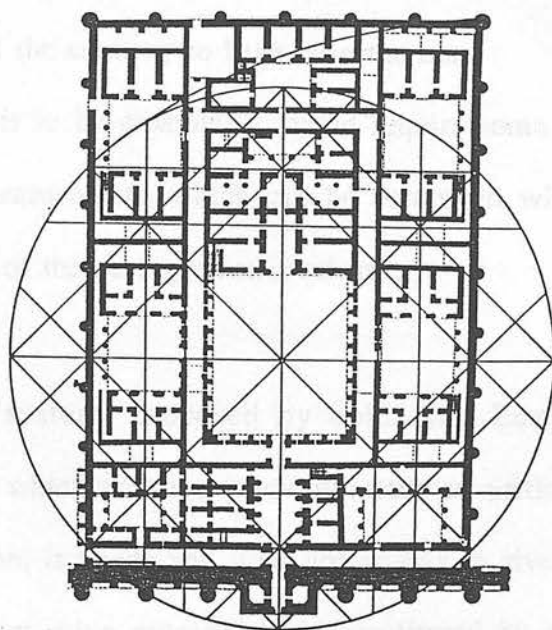


Fig. 4.7.2

3. Although the geometric scheme which was superimposed on al-Ukhaydir by El-Said and Parman appears in some instances to align with parts of the inner building, it does not in any way confirm the proportion of the courtyard, nor does it sympathize geometrically with the line of the external wall of the palace complex.
4. The arch shown starts from the top left-hand corner, intersects the extended side b-c, and results in the main rectangle abef. This positive observation did not require the complex geometric diagram used for the analysis.
5. The way El-Said and Parman attempted to analyse al-Ukhaydir could give the misleading impression that the inner building was conceived separate from the main great square enclosure, including a sizeable annex. However,

we have proved the contrary to have been the case.¹

6. For this analysis to be credible, it would require some mathematical check using actual measurements, which can be compared with figures reflecting the proportions of the pure geometric scheme.

The other system, proposed by Soler and Zozaya,² for analysing al-Ukhayḍir is one which is based predominantly on arithmetic and it is one which, on our opinion, is vague and does not appear to give any accurate results like those obtained by using geometric grids confirmed by actual measurements. Although Soler and Zozaya concentrate mainly on the analysis of Qaṣr al-Ḥayr, but according to their findings, the system applies also in the case of al-Ukhayḍir, Qaṣr al-Minya, Mshattā, Dār al-Imāra in Kufa, and most of the Umayyad buildings, with their typically square or rectangular ground plans.

Basically, the analysis transmits or represents the proportions or characteristics of a ground plan, independent of dimensions, onto a square grid of 'numbers', usually known as the Vedic square.³

One square is formed on a nine-by-nine grid of numbers, one to nine horizontally and vertically from the top left corner. At the intersection of each of the vertical and horizontal columns is the product of the two numbers. When

1. See analysis, 4.7 Ukhayḍir, vol. II, p.41 of thesis (drawings)

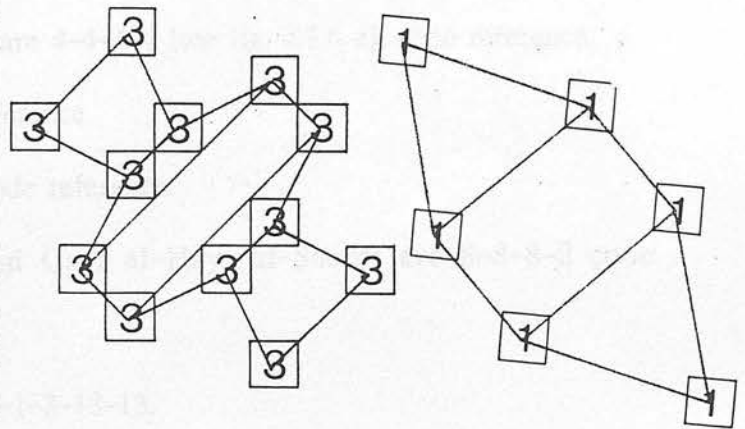
2. *Op. cit.*

3. K. Albarn, *The Language of Pattern* (London, 1974) p.8-12.

the product exceeds nine, the two digits are added together to form a simple digit (fig. 4.7.3a) and the grid is filled. As we shall see, this cabbalistic reduction of numbers larger than nine has many uses.

When we look at the completed square, we are aware of numerical relationships revealing characteristic visual patterns. Each number, line of numbers, or interconnection of like numbers has a specific form. Crucial to the understanding of the square is the recognition of the reflection of one and eight, two and seven, three and six, four and five: all pairs which, when added together, equal the remaining number, nine. Each vertical column of numbers has its identical partner in the horizontal column. Like numbers, once joined (figs. 4.7.3 b & c), reveal a similar pattern of reflection and, furthermore, these figures enclose a group of numbers which, when added up, can be reduced (by cabbalistic reduction) to that figure (excepting two and seven) whose enclosed squares total twice the figure.

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9



Figs. 4.7.3 a, b & c

The reflecting numbers have a unique character. The two and seven also form the centre of the one-to-eight square and, after reduction, total nine. Seven is the centre of the Vedic square, which may indicate the origin of its importance. (Seven is said to be the most commonly chosen number lower than nine, possibly echoing its one-time significance.) Two, of course, is the dual of seven. It is interesting to note that seven and two form the angle of the pentagon. Twenty degrees is the angle of a double nonagon. A figure squared employs a two. The diagonal of the square is equal to the long side of a root-two rectangle, with the side of the square as the shorter side. Twenty-two over seven is π .

What Soler and Zozaya did in the case of early Muslim castles was to transmit the plans of those buildings onto the square, using the number of towers in each building as reference or code. For example,

1. Minya, al-Gharbī, and Khirbat al-Mafjar are 3-3-3-3 (see fig. 4.7.6 e) code reference.
2. Al-Sharqī and Qaṣr al-Tuba are 4-4-4-4 (see fig. 4.7.6 e); code reference.
3. Mushattā is 7-7-7-7 code reference.
4. Al-Ukhayḍir is 7-10-7-10 code reference.
5. Dār al-Imāra in Kufa and Qaṣr al-Ḥayr al-Sharqī are 8-8-8-8 code reference.
6. Al-Ukhayḍir total plan is 13-1-3-13-13.

The system's main criterion is that a plan with corner towers, when transmitted

onto the square, will be counted twice. An entrance flanked by two towers will be counted as one. For example, the grand plan of Qaṣr al-Ḥayr may appear to be 3-3-3-4, but is in fact a 3-3-3-3 plan. Mshattā is 7-7-7-7, and so on.

Soler and Zozaya's work was inspired by two medieval miniatures¹ (see figs. 4.7.4 & 4.7.5) relating to Umayyad palace architecture in Spain. Soler and Zozaya believed that the peculiar manner in which the plan was drawn echoes the above-mentioned system. Figs. 1 and 2 show corner towers as two units, as they merely represent a technique reflecting some type of grid system.

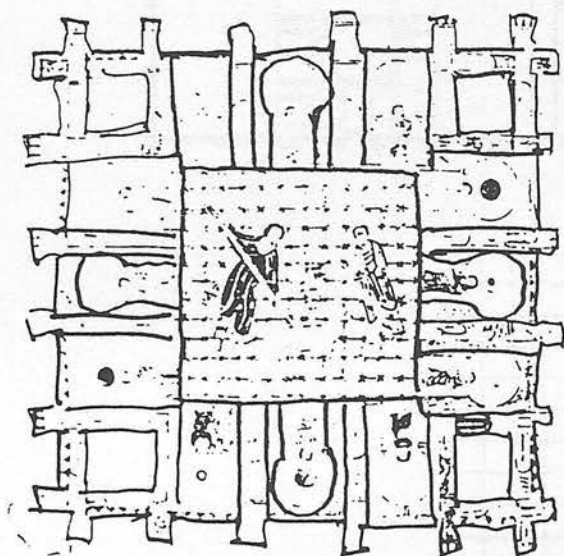


Fig. 4.7.4

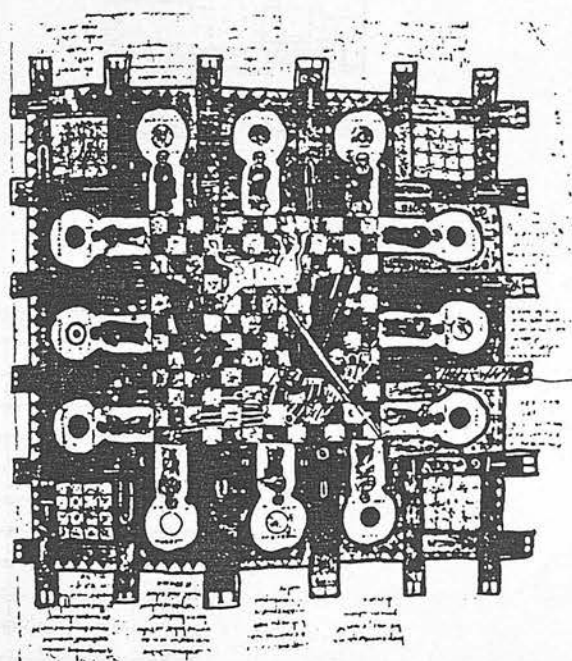
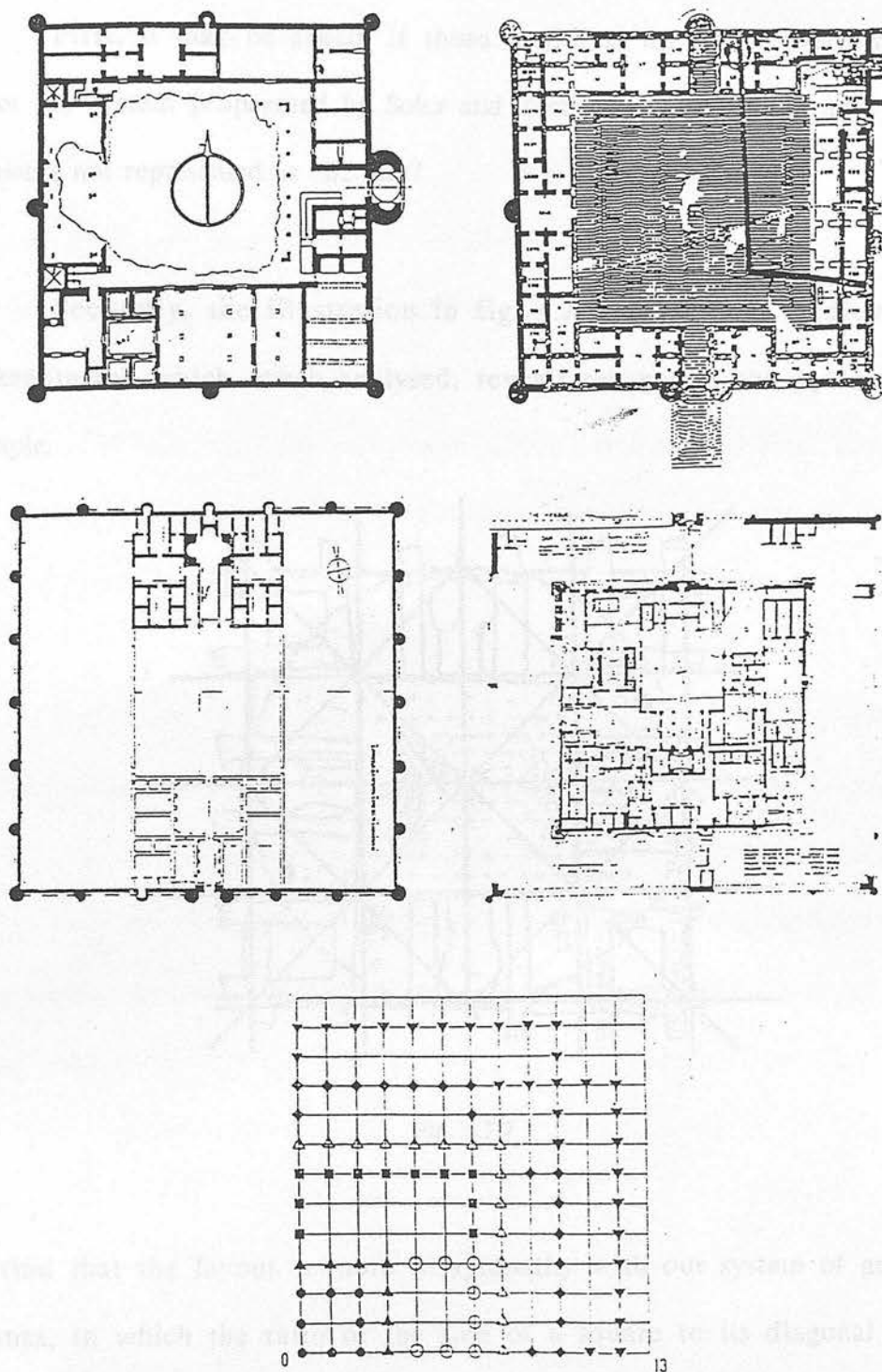


Fig. 4.7.5

1. Cited by Soler and Zozaya in Beatos Morgan, ref. F² 222 V.



Figs. 4.7.6/a Qaṣr al-Minya; 4.7.6/b Qaṣr al-Ḥayr al-Sharqī; 4.7.6/c Mshattā; 4.7.6/d Dār al-Imārah, Kufa; and 4.7.6/e Code reference

First, it may be asked, if those medieval miniatures were meant to reflect the system propagated by Soler and Zozaya, why is the entrance flanked by towers not represented as one unit?

Secondly, the illustration in fig. 4.7.7 indicates a medieval plan representation which, when analysed, reveals the use of the squared-circle principle.

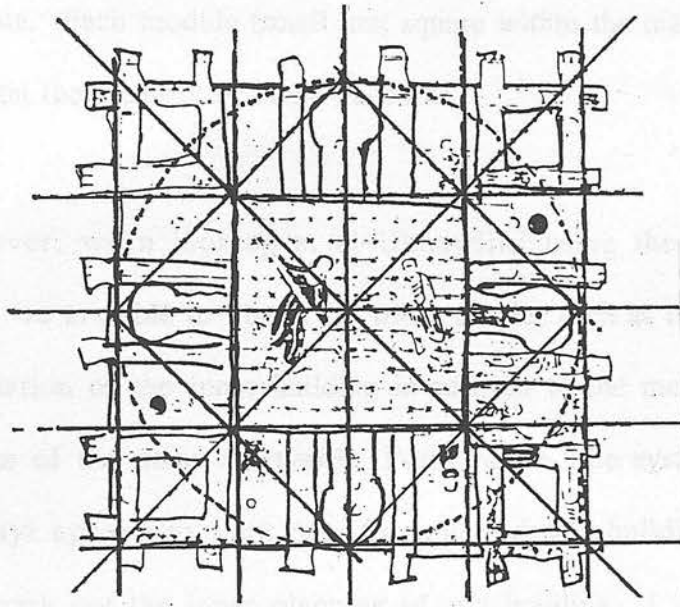


Fig. 4.7.7

We find that the layout is more in sympathy with our system of geometric schemes, in which the ratio of the side of a square to its diagonal is $1:\sqrt{2}$, especially when using the further extended sub-lines suggested by El-Said and

Parman¹ (see fig. 4.7.7).

In conclusion, we would very much like to accept this system, with its inherent simplicity, as explained by its authors. If the Caliph wanted to build a palace within a given site, the architect, in his wisdom, would simply have decided whether it would be according to the 6-6-6-6 module or the 13-13-13-13 module. As Soler and Zozaya explained, this module is a proportional one. Each module (small unit square within the main square) can be of any dimension the architect or client chooses.

However, when looking at al-Ukhaydir,² using the principle of the squared circle, we are able to obtain far more details, such as the position of the arcade, the relation of the inner building in relation to the main enclosure, and the proportions of the main courtyard. Furthermore, the system proposed by Soler and Zozaya appears to work only for square desert buildings and does not at any stage work out the inner planning of any building. I would like to see this system decode a plan of the complexity of Qayrawān or the Dome of the Rock.

1. *Geometric Concepts in Islamic Art*, p.

2. See our geometrical check on the dimensions in thesis, vol. I, chapter IV, part 7; and vol. II (drawing, ref. UKHA/A3).

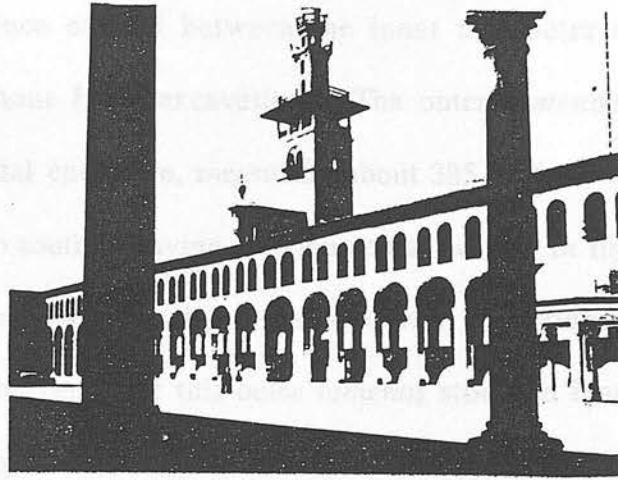
4.8 THE GREAT MOSQUE OF DAMASCUS

When Damascus became the capital of the Islamic caliphate in A.D. 661, the Muslim population was increasing, partly by immigration and partly by conversion to Islam. Under these circumstances, one of al-Walid I's first acts in the eighth century was the provision of a congregational mosque which would not only be adequate in size, but also rival the finest Christian churches in Syria. For its construction he gathered together skilful artisans from Persia, India, North Africa, and Byzantium.¹

Although the foundation inscription of the Great Mosque no longer exist, its text has been preserved by al-Mas'ūdī, who saw it in 332/934. It stated that the order to construct the mosque was given by al-Walid in 87/706.² At the time of the conquest of Damascus in A.D. 635, the inner colonnaded *temenos* of what had been the early first-century Temple of Jupiter Damascenus *surrounded*³ the Theodosian Church of St. John on or near the site of the original temple.

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1. Creswell, *Early Muslim Architecture*, vol.I, pt.i, p.151, where al-Maqqisī is cited as the principal historical authority.
 2. *Ibid.*, p.153.
 3. J.D. Hoag, *Islamic Architecture*, p.22. If we adopt the view that the present prayer hall is actually the old Church of St. John, this means that the present sanctuary should be surrounded by a colonnaded *temenos* on all sides.

Late in A.D. 706, al-Walīd obtained the church from the Christians and proceeded to demolish it with the exception of the four walls.¹



The prayer hall was divided into three broad aisles with gabled roofs and parallel to the south wall of the old inner *temenos* the *qiblah* was sited. Hoag² believes that this arrangement may well have evolved from the frequent habit of converting Syrian basilicas into mosques: 'Because Mecca was due south, the Muslim had only to pray across the aisles of a structure that normally pointed east. Except, the theory which states that the present prayer hall was the original Church of St. John appears to have long been discarded.'³

However, as we shall see later, there appears now to be sufficient new evidence for the case of the Church of St. John to be reopened.⁴

1. Creswell, *Early Muslim Architecture*, vol.I, pt.i, p.190.

2. *Islamic Architecture*, p.22.

3. *Ibid.*, p.25.

4. See Creswell, *Early Muslim Architecture*, vol.I, pt.i, p.183.

The Ancient Plan

What once existed between the inner and outer *temenos* cannot be determined without fresh excavations. The outer *temenos* was an immense, slightly trapezoidal enclosure, measuring about 385 m. from east to west and 305 m. from north to south,¹ having a monumental portico in the centre of the west side. A bazaar, sheltered by a portico, ran all around the interior, and presumably in the centre of this outer *temenos* stood an inner *temenos*, a great oblong measuring 157.5 m. from east to west and nearly 100 m. from north to south. At each corner of the inner *temenos* there was a square tower, of which only the south-western one remains intact. Also, the west side has been completely preserved, together with a considerable portion of the south and east walls and about 21 m. of the north. The southern part of the eastern wall is believed to have been entirely rebuilt, but the northern half has been partly preserved.² Creswell refers to Phené Spiers's modified opinion expressed in 1903 stating that

it is pre-Roman, and belongs to that type of Syro-Greek work which is found throughout Palestine...We have assumed that it may have been erected by Antiochus Cyzicenus (125-96 B.C.) because he seems to have been the first of the Seleucidae to make Damascus his capital [111 B.C.], and is likely therefore to have carried out these important works there.

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1. Creswell, *Early Muslim Architecture*, vol.I, pt.ii, p.156.
 2. *Ibid.*, p.157.

The Mosque

Creswell describes the features of the Islamic mosque constructed on the site of the pagan temple as follows.

THE EAST ENTRANCE. In addition to the triple entrance on the south side, there were three others, one in each face, the chief one being obviously that on the east side now known as the Bāb Jairūn. A great central doorway and two small side ones were placed under an enormous portico almost 16 m. deep and nearly 28 m. wide, preceded by a broad flight of steps.

The columns at the eastern entrance, like those of the interior, are Corinthian and are believed to have fallen down in 1858.¹ During works on the portico of the eastern entrance, carried out by the Syrian Department of Antiquities, it was found that the ancient street level was about 5 m. below the floor of the mosque. This means that the *temenos*, like that of Palmyra, stood on a plinth about 5 m. high.² Creswell describes the north entrance as follows:

THE NORTH ENTRANCE. The present northern entrance and the greater part of the north wall are not original for an inscription at A records the rebuilding, presumably of the eastern half, in 482 H. (1089)...The original northern entrance must have been in almost exactly the same position as the present one, for the axis of the door is 86.92 m. from the north-west corner, compared with 86.32 for the axis of the central southern doorway, a difference of 60 cm. only.³

1. *Ibid.*, p.161.

2. *Ibid.*

3. *Ibid.*, p.162.

Location of Original Church

There are a number of theories on which architectural historians appear to base their argument regarding the exact location of the original Church of St. John. Creswell,¹ for instance concludes that the present prayer hall of the Great Mosque of Damascus is far removed from the original site of the old church and that an independent building once stood in the middle of the inner *temenos*.

However, the more one analyzes the above statement in relation to the present plan, the more one finds it difficult to agree with Creswell, and the easier it becomes to identify with the theory of Dussaud, Watzinger, and Wulzinger that the present prayer hall is actually the old Church of St. John.²

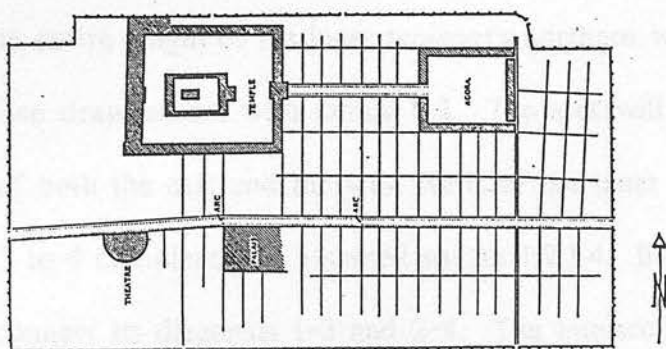


Fig. 4.8.1

1. *Ibid.*, p.191f

2. *Ibid.*, vol.I, pt.i, p.181.

Geometric Analysis

If we attempt to locate the centre of the present dome above the sanctuary, in relation to the entire complex, we find that it lies at the intersection of 45° diagonals projecting from each of the northern corners. Such 45° diagonals assume, however, that the temple (inner *temenos*) was a square and not a rectangle. In other words, geometry suggests that the present *qiblah* wall was not necessarily the limit of the ancient temple, though of course this is not to deny that the *qiblah* wall was part of that temple.

Geometrically, it is not difficult to create such accurate 45° diagonals on the ground. The first step is to create a square out of the present rectangular inner *temenos*. As shown in analysis drawing DAMA/A5, we complete this square by first drawing an arc from corner no.2 with radius 2-1. This distance represents the entire length of the inner *temenos*'s northern wall. Similarly, from corner no.1 we draw an arc with radius 1-2. The arcs will meet the extensions southwards of both the east and the west walls of the inner *temenos* at 3 and 4. Connecting 3 to 4 completes the assumed square 1,2,3,4. In order to establish its centre, we connect its diagonals 1-3 and 2-4. The intersection of the diagonals, as well as indicating the centre of the assumed square 1,2,3,4, also appears to correspond to the centre of the present sanctuary dome. The intersection points of the diagonals also tie in with the *qiblah* axis. Aligned with this axis are a number of elements within the prayer hall, such as the centre of the *mihrāb*, the

central aisle, the main entrance to the prayer hall, and a water fountain within the courtyard. Such a fountain is often found on the main *qiblah* axis in other mosques.

The *qiblah* axis can be arrived at by first drawing a circle within the assumed square 1,2,3,4 (see analysis drawing DAMA/A6). With the same radius, we draw arcs from each of corners 2 and 4. These meet the sides of the assumed square 1,2,3,4 at 5, 6, 7, and 8. The *qiblah* axis can now be established by simply connecting points 7 and 5. Points 7 and 8 appear to represent the longitudinal east-west axis of the present mosque, which is also the horizontal X-X1 axis of the assumed square 1,2,3,4.

Progressing with our theory of the assumed square 1,2,3,4 and by adding in our standard pattern of concentric alternate and consecutive squares, we immediately observe that square 21,22,23,24 corresponds with the width of the present sanctuary (see analysis drawings DAMA/A7 and A/8). This fact can be checked arithmetically using our standard formula of concentric squares in which consecutive parallel sides relate in a $1: \sqrt{2}$ proportion, while alternate parallel sides relate in a $1: 2$ proportion.

Based on Creswell's survey,¹ the internal width of the sanctuary = 38.3 m. - 1.7 m. (colonnade depth) = 36.6 m. $36.6 \text{ m.} / 2 = 18.3 \text{ m.}$, which gives us distance A (see analysis drawing DAMA/A7). $4A = 18.3 \times 4 = 73.2 \text{ m.}$ The distance which represents 4A in Creswell's survey reads as $19.15 + 47.87 + 6.2 = 73.22 \text{ m.}$ In other words, inductive geometry produces the same result as direct measurement to within 2 cm.

By proceeding inwards with our concentric squares, we find that square 33,34,35,36 corresponds with the square of the sanctuary dome (see analysis drawing DAMA/8). Again, this can be checked arithmetically using the above formula.

The side of the square which carries the dome measures, according to Creswell's detailed measured survey,² 12.53 m. This is arrived at by adding $4.59 \times 2 = 9.18$ (width of pilaster supporting the dome). $21.71 - 9.18 = 12.53 =$ distance B, which represents the diameter of the dome (see analysis drawing DAMA/A8).

As indicated earlier, the width of the sanctuary is 36.6 m. $36.6 \text{ m.} - 0.3 \text{ m.}$ (forward projection of the *mihrāb*) = 36.3 m., which we shall call distance A1. $A1/B = 36.3/12.53$ (diameter of dome) = 2.85. $(\sqrt{2})^2 = 2.82$. Once again, therefore,

1. *Early Muslim Architecture*, fig. 90.

2. *Ibid.*, fig. 81.

the correlation between inductive geometry and actual measurement corresponds to within only 2 cm.

The Geometric Relationships between the Outer and Inner Temene

In order to establish the existence of any geometric relationship between the outer and inner *temene*, presuming that the inner *temenos* stood in the centre of the outer enclosure,¹ we need to return to our assumed square 1,2,3,4. In applying the same principle of concentric squares which we proved earlier to be of significance in the proportional relationship between the inner *temenos* and the mosque proper, as well as the proportional relationship between the depth of the prayer hall and the diameter of its dome; we arrive at a great square measuring an area almost equalling Watzinger and Wulzinger's² greater *temenos* believed to have surrounded the present mosque compound.

As demonstrated in analysis drawing DAMA/A9, we establish a set of external alternate squares Y1,X1,Y,X and X,W,Y,Z, where X-W is twice the length of 1-2. 1-2, which we shall call A, represents the northern wall of the inner *temenos* measuring, according to Creswell's survey,³ 160 m. including wall thickness. $2A = 320$ m. This means that using a simple geometric progression starting from the diameter of the dome, we arrive at an outer *temenos* enclosure

1. See plan in V.S. Bianca, *Architektur und Lebensform* (1979) p.29.

2. *Die antike Stadt*. See also Creswell, *Early Muslim Architecture*, vol.I, pt.ii, p.156.

3. *Early Muslim Architecture*, vol.I, pt.i, fig. 90.

of 320 m.². Although Watzinger and Wulzinger believe in an outer *temenos* slightly trapezoidal measuring 305 m. x 335 m., we think our geometric result of 320 m.² worthy of future examination.

Conclusion

Contrary to the popular view that Creswell has provided more than sufficient evidence to dismiss the theory of Dussaud, Watzinger, and Wulzinger regarding the location of the old Church of St. John,¹ we find that Creswell leaves us with more unanswered questions than convincing conclusions.

For example, (1) what is the full story behind the projecting part of a wall at the south-west corner of the present mosque shown in Creswell's survey,² which he has classified as part of al-Walīd's work? This wall, which is more emphasized on Bianca's plan,³ could be significant to our proposals that the east and west colonnades of the inner *temenos* must have projected further south.

(2) Does Creswell contradict himself when stating that al-Walīd demolished all that was in the inner *temenos* apart from the four corner towers?

1. Creswell, *Early Muslim Architecture*, vol. I, pt. i, p. 181.

2. *Ibid.*, p.127.

3. *Architektur und Lebensform*, p.29.

For in one of his plans¹ he dates the prayer hall entrance porch as pre-Islamic.

(3) How would Creswell explain the second *mihrāb* in the Great Mosque known as the *mihrāb* of the Prophet (p.b.u.h.)? It seems reasonable to accept the view of Dussaud that it was the original *mihrāb* dating from the time when the church was shared between Muslims and Christians.

(4) Having examined many photographs of the Great Mosque of Damascus, it is difficult to imagine how a church in Damascus could have fitted in the present inner *temenos* with spare space around. Creswell's plan² which shows a proposed location for the old church is unrealistic. In order to appear to have sufficient space around it, Creswell so diminished the church's proportions that it came to almost equal in size the entrance porch on the east side.

(5) When Creswell states that the south-west corner tower remained intact, does he mean the bottom section (arcade) or the top section which is now a minaret?

Our proposal of a square inner *temenos* does not, of course, deny that the present south wall of the mosque is of ancient, pre-Islamic workmanship. What is being suggested here is that the original temple extended well to the

1. *Early Muslim Architecture*, vol. I, pt. i, fig. 90.

2. *Ibid.*, p.194.

south of that ancient wall and formed a perfect square. At some indeterminate time the present south wall of the inner *temenos* (now the *qiblah* wall of the mosque) was built, thereby reducing the putative original square into the present rectangle.

Having arrived at the conclusion of a square inner *temenos*, the only explanation we can offer is that the present prayer hall of the Great Mosque of Damascus must have once formed a type of stoa common in Greek architecture (see fig. 4.8.2).

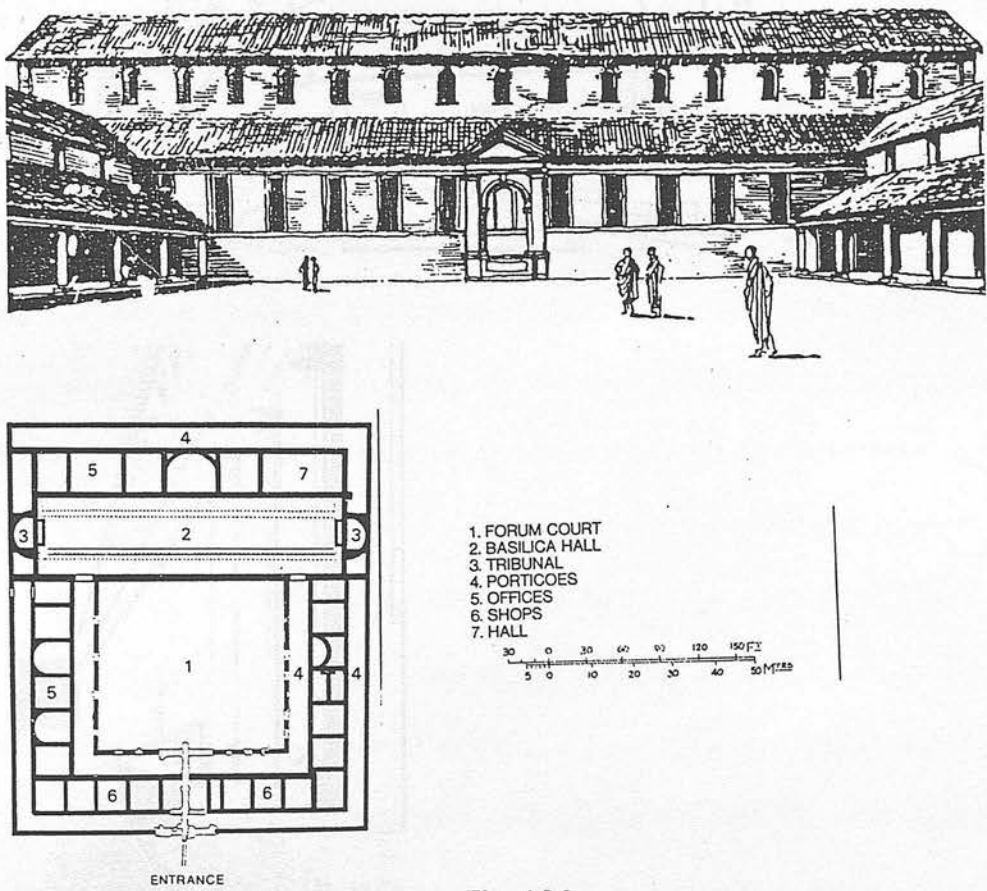


Fig. 4.8.2

The structure could also have stretched from east to west, resembling a Roman basilica (see fig. 4.8.3).

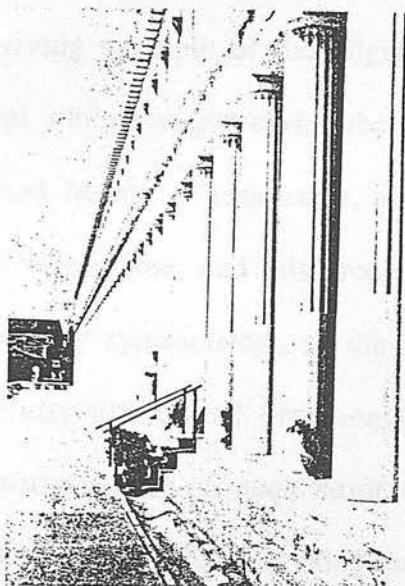
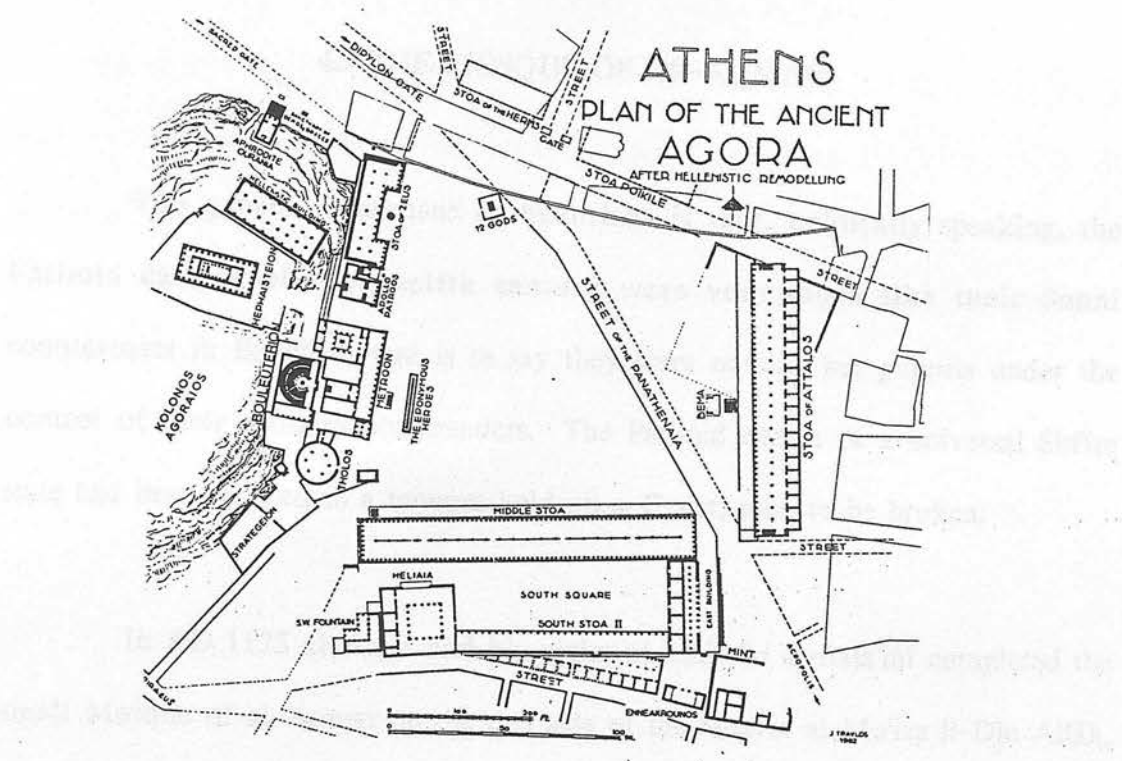


Fig. 4.8.3

4.9 THE MOSQUE OF AL-AQMAR

The general consensus of historians is that, politically speaking, the Fāṭimid caliphs of the twelfth century were very much like their Sunni counterparts in Baghdad, that is to say they were nothing but puppets under the control of their military commanders. The Fāṭimid dream of a universal Shī'ite state had been reduced to a tenuous hold upon Egypt, soon to be broken.

In AD 1125 al-Āmir and his vizier al-Ma'mūn al-Baṭā'ihī completed the small Mosque of al-Aqmar on the east side of the Shari'at al-Mu'izz li-Dīn Allāh, in the north-east quadrant of Jawhar's original enclosure. This is the earliest surviving example of the alignment of the principal façade with a pre-existing street while also preserving the required internal orientation of the mosque proper toward Mecca.¹ Presumably, as the city within the walls became more crowded land values rose, and this procedure became increasingly necessary. The façade – originally symmetrical, as the plan shows – is the first in Egypt to exhibit a carefully articulated ornamental scheme. There is also a more complex and sophisticated use of *muqarnaṣāt*, here defining for the first time the square heads of the shallow niches flanking the portal. This niched portal, set into a projecting

1. Hoag, *Islamic Architecture*, p.146.

porch and reminiscent of al-Ḥākim's but shallower, begins to exhibit the keel-shaped arch so characteristic of later Fātimid architecture. This type of arch is fully developed in the arcades of the *ṣaḥn*. Owing to the siting of this mosque between narrow alleys, the visible left corner of the façade has been chamfered, and a double register of *muqārnaṣ* cells employed to join the lower with the upper advanced levels of the walls. Owing likewise to the parallel placing of the mosque front with the street outside, the vaulted entrance passage is made to change direction slightly so as to become aligned with the sanctuary.

Geometric Analysis

In order to analyze the layout of al-Aqmar Mosque, we must begin by examining the central courtyard, firstly because it is the most striking geometric shape within the composition, and secondly, because most courtyards in Muslim buildings, particularly if they contain water sources, have significant importance in discovering any hidden geometric system (see chap. 3, § 10, 12, & 16 of this thesis).

In geometric analysis drawing no. AQM/A2. the major diagonals 18-20 and 17-19 are drawn in order to establish the exact centre of the courtyard (centre O). Then the vertical and horizontal axes follow by drawing arcs from each corner of the square. The next step is to draw a circle from centre A, touching the edge of the courtyard which naturally squares the circle, i.e.

17,18,90,20.

From the same centre O, a larger circle is drawn touching the corners of the previous square. When this circle is squared, square 13,14,15,16 is produced.

From the same centre O, a still larger circle is drawn touching the corners of the previous square. When the circle is squared, the result is square 9,10,11,12. Drawing no. AQM/A2 illustrates how three sides of this square line up with the external walls of the mosque. The fourth side, i.e. the *qiblah* side, lines up with the row of columns within the prayer hall (see drawing AQM/A2).

A larger circle can now be drawn from the same centre A touching the corners of the previous square. When the circle is squared, square 5,6,7,8 is produced (see drawing AQM/A3). Side 8-5 appears to intersect the centre point at the bent entrance access (see drawing AQM/A4).

The final circle is again drawn from the same centre O touching the preceding square and this results, when squared once more, in the master square 1,2,3,4. Side 1-4 of the master square (drawing AQM/A5) appears to line up with the outer face of the external wall of the mosque on the *qiblah* side.

Drawing AQM/A2 shows that if we were to connect points of intersection between the circle which gave square 17,18,19,20 and the main

diagonals, we would achieve square 21,22,23,24 and by repeating this procedure would arrive at square 29,30,31,32 as shown in drawing AQM/A5.

This square appears to be the unit square, for it matches the prayer hall bays and the bays of the surrounding ancillary spaces around the courtyard. Drawing AQM/A6 shows the same area (represented by square 29,30,31,32) appearing to form a kind of central corridor from the entrance right through to the *mihrāb*'s external width.

Drawing a hexagon 40,41,42,43,44,45 (see drawing AQM/A7) within the master circle will help us to appreciate the true plan of the mosque before it had to bend at the front to line up with the street.

This is a good example of late 'early Muslim architecture' in which the geometry is made to work even within an existing town fabric. It is an interesting problem which the architect appears to have dealt with in a very clever way. On the one hand, the mosque had to face Mecca, i.e. due south-east, while on the other, the main front elevation had to line up with the street grid which appears to run north-south, east-west.

Arithmetical Check

According to Creswell's measured dimensions, line 9-12 (shown in

drawing AQM/A9 in vol. II of thesis)

$$6.42 + 7.1 + 6.42 = 19.94 \text{ m.}^1 \text{ (actual width of building)}$$

$$9.97 \times 3 \text{ units} = 29.91 \text{ m. (geometric length of building).}$$

Compared with the actual length of the building,

$$\text{line 9-4A (drawing AQM/A9)} = 29.85 \text{ m.,}^2$$

producing a margin of 60 mm.

Another way to check the proposed geometric scheme is to consider side 17-18 of the courtyard, the actual dimensions of which, according to Creswell, are 10.17 m. and 9.77 m. This gives an average courtyard width of

$$\frac{10.17 + 9.77}{2} = 9.97 \text{ m. (actual dimension)}$$

Square 9,10,11,12 relates to square 17,18,19,20 in a 1:2 proportion (second concentric square).³ Therefore, side 9-12 should equal twice the side 17-20 indicated in our drawing AQM/A9 in vol.II of the thesis.

$$9.97 \times 2 = 19.94 \text{ m. (geometric dimension)}$$

$$19.94 \text{ m. (actual dimension),}$$

giving a result of 0 margin.

1. Creswell, *Early Muslim Architecture*, vol. II, p. 242, fig. 141: Mosque of al-Aqmar.

2. *Ibid.*

3. See section 3.5 of thesis (Two-dimensional architectural design).

4.10.1 THE GREAT MOSQUE OF SĀMARRĀ'

Introduction

Completely dwarfing all mosques that have been built either up to the present century, al-Malwiya Mosque (named with reference to its spiral minaret) was begun in A.D. 847 by the Caliph al-Mutawakkil. Nine bays deep and twenty-five bays wide, its total dimensions covered 784 x 512 feet.¹

The mosque's roof was supported without arches, using twenty-four rows of brick piers which had clusters of four-column sections of coloured marble rising to the ceiling with bases and capitals. The Great Mosque was built of burnt brick and the outer walls were 2.65 m. thick and of light-red bricks 25-27 cm. square. Outside the first enclosure,² and linked to the main structure of the mosque at its northern end, stands the enigmatic spiral ramp minaret, perhaps following in design the scheme of the temple towers or ziggurats of ancient Assyria.³ It is attached by means of a ramp to the exact centre of the north side, the ramp spiralling around the exterior of the tower, steepening as it ascends.

-
1. D. Talbot Rice, *Islamic Art* (1975) p.36. B. Fletcher, however, gives the dimensions as 780 x 510 feet, also confirmed by Creswell. See short account by J.W. Allan, *A Short Account of Early Muslim Architecture*, p.359
 2. See geometric analysis.
 3. Creswell, *Short Account*, rev. Creswell, p. 362.

The History of the Site

Ja'far al-Mutawakkil ibn al-Mu'taṣim succeeded al-Wāthiq in A.D. 847, taking up his residence in al-Hārūnī Palace. After ensuring that his sons took possession of the palaces of al-Mu'taṣim in a continuous urban form from Balkuwāra in the south to a place called al-Dūt in the north, al-Mutawakkil then added *Shārī' al-Askar* (Army Street) and *al-Shārī' al-Jadīd* (New Street) to the streets of al-Ḥā'ir.¹ He then built the Great Mosque at the beginning of al-Ḥā'ir in a broad space beyond the houses and not in contact with any allotment or market.

The site was part of an existing fabric. Al-Ḥā'ir was already a built-up area and had been developed extensively by al-Mu'taṣim. Therefore the natural qualities of the site are obviously linked to the siting of the whole of Sāmarrā'.

Geometric Analysis

The proportions of the Great Mosque of Sāmarrā' are best expressed in a step-by-step geometric reconstruction of its elements including its additions (*ziyādāt*).

Stage 1: We draw a circle of a given size with centre O, intersecting its Y-Y1

1. *Ibid.*

axis at P and Q (see analysis drawing MALW/A1).

Stage 2: From P we draw another circle of the same radius, intersecting the Y-Y1 axis at R. By enclosing the two interlocking circles from centres P and Q within rectangle 1,2,3,4, we arrive at the proportion of the Great Mosque (2-3) (see analysis drawing MALW/A2).

Stage 3: In order to arrive at the proportions of the outer enclosures, we draw a third circle from centre Q with the same radius of O and P intersecting the Y-Y1 axis at S. We then draw a master circle from centre O, enclosing the three previous interlocking circles with centres O, P, and Q (see analysis drawing MALW/A3), where e, f, and n determine the shape of the courtyard.

Stage 4: By squaring master circle O with square 5,6,7,8 (see analysis drawing MALW/A4), the intersections of master circle O with arc 7-0/centre R and with arc 5-0/centre 5, determine the location of the two parallel lines 10-11 and 12-9. These appear to correspond to the side wall of the first enclosure.

Stage 5: Circling square 5,6,7,8, this square with radius 0-7 intersects the Y-Y1 axis at T and U and intersects the X-X1 axis at VW. An arc from V, with radius V-O, will intersect the super-circle at B and similarly an arc from W, with radius W-O, will intersect the super-circle at B1 (see analysis drawing MALW/A5). By connecting points B-B1, this appears to correspond to the mosque's southernmost enclosure.

Stage 6: Squaring super-circle O (see analysis drawing MALW/A6), it intersects T-11 at C and T-12 at C1. C and C1 determine the location of two parallel lines which appear to correspond to the furthestmost east and west enclosures.

Arithmetic check: Without having seen any measured drawings, we are able to determine the dimension of the external mosque enclosure.

Without the benefit of measured drawings of the entire complex, our geometric estimation based on the same scale gives an overall dimension of 376 x 436 m. Creswell arrives at a great enclosure of 376 x 444 m.,¹

The Proportioning System of the Minaret

In Plan: By squaring the generative circle O with square 3,4,13,14 (see analysis drawings MALW/A7 & A7.1), we establish a series of diminishing concentric alternate consecutive squares. We discover that square 19,20,21,22 corresponds to the area of the minaret. The distance from the minaret to the northern external wall appears equal the width of the minaret itself, achieved by diagonals 23-26, intersecting the extension of side 21-20. Within square 24,25,26,27, which forms the base of the minaret, a further subdivision can be established in order to determine the width of the linking ramp (see drawing MALW/A9).

In section: Drawings MALW/A8 and A9 indicate how by placing the minaret plan

1. *Ibid.*, vol.II, p.259.

24,25,26,27 beneath its section using consecutive square 28,29,30,31 as a base, we find that arc /centre 29 and radius 29-A intersects the minaret's vertical axis halfway up the main spiral ramp. Arc from centre 29 with radius 29-B intersects the minaret's vertical axis at the minaret's peak.

Arithmetical Check

Having examined the plan of this immense rectangular building and concluded geometrically its proportion, it is somewhat reassuring to discover that Creswell¹ also finds that the plan of the Great Mosque adheres to the 2:3 proportion.

If we take the length of the building and divide it by three, then multiply the result by two, this should give us the width of the building (proportionally). We compare this with the actual width to see how close our geometric conclusion is to the building as it stands.

$$239.68 \text{ m.} + 5.3 \text{ m. (wall thickness)} = 244.98 \text{ m. (actual length)}$$

$$244.98/3 = 81.66 \text{ m.}$$

$$81.66 \times 2 = 163.3 \text{ m. (geometric width)}$$

$$156.37 \text{ m.} + 5.3 \text{ m. (wall thickness)} = 161.67 \text{ m. (actual width)}$$

$$163.3 - 161.67 = 1.63 \text{ m. margin.}$$

1. *Early Muslim Architecture*, vol. II, p. 254.

1.63 m. margin between geometric lines¹ and actual dimensions in a building which has an area of 38,000 sq. m., not including the *ziyādahs*, is a good result considering the size of the building.

A further arithmetical check can be carried out on our geometric hypothesis proposed in drawing MALW/A7.1 of vol. II of the thesis. This states that the concentric square 19,20,21,22 has a proportional relationship with square 3,4,13,14 and also equals in area the base of the minaret represented by square 24,25,26,27.

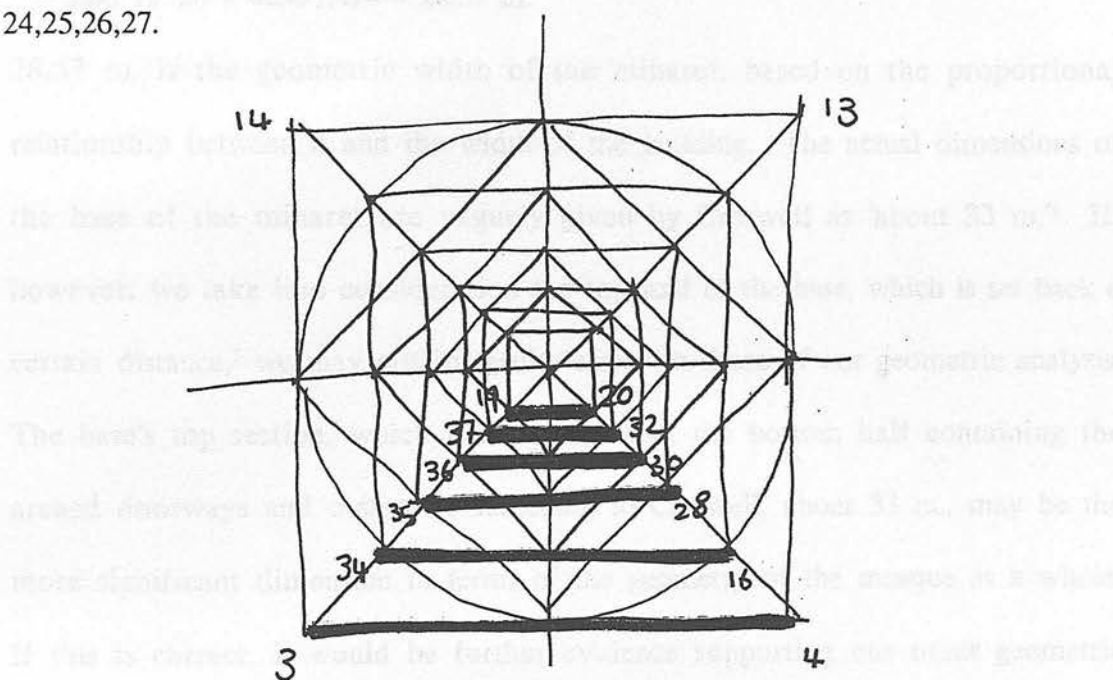


Fig. 4.10.1.1

As explained in section 3.4, within the squared circle diagram, consecutive concentric squares relate in a $1:\sqrt{2}$ proportion, whereas alternate

1. See vol. II of thesis, p. 61-65.

squares relate in a 1:2 proportion. Therefore, as we already know the actual dimension of side 3-4 (see fig. 4.10.1.1), as it is given by Creswell,¹

side 3-4 = 161.67 m. (including wall width)

side 34-16 = $161.67/1.414 = 114.3$ m.

side 35-28 = $161.67/2 = 80.8$ m.

side 36-30 = $80.8/1.414 = 57.14$ m.

side 37-32 = $80.8/2 = 40.4$ m.

side 19-20 = $40.4/1.414 = 28.57$ m.

28.57 m. is the geometric width of the minaret, based on the proportional relationship between it and the width of the building. The actual dimensions of the base of the minaret are vaguely given by Creswell as 'about 33 m.'² If, however, we take into consideration the top half of the base, which is set back a certain distance,³ we may obtain results closer to those of our geometric analysis. The base's top section, which is set back from the bottom half containing the arched doorways and measures, according to Creswell, about 33 m., may be the more significant dimension in terms of the geometry of the mosque as a whole. If this is correct, it would be further evidence supporting our other geometric supposition, shown in drawing MALW/A7.1 (in vol. II), that the distance between the minaret and the mosque's northern wall ought to be equal to the width of the actual minaret.

1. *Early Muslim Architecture*, vol. II, p. 260, fig. 205.

2. *Early Muslim Architecture*, vol. II, p. 259.

3. See J. Hoag, *Islamic Architecture*, p. 54, pl. 54. The base of the minaret was restored by the Iraqi Department of Antiquities in 1936/7.

Side 23-27 = side 27-24 (drawing MALW/A7.1).

This can be further checked using the minaret's separate geometric analysis (see drawing MALW/A8 in vol. II). Having circled the square base of the minaret and then squared it, our supposition is that this square (i.e. 28,29,30,31) equals the height of the minaret minus the topmost arched storey. If we take the dimension of the lower section of the base, i.e. 33 m., as the basis for calculations, we find:

$33 \times 1.414^1 = 46.6$ m. (geometric height of minaret based on 6.1×6 (less top section of minaret) = 36.6 m. (actual height).²

However, if we take the set-back upper section of the base as the basis for calculation, i.e. 27.25 m.,³ taken to equal the distance between the minaret and the northern wall, we discover the following:

$27.25 \times 1.414 = 38.5$

6.1×6 (less top section of minaret) = 36.6.

$38.5/36.6 = 1.9$ m. margin, or approximately 3 % when considering the total height of the minaret.

In conclusion, we are able to confirm that the Great Mosque of Sāmarrā' is built according to a typical 2:3 proportion and could not have developed to that complexity including the *ziyādahs*, without such a system.

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1. Side 28-29 relates to side 25-26 (the base of the minaret) in a $1:\sqrt{2}$ proportion.
 2. Creswell, *Early Muslim Architecture*, vol. II, p. 261.
 3. *Ibid.*, p. 259.

4.10.2 DĀR AL-IMĀRAH, KŪFA

Introduction

The Dār al-Imārah ("House of Government") at Kūfa was perhaps known as such as early as A.D. 638 when Sa'd ibn Abī Waqqās marked out Kūfa's first mosque. The Dār al-Imārah served both as a residence and administrative centre.

After his appointment to Baṣra, Ziyād ibn Abīhī, who at first served 'Alī and then transferred his allegiance to Mu'āwiya, was also governor of Kūfa. There in A.D. 670 he rebuilt the Congregational Mosque on stone columns 51 feet tall supporting a flat roof of teak.¹

Two major points make the analysis of this building rather difficult. First is the fact that the building, as a result of various excavations, appears to contain the remains of a number of structures erected one above the other. The chronological sequence is as follows.

1. J.D. Hoag, *Islamic Architecture*, p.15.

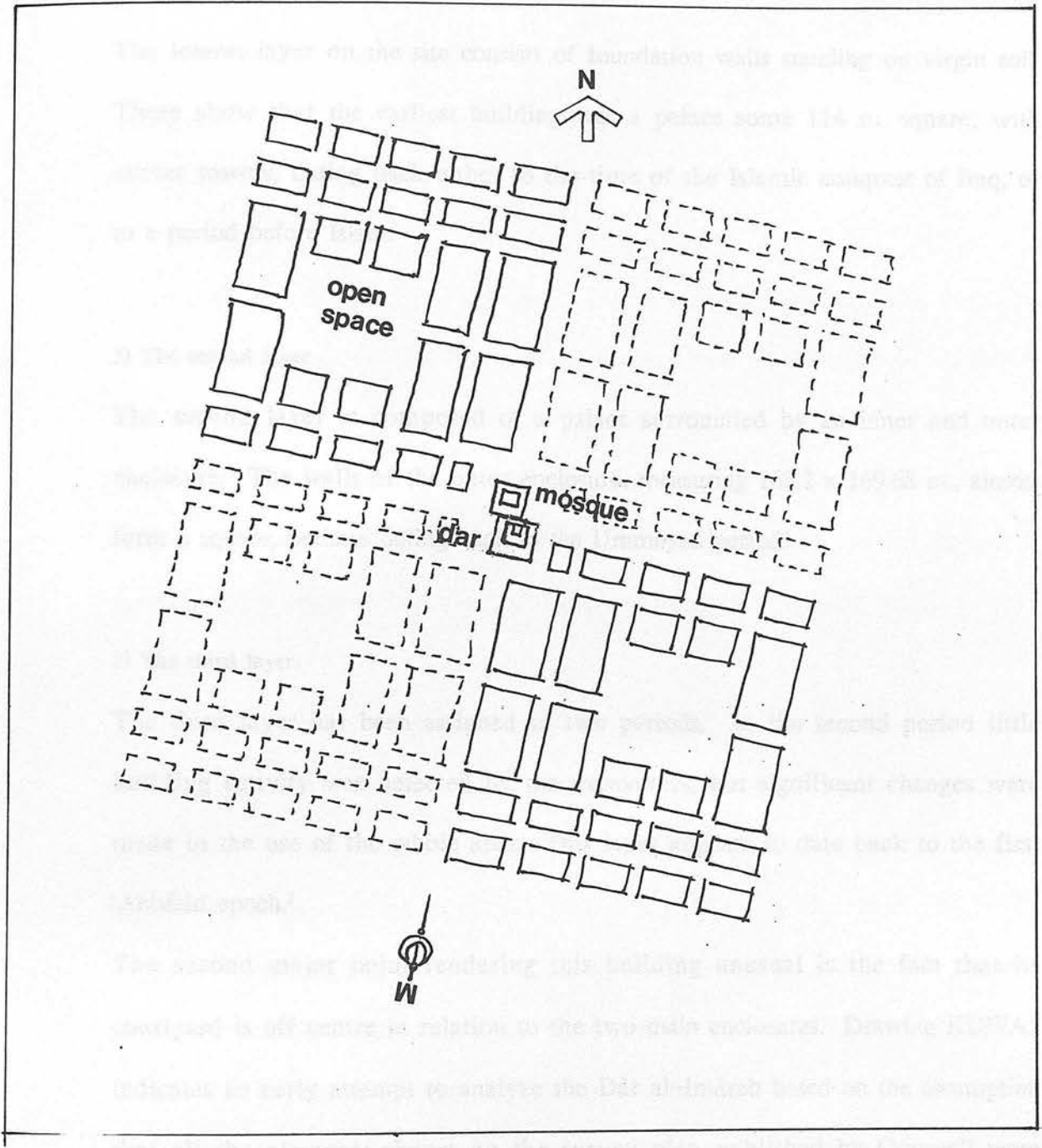


Fig. 4.10.2.1

Plan of Kūfa

1) The lowest layer

The lowest layer on the site consists of foundation walls standing on virgin soil. These show that the earliest building was a palace some 114 m. square, with corner towers, dating back either to the time of the Islamic conquest of Iraq, or to a period before Islam.¹

2) The second layer

The second layer is composed of a palace surrounded by an inner and outer enclosure. The walls of the outer enclosure, measuring 168.2 x 169.68 m., almost form a square, perhaps dating back to the Ummayyad period.

3) The third layer

The third layer has been assigned to two periods. In the second period little building activity was detected by the excavators, but significant changes were made in the use of the whole area. This layer appears to date back to the first 'Abbāsid epoch.²

The second major point rendering this building unusual is the fact that its courtyard is off centre in relation to the two main enclosures. Drawing KUF/A5 indicates an early attempt to analyze the Dār al-Imārah based on the assumption that all the elements shown on the survey plan published by Creswell were constructed within the same period. This analysis did not unfortunately produce a

1. Creswell, *Early Muslim Architecture*, p.14.

2. *Ibid.*

convincing result, particularly in relation to the juxtaposition of the central courtyard to the outer enclosure. The way the four porticos or *īwāns* were arranged around the central courtyard is reminiscent of the architecture of Mesopotamia in the centuries just before the coming of Islam.¹

In drawing KUF/A1 we draw a circle from centre O with a radius equalling the distance from centre O to the front face of the portico. Squaring this circle results in the square 1,2,3,4. Using the same radius and drawing an arc from each of the corners of the generating square 1,2,3,4, we achieve a generating square at 5,6,7,8,9,10,11,12, the width of each of the segments appearing to correspond to the width of each of the porticos off the courtyard.

Let us circle the square 1,2,3,4 using its diagonal O-3 as the radius. The resulting circle, when squared in its turn, produces the larger square 13,14,15,16. Let us now circle the next square with radius O-15. The resultant circle, when squared once more, gives the still larger square 17,18,19,20, where side 19-20 appears to correspond to the inner enclosure's northern wall. This square, contrary to the interpretations of the excavation in Creswell,² appears to point to an identifiable building campaign and a turning point in the design of the

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1. G. Herrmann, *The Making of the Past: the Iranian Revival* (Oxford, 1977) p.57.; E.J. Keall, "Some Thoughts on the Early Eyvan", in D. Kouymjian (ed.), *Near Eastern Numismatics, Iconography, Epigraphy and History: Studies in Honor of G.C. Miles* (Beirut, 1974) p.123-130.
 2. *Early Muslim Architecture*, vol. I, pt. i, p. 53.

building as a whole.¹ In other words, the four-*īwān* building is totally distinct from the rest of the plan, i.e. it has its own separate centre. The rest of the building is struck from a different centre. This is the proof that the monument, as it stands excavated, is not the result of a single building campaign.

It seems that after this four-*īwān* plan was completed, the building grew outwards except from the north. This process can be explained as indicated on drawing KUF/A2 by drawing an arc from centre O with radius O-19 crossing the horizontal axis at c and similarly arc O-20 crossing the horizontal axis at d. The tangents at c and d will intersect the extension from 19-20 at 21 and 22. These tangents appear to correspond to the east and west external walls of the inner enclosure.

In order to locate the external south wall of the inner enclosure (see drawing KUF/A3), we draw an arc from 21 with radius 21-22 intersecting tangent C at 26 and similarly an arc from 22 intersecting tangent D at 25. The resultant square 21,22,25,26 now corresponds exactly with the inner enclosure of the Dār al-Imārah.

The outer and inner enclosures do not share the same centre. The only way that a proportion of $1:\sqrt{2}$ (which operates between the inner and outer

1. Note Creswell's comments on the change in area 100 shown in his plan, which is in line with the sides of our square 17,18,19,20.

enclosures – see drawing KUF/A5) can be achieved is if a second centre point for the building was devised, thus creating a dual-centred plan instead of the single-centred design encountered in most of the buildings analyzed in this thesis.

The centre of the second design can be obtained by drawing the diagonals of the resulting new square 21,22,25,26, which intersect at the mid-point of the front of the south *īwān* which apparently leads to the throne room (see drawing KUF/A4). We obtain the centre for the new square by first circling square 21,22,25,26 with radius O-21. This circle, when squared in its turn, produces the still larger square 27,28,29,30 which corresponds exactly to the perimeter of the outer enclosure.

The building history outlined above is not readily susceptible of proof. Indeed, the sequence could even be reversed, with the four-*īwān* plan inserted later into a pre-existing structure.

Arithmetical Check

According to Creswell,¹ the outer enclosure indicated in our drawing KUF/A5 as square 27,28,29,30, measures 168.2 m. x 169.68 m. The inner enclosure, indicated in our drawing KUF/A4 as square 25,26,21,22, measures 110.36 m. x 110.24 m. internally. According to our geometric principle of

1. *Early Muslim Architecture*, vol. I, p. 48.

squaring the circle, both squares ought to relate in a proportion of $1:\sqrt{2}$. To check this, we carry out the following calculations:

$$110.36 + 1.82 + 1.82 = 114 \text{ m. (side 21-22 externally)}$$

$$168.2/114 = 1.47$$

$$\sqrt{2} = 1.414.$$

A further check can be carried out on the proportion of the central courtyard indicated in our drawing KUF/A2 as square 1,2,3,4 (which, according to Creswell,¹ measures 37 m.) and the area including the four *īwāns* (which, according to Creswell, measure 74 sq. m.). As is shown in our geometric analysis, both squares ought to relate in a 1:2 proportion.

$$37 \text{ m.} \times 2 = 74 \text{ m. (geometric dimension).}$$

1. *Ibid.*, p. 51.

4.10.3 BALKUWĀRA PALACE

Introduction

In contrast to Jawsaq al-Khaqani, the palace of Balkuwāra has defined boundaries and it is entered from the north-east rather than from the Tigris (see fig. 4.10.3.1). This complex, in which the name of al-Mu'tazz is believed to appear on a beam over the throne room,¹ cannot have been built much after A.D. 859 when the construction of the Ja'fariyya district began.

The Site Location

Al-Mutawakkil extended the city northward along the Tigris and built a new palace, the Qaṣr al-Ja'farī, and to the south a residence for his son, Prince al-Mu'tazz. This is the Balkuwāra Palace.

1. Creswell, *Early Muslim Architecture*, vol.II, p.270.

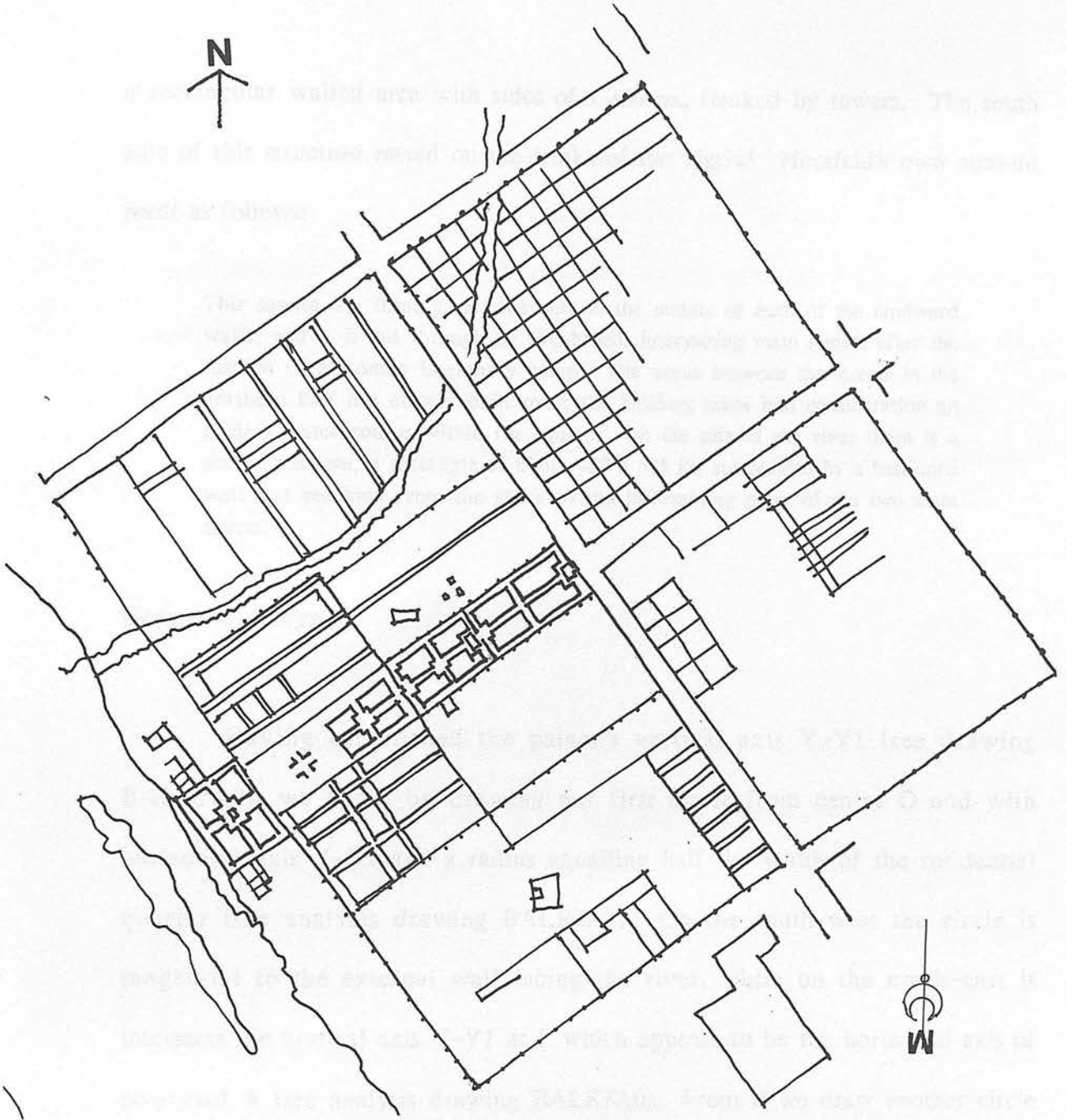


Fig. 4.10.3.1

The site is in the section known as al-Manqur, about 6 km. south of modern Sāmarrā'. Creswell notes that when Herzfeld excavated the area in 1911 he soon realized that he had to deal with an immensely large palace consisting of

a rectangular walled area with sides of 1,250 m., flanked by towers. The south side of this structure rested on the banks of the Tigris.¹ Herzfeld's own account reads as follows:

This square has three gates, one only in the middle of each of the landward walls, and it is cut through by two broad, intersecting main streets after the fashion of a Roman Legionary camp. The areas between the streets in the northern half are closely built over; the building takes into consideration an ancient water course within the square. On the side of the river there is a second castrum, a rectangle of about 460 x 575 m., surrounded by a bastioned wall and reaching from the shore to the intersecting point of the two main streets.

Geometric Analysis

Having established the palace's vertical axis Y-Y1 (see drawing BALK/A0), we begin by drawing our first circle from centre O and with horizontal axis X-X1 and a radius equalling half the width of the residential quarter (see analysis drawing BALK/A1). On the south-west the circle is tangential to the external wall facing the river, while on the north-east it intersects the vertical axis Y-Y1 at P which appears to be the horizontal axis of courtyard A (see analysis drawing BALK/A0). From P we draw another circle with the same radius, intersecting the Y-Y1 axis in the north-east at R.

Sides 1-4 and 2-3 of square 1,2,3,4, which squares circle O, appear to correspond with the south-east and north-west side walls of the palace.

1. *Ibid.*, vol.II, p.265.

By squaring circle P, which is the second circle of our initial analysis and sub-dividing square 5,6,7,8 in a series of concentric alternate and consecutive squares (see analysis drawing BALK/A2), we discover that square 9,10,11,12 equals the width of courtyard D adjacent to the river. We also observe that square 13,14,15,16 corresponds in width to the central line of courts A and B. These concentric squares also appear to regulate the division within these central courts (see analysis drawing BALK/A3).

Arithmetical Check

Our geometric theory for Balkuwāra proposes a regular square measuring 1,130 m. on each side (see drawings BALK/A0–A4 in vol. II of thesis). This does not, however, agree with the measurements provided by Creswell,¹ i.e. 1,250 m. per side. This discrepancy can be explained as follows. Creswell tells us that when Herzfeld excavated in the area of Balkuwāra in 1911, he was dealing with ‘an immensely large palace, consisting of a rectangular walled area of 1,250 m. a side, flanked by towers, *the south side of which rested on the banks of the Tigris.*’ The words I have italicized in this quotation lead us to conclude that Herzfeld’s 1,250 m. must have included the projecting courtyard on the south-west side of the palace, because if we deduct the depth of this projecting court from Herzfeld’s measurement (1,250 – 120), we arrive at exactly 1,130 m.

1. *Early Muslim Architecture*, vol. II, p. 265.

for each side of the immense square, not including the projecting south-west court.

Furthermore, our geometric scheme suggests that the inner palace's plan¹ measures, according to Creswell, 460 x 575 m. and occupies exactly half of the great square enclosure. Half of the length of the great square enclosure is

$$1,130/2 = 565 \text{ m. (geometric depth of inner palace)}$$

$$575 \text{ m. actual depth} - 8 \text{ m. (two walls' thickness)} = 567 \text{ m.}$$

$$567 - 565 = 2 \text{ m.}$$

Thus, we are able to prove our geometric scheme (see drawing BALK/A4) to within a 2 m. margin or error in the total length of 1,130 m.

Our geometric analysis of the Palace of Balkuwāra, not only on account of its size but also in the impressive way by which its geometry organized the laying out of its courts and the relationship between the main enclosure and the elements within it, confirms Creswell's conclusion² that in its layout Balkuwāra has close analogies with the palaces of Mshattā and Ukhaydir, except that here, owing to the building's colossal size, every element is enlarged and multiplied many times over.

1. Shown in Creswell, *Early Muslim Architecture*, vol. II, p. 266, fig. 214.

2. *Ibid.*, p. 270.

4.10.4 THE *MIHRĀB* OF THE MOSQUE OF AL-MANŞŪR

It is believed that the marble *mihrāb* presently to be seen in Jāmi' al-Khāṣakī in East Baghdad is probably the *mihrāb* of the Mosque of al-Manşūr,¹ which he had ordered for his mosque at the centre of the round city. This mosque was first constructed c. A.D. 766. The marble in which it is constructed is apparently different from the marble of Mosul and was probably imported from Syria where it was carved before it was transported to Baghdad.²

Geometric Analysis

Here we see a classic Islamic proportioning system with three interlacing circles giving the composition a 1:2 apportionment reminiscent of the Ka'ba, al-Malwiyya Mosque in Sāmarrā', al-Qarawiyyīn Mosque in Fez, and the final stage of the development of the Friday Mosque in Işfahān.

Having established the *mihrāb*'s central axis, we begin the geometric analysis at the most logical starting point, that is by finding the centre which

1. See Creswell, *Early Muslim Architecture*, p.35.

2. *Ibid.*, p.36.

generates the shell design crowning the *mihrāb* (see analysis drawing MIH/A1).

We immediately notice that circle A, the closest to the shape of the shell, is in fact tangent to the central axis to each of the columns flanking the *mihrāb* and which intersect the *mihrāb*'s central axis at B. From B we draw another circle of the same radius intersecting the *mihrāb*'s central axis, this time at C. From C we draw a third circle of the same radius intersecting the central axis at D.

Point D takes us down to just above the base of the two flanking columns. This point proves significant only when the analysis of the elevation is read in conjunction with the analysis of the plan.

Before we do this, however, let us examine the plan of the *mihrāb* independently (see analysis drawing MIH/A1 & A2). By adopting the same circle of the shell and squaring it, we arrive at square 1,2,3,4. Then, by establishing a series of alternate squares, the consecutive square to square 13,14,15,16, we discover that square 17,18,19,20 corresponds to the projecting decorative band which runs vertically along its central axis of the *mihrāb*.

The axis which governs the curvature of the *mihrāb* can be obtained from the following centres:

Centre M. resulting from the intersection of side 21-22 with the Y-Y1

axis, produces arc 29–30.

Centre L, resulting from the intersection of side 5–6 with the Y–Y1 axis, produces arc 29–31.

Centre K, resulting from the intersection of side 9–10 with the Y–Y1 axis, produces arc 31–32.

Centre N, resulting from the intersection of side 1–4 with extended side 10–9, produces circle N which forms the circular flanking columns on each side of the *mihrāb*.

By placing side 27–28 of master square 25,26,27,28 developed from the generative square 1,2,3,4 centrally just above the base of the flanking columns, we find that we are now able to read the *mihrāb* from all aspects simultaneously (see analysis drawing MIH/A1 & A2).

If we draw an arc from centre 27 with radius 27–0, the arc will intersect the *mihrāb*'s central axis at B (the centre of the second interlocking circle).

Another arc drawn from centre 27 with radius 27–0 will intersect the *mihrāb*'s central axis at E, which appears to correspond with the topmost feature of the *mihrāb*.

Arithmetical Check

As indicated in drawing MIH/A1 in vol. II of the thesis, the *mihrāb* appears to have been designed in a 2:4 or 1:2 proportion, i.e. three interlocking circles make up the height of the *mihrāb* from the top of the column's base on each side to the top of the semi-circular arch (A), and also the width of the *mihrāb* between the centres of the flanking columns (B).

The setting out of the width is initiated by the first circle, half of which forms the shell hood.

Actual width (B) = 840 mm.¹

Geometric width (B) = height (A)/4 x 2

$$1,910 - 210 = 1,700 \text{ mm.}$$

$$1,700/4 = 425 \text{ mm.}$$

$$425 \times 2 = 850 \text{ (geometric width B)}$$

$$850 - 840 = 10 \text{ mm.}$$

Thus, we have a result with a difference of 10 mm. between the actual measurement and the geometric supposition.

A further check can be carried out with the aid of our drawings MIH/A1 and A2 in vol. II of the thesis. According to our geometric scheme for the *mihrāb*, we propose that the width of the central band of ornament running

1. Creswell, *Early Muslim Architecture*, vol. II, p. 35.

up the back of the concave part of the *mihrāb* has a direct proportional relationship with the geometry which controlled the width and height of the *mihrāb*.

Actual height (A) = 1,700 mm. = side 26-27

side 26-27 = $1,700/2 = 850$ (side 2-3)

side 2-3 = $850/2 = 425$ (side 10-11)

side 10-11 = $425/2 = 212.5$ (side 14-15)

side 14-15 = $212.5/1.414^1 = 150.28$ (side 18-19)

150.28 is the geometric width of the decorative band.

160^2 is the actual width of the band.

This produces a 6 % margin of error in relation to the actual width, but in relation to the height, we have a 0.055 % margin of error.

-
1. Side 14-15 relates (according to our principle of squared circles) to side 18-19 in a $1:\sqrt{2}$ proportion.
 2. Creswell, *Early Muslim Architecture*, vol. II, p. 36.

4.10.5 QARAWIYYĪN MOSQUE, FEZ

Begun in A.D. 857 by the daughter of a wealthy Arab immigrant from Qayrawān, the mosque's prayer hall originally consisted of four aisles parallel to the *qiblah* and was twelve bays wide¹ (see analysis drawing QARA/A1).

It is believed that the present outer arcades within the prayer hall and perpendicular to the *qiblah* mark the former external walls.² When Fez returned to Sunni hands, increased trade and peace encouraged population growth, and an expansion of the mosque was approved and funded by 'Abd al-Raḥmān. The other additions probably date from A.D. 956. Without moving the *qiblah*, the prayer hall was enlarged east and west to its present limit and also to the north, occupying the old *ṣaḥn*. A new *ṣaḥn* was added which some historians believe must have been centred on the previous minaret opposite the *miḥrāb* aisles as in Qayrawān.³

The final overall expansion of the mosque was carried out under the instructions of Sultan 'Alī ibn Yūsuf in A.D. 1134. He added three aisles the full

1. J.D. Hoag, *Islamic Architecture*, p.98.

2. *Ibid.*, p.103.

3. *Ibid.*

width of the earlier building, south of the *qiblah* wall.

The last major additions to the Qarawiyyīn mosque were the two pavilions projecting into the *ṣaḥn* built by the Sa'dī Sultan 'Abd Allāh ibn al-Shaykh (*reg.* 1613–24). They contain fountains for ablutions, the design echoing that of the Alhambra's court of lion pavilions.

Geometric Analysis

Having established the *qiblah* axis, the central aisle (see analysis drawing QARA/A2¹) appears to consist of a series of circular floor patterns, one of which is made up of two rings. By placing our compass at the centre of this design and drawing a circle at a tangent to the sanctuary's present entrance arcade, we find that it also touches the north and south internal arcades perpendicularly to the *qiblah* wall. Although in the south-east, the circle goes beyond the *qiblah* wall and appears to correspond with the depth of the *mihrāb* which projects outwards.

By squaring the generative centre O with square 1,2,3,4 and establishing a series of alternate squares, we find that the extension of side 30–31 in square 29,30,31,32 corresponds with the presumed limit of the original mosque (see drawing QARA/A2).

1. Plan after Hoag.

Side 9–12 in square 9,10,11,12 (see drawing QARA/A3) seems to correspond with the limit of the second extension of the mosque. Consecutive square 29,30,31,32 gives us the width of the central aisle, whereas square 25,26,27,28, when extended towards the *ṣaḥn*, seems to be in line with the sides of the fountains that feature in the inner square (see analysis drawing QARA/A3).

By circling generative square 1,2,3,4 and squaring it, we find that side 17–20 of square 17,18,19,20 forms the vertical axis of the present *ṣaḥn* (see analysis drawing QARA/A4). Circling the latter square and then squaring it, we find that side 21–24 of square 21,22,23,24 corresponds with most of the mosque's western external wall. Also, by inscribing a hexagon within the above master square, side 25–26 corresponds to the mosque's external south-west wall (see analysis drawing QARA/A6).

Arithmetical Check

Among our proposals in drawings QARA/A1–6 (in vol. II of the thesis), we include evidence that the width of the central aisle leading to the *mihṛāb* and shown in drawing QARA/A3 as 26–27 relates proportionally to the width of what is known as stage I of the mosque, shown in drawing QARA/A2 as square 1,2,3,4.

Side 1-4 = 46.5 m.¹ (actual width)

$46.5/2 = 23.25$ m. (side 10-11)

$23.25/2 = 11.625$ m. (side 18-19)

$11.625/2 = 5.8$ m. (side 26-27) – geometric width of aisle

5.5 m.² = actual width of aisle.

$5.8 - 5.5 = 300$ mm., i.e. a 5.5 % margin or error.

CHAPTER FIVE

ALTERNATIVE SYSTEMS OF ANALYSIS

5.1 M. Vachère on the Dome of the Rock and the Principles of Statics

Design in Islamic Architecture

5.2 Rotative Spherical Examples

5.3 Leon Goltschick on Timurid Buildings

5.4 Erik Ferné on St. Andrew's Crypt, Canterbury and Fly Cathedral

5.5 C. Robert on the Mosques of Timgad (Morocco) and Casula di Lincea

-
1. H. Terrasse, *La Mosquée Al-Qaraouiyyin à Fès* (Paris, 1968) p. 99, fig. 6.
 2. *Ibid.*, p. 106, fig. 20.

5.1 M. Ecochard on the Dome of the Rock and the Principles of Stalactite Design in Islamic Architecture

If there was any proof of the geometric system we proposed in this thesis, i.e. the squared circle resulting in a square rotated 45 degrees, as a model of understanding the principles of design in early Muslim architecture, it would be that ours, originally presented by Ecochard, particularly as he carefully considered the geometric scheme of the

CHAPTER FIVE

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5.1 M. Ecochard on the Dome of the Rock and the Principles of Stalactite Design in Islamic Architecture

5.2 Bulatov: Selected Examples

5.3 Lisa Golombek on Timurid Buildings

5.4 Eric Fernie on St. Anselm's Crypt, Canterbury and Ely Cathedral

5.5 C. Ewert on the Mosque of Tinmal (Morocco) and Castelo de Lousa

Having examined the plan of the Dome of the Rock, Ecochard came to

1. *Principes de l'architecture arabe, byzantine et islamique, aux sources de géométrie* (Paris 1977) p. 109-110.

2. *Opus de Géométrie*, vol. 1, par. 1.

3. *Id.*

5.1 M. Ecochard on the Dome of the Rock and the Principles of Stalactite

Design in Islamic Architecture

If there was any proof of the geometric system we propose in this thesis, i.e. the squared circle resulting in a square rotated 45 degrees, as a means of understanding the principles of design in early Muslim architecture, it would be that most articulately presented by Ecochard, particularly as he carefully considers the geometric scheme of the Dome of the Rock as a common approach within the area of Byzantine Palestine.¹

Many opinions have been expressed with regard to the architectural origins of the plan of Qubbat al-Şakhra. De Vogué (1864)² connected it with that of the churches at Bosra and Ezra, which are in turn linked by the Golden Octagon of Constantine at Antioch and Santa Costanza at Rome. Adler (1873) considered that its plan was based on that of the Anastasis. Ashbee³ has suggested that the remains of the temple which Hadrian is said to have built on the same spot in honour of Jupiter may have influenced the plan, while others have insisted that it was a creation of Roman-type architecture.

Having examined the plan of the Dome of the Rock, Ecochard came to

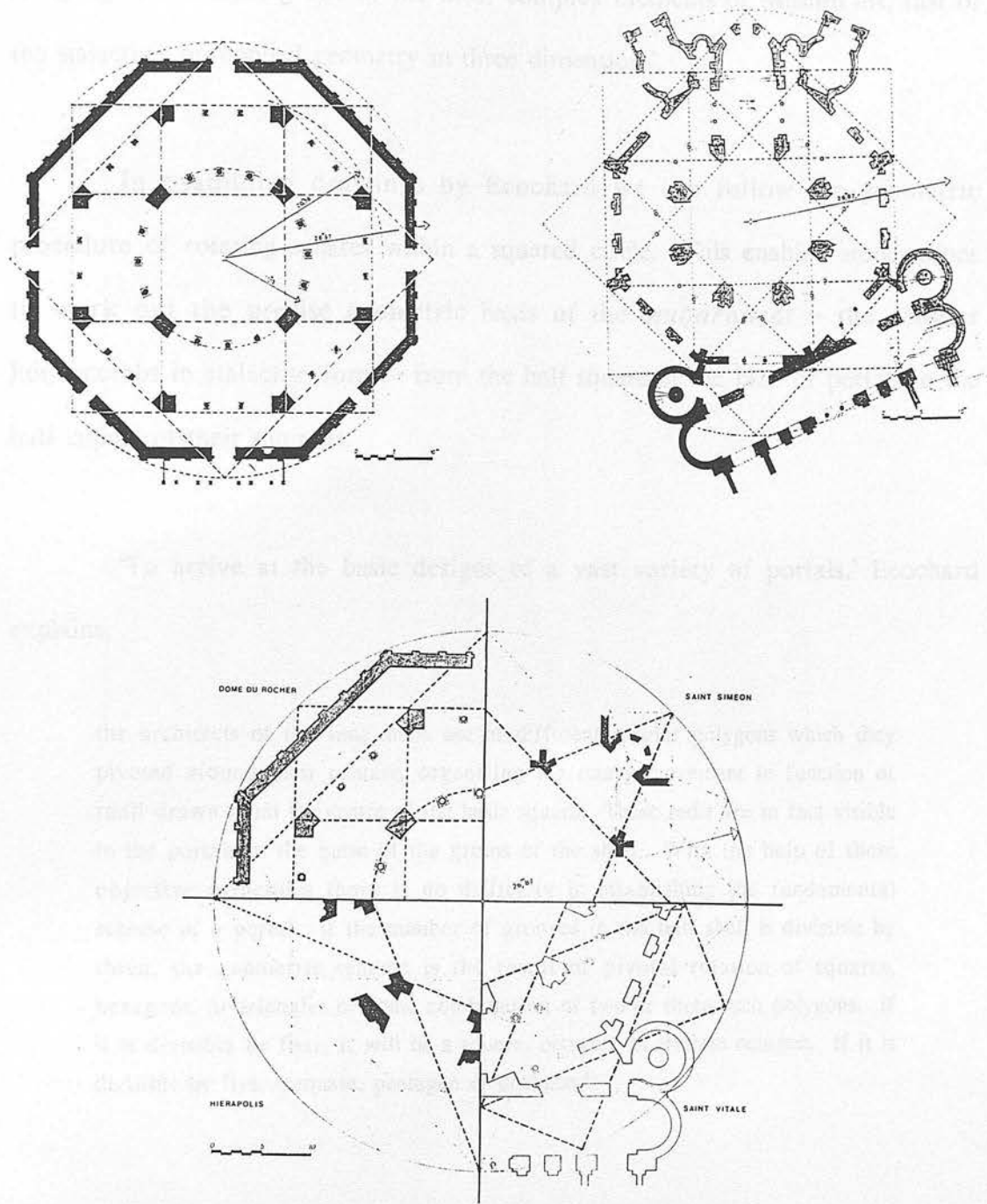
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1. *Filiation de Monuments grecs, byzantins, et islamiques: une question de géométrie* (Paris, 1977) p. 18-38.
 2. Cited by Creswell, vol. I, part I.
 3. *Op. cit.*

the conclusion that its shape followed the lines of an extremely precise geometric scheme. However, there appear to have been other monuments which followed the same geometric rules. What is more intriguing is that some of those monuments have exactly the same dimensions within a margin of 6 mm.'s difference. This, of course, could not, in Ecochard's view, have been coincidental.

In examining and comparing the plans from St. Vitale at Ravenna, the Church of the Ascension at Jerusalem, the Cathedral at Bosra, the Sanctuary of Pieria at Seleucia, and the Basilica of San Lorenzo at Milan, Ecochard confirmed his hypothesis that these plans had similar geometric schemes, all enclosed within a circle of the same dimensions. There were other monuments examined by Ecochard in which, although the plan was seen to be based on the same geometry, their proportions were reduced by half. Figs. 5.1.1 a, b, & c illustrates Ecochard's comparison between the geometry of the Dome of the Rock and that of St. Vitale.



Fig. 5.1.1 (a) Dome of the Rock, (b) St. Vitale, & (c) comparison of their analyses



Figs. 5.1.1 (a) Dome of the Rock, (b) Saint Vital, & (c) composition of four analyses

Michel Ecochard dedicated a great deal of his time and effort to studying and analyzing one of the most complex elements of Muslim art, that of the stalactites or 'applied geometry in three dimensions'.

In examining drawings by Ecochard we can follow the geometric procedure of rotating squares within a squared circle. This enabled stonemasons to work out the precise geometric basis of the *muqarnaṣāt* – the cellular honeycombs in stalactite form – from the half square at the base of portals to the half cupola of their summits.

'To arrive at the basic designs of a vast variety of portals,' Ecochard explains,

the architects of the time made use of different regular polygons which they pivoted around their centers, organizing the rotary movement in function of radii drawn from the centre of the basic square. These radii are in fact visible in the portals in the guise of the groins of the shell. With the help of these objective principles there is no difficulty in establishing the fundamental scheme of a portal. If the number of grooves in the half shell is divisible by three, the geometric scheme is the result of pivotal rotation of squares, hexagons, or triangles or some combination of two or three such polygons. If it is divisible by four, it will be a square, octagon, or stellate octagon. If it is divisible by five, a square, pentagon or decagon.¹

1. See fig. 5.1.2.

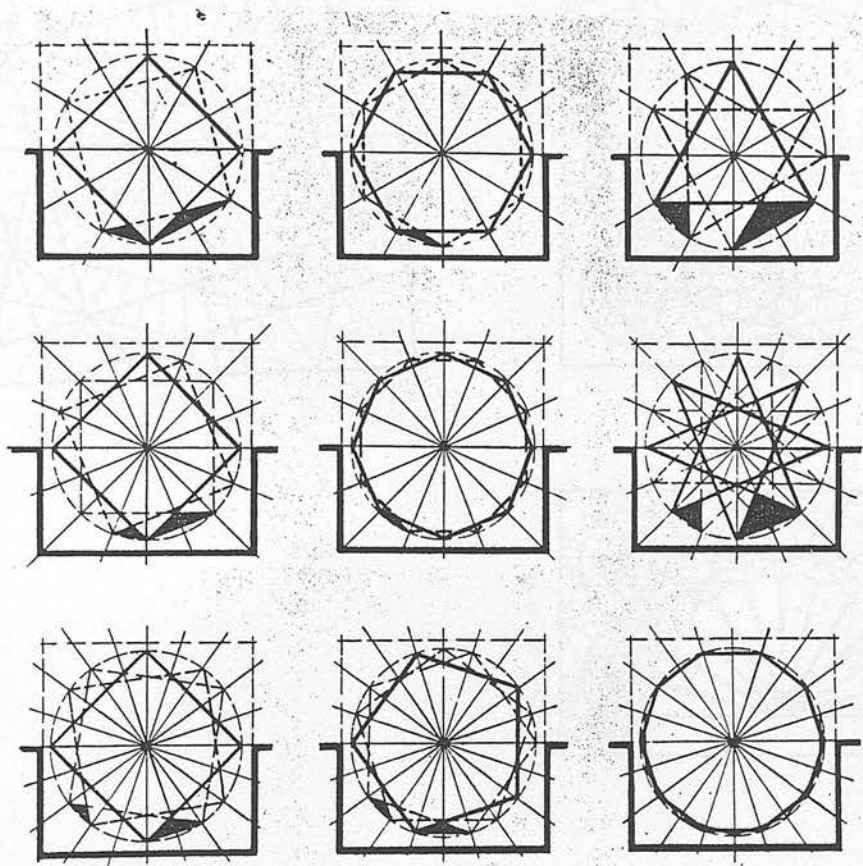
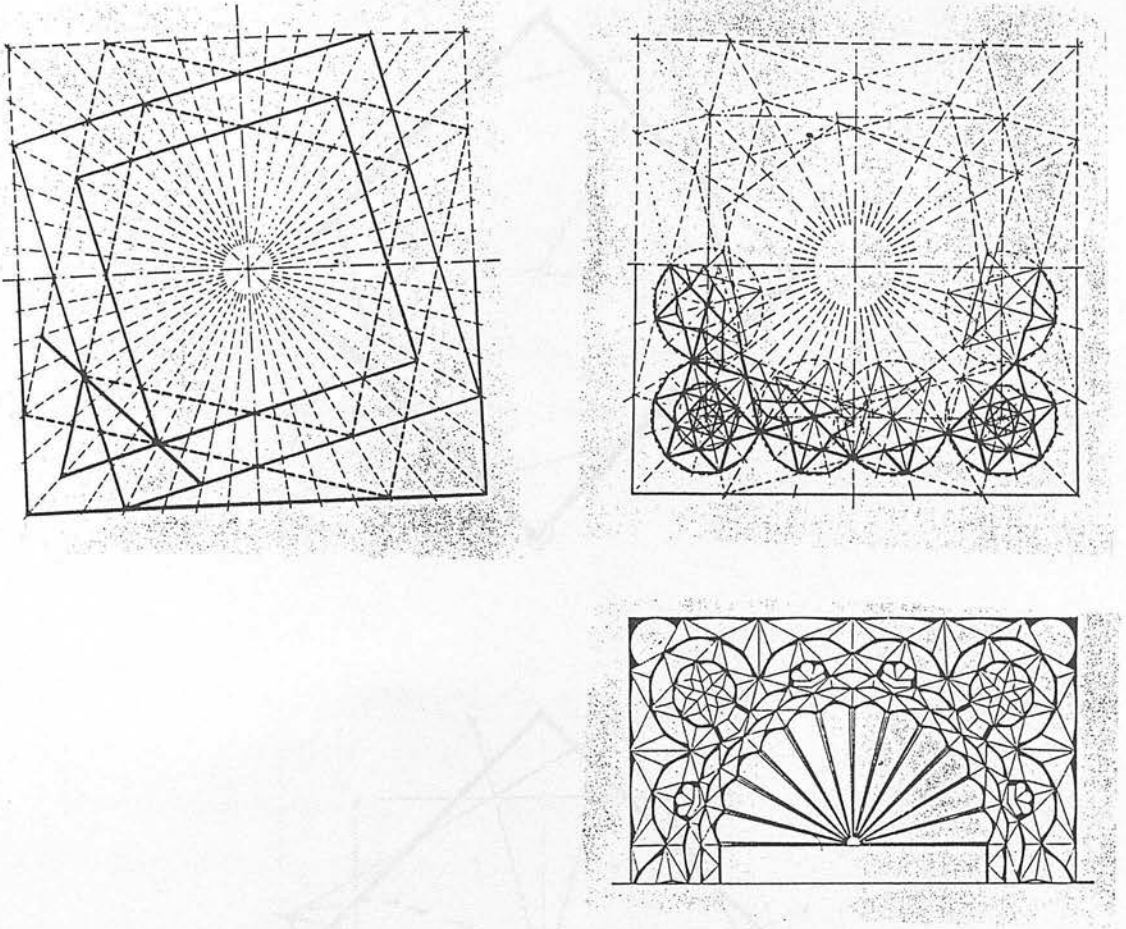


Fig. 5.1.2

The following diagrams (see figs. 5.1.3 a, b, & c) indicate how each of these geometric systems of rotating squares can be initiated. Note how although Ecochard does not indicate this in his A,B,C figures, the circle had to be accurately squared in order to obtain the initial divisions.

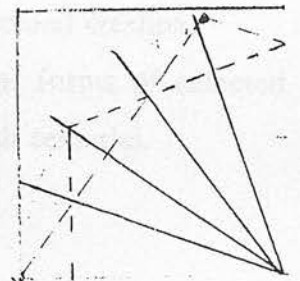
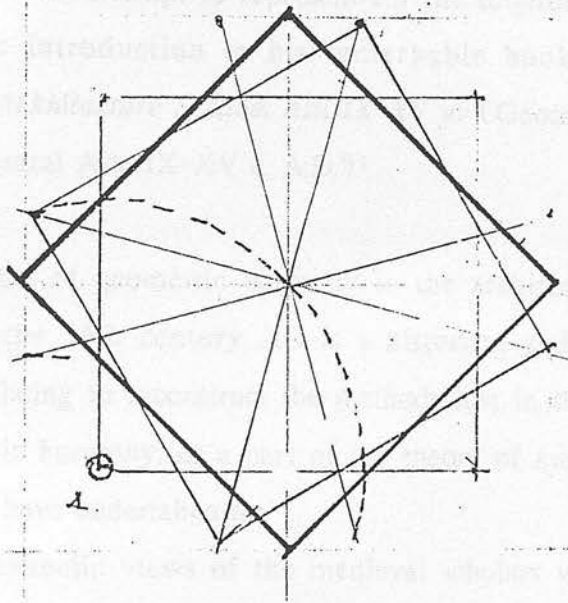
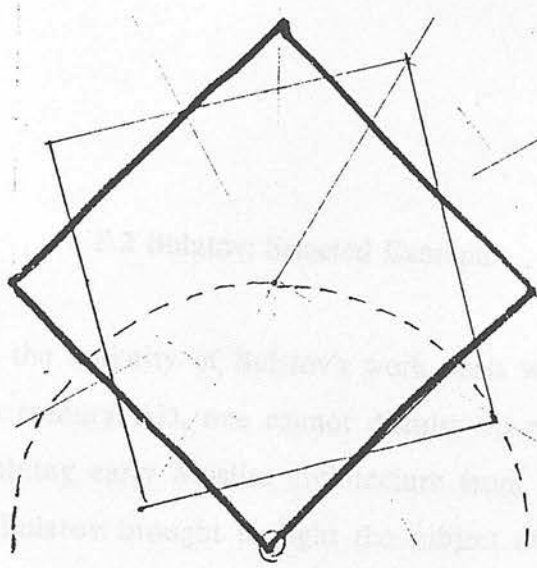


Figs. 5.1.3 a, b, & c

The remarkable working drawings of Ecochard were more than a mental exercise in complex patterns of geometry. In fact, they have served as means for accurate restoration of Syrian portals¹ dating from the twelfth to the fourteenth century.

The following analyses by Ecochard served for the reconstruction of the original portal in the Madrasa Zāhiriyya of Damascus, again basing the design on the rotation of a square within a squared circle (see figs. 5.1.4 a, b, & c).

1. Papadopoulo, *Islam and Muslim Art*, at end.



Figs. 5.1.4 a, b, & c

5.2 Bulatov: Selected Examples

Although the majority of Bulatov's work deals with Islamic architecture after the eleventh century AD, one cannot dismiss his major role in setting a precedent in examining early Muslim architecture from a completely new and scientific angle. Bulatov brought to light the subject of 'applied geometry of medieval Islam'.

Here we shall attempt to represent for the English reader the arguments contained in the introduction to his remarkable book *Geometricheskaia Garmonizatsia v Arkhitekture Srednei Azii IX-XV vv* ('Geometric Harmony in the Architecture of Central Asia IX-XV c. A.D.').¹

The subject of geometric harmony in the architecture of Central Asia from the 9th to the 15th century AD is a historical and theoretical piece of research, its aim being to reconstruct the methods lost in the course of time for obtaining geometric harmony, as a part of the theory of medieval architecture of Central Asia. We have undertaken to:

- A) Examine the aesthetic views of the medieval scholars who exerted influence on the architectural theory and practice of the Middle East.
- B) Reveal the association of applied mathematics with architectural creation.
- C) Analyze the method of construction of architectural forms of selected examples from the architecture of Central Asia of the 9th–15th centuries.

1. Published in Moscow, 1985.

Such work is conducted for the first time and introduces into scientific practice little known data which allow one to look at the subject in a completely different way, as the architectural science of the Middle Ages in the countries of the Middle and Near East, to speak not only of its existence but also of its high degree of development.

Our research envelopes the period covering the 9th–15th centuries AD, which were characterized by a great flourishing of science, culture, and the arts in medieval Central Asia. We deliberately do not touch on the problem of the high renaissance because it continues to be under discussion and also by virtue of the fact that its examination would lead us away from our basic theme, although the content of our research may to a certain degree further the idea of widening the phenomena.

In this work the author does not set himself the task of embracing all aspects of the problem of geometric harmonization with the desired fulness. It is not possible to carry out a wide comparative analysis of the structures of these architectural forms of the Near and Middle East because of the lack of an adequate number of published materials with surveyed measurements, which means that such work could only be carried out in a selective and fragmentary way.

Selected Examples

From Bulatov's work *Geometricheskaia Garmonizatsia* we choose to examine four particular areas:

1. The geometric analysis of the Mausoleum of 'Ā'ishah Bībī.¹
2. The geometric analysis of Masjid-i Jāmi' in Samarqand.²
3. The geometry of space between individual buildings.³
4. The geometry of decorative pattern.⁴

The Mausoleum of 'Ā'ishah Bībī (see fig. 5.2.6)

At first glance one gains the impression that Bulatov used the same system of concentric squares to analyze his structure as we have used in the examples of chapter 3 of the present work. But then when we draw the circle which would be squared by the overall plan of the mausoleum, we see that the first points where the circle intersects the diagonals have no relevance to the analysis. Also, if we continue the sides of Bulatov's first concentric square represented by our square 5,6,7,8 (see fig. 5.2.1), we are left with the central space of the mausoleum still unexplained geometrically.

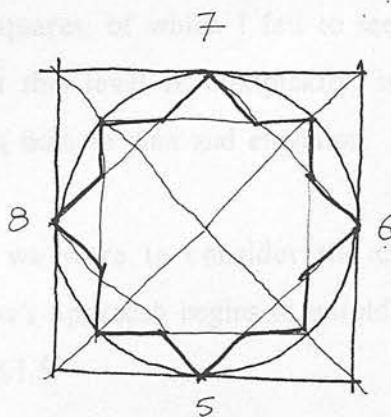


Fig. 5.2.1

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1. *Geometricheskaia Garmonizatsia*, p.80.
 2. *Ibid.*, p.160.
 3. *Ibid.*, p.210.
 4. *Ibid.*, p.287.

However, by adopting a different principle of proportion, we see it often used in the architecture of Central Asia.¹ This is usually formed by dividing the square by drawing the diagonals of the two sets of semi-squares, arriving at a slanted central square whose side is $1/\sqrt{5}$ and where the diagonal itself is $\sqrt{5}/2$ (see fig. 5.2.2).

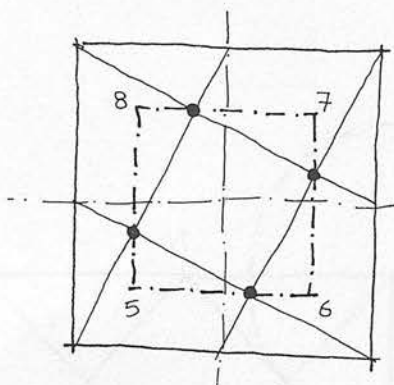


Fig. 5.2.2

Both the line of the external walls and the central space of the mausoleum can now be explained in relation to Bulatov's analysis. From this square Bulatov draws his concentric squares, of which I fail to see any significance because the diagram itself, without this level of complexity, is adequate to place the main elements of the building both in plan and elevation.

However, if we were to consider the central square 5,6,7,8 as the generative unit, Bulatov's approach begins to unfold as follows, as represented the following figs.: 4.1.3 – 4.1.5.

1. L. Golombek, *Timurid Architecture*, p.142, 144.

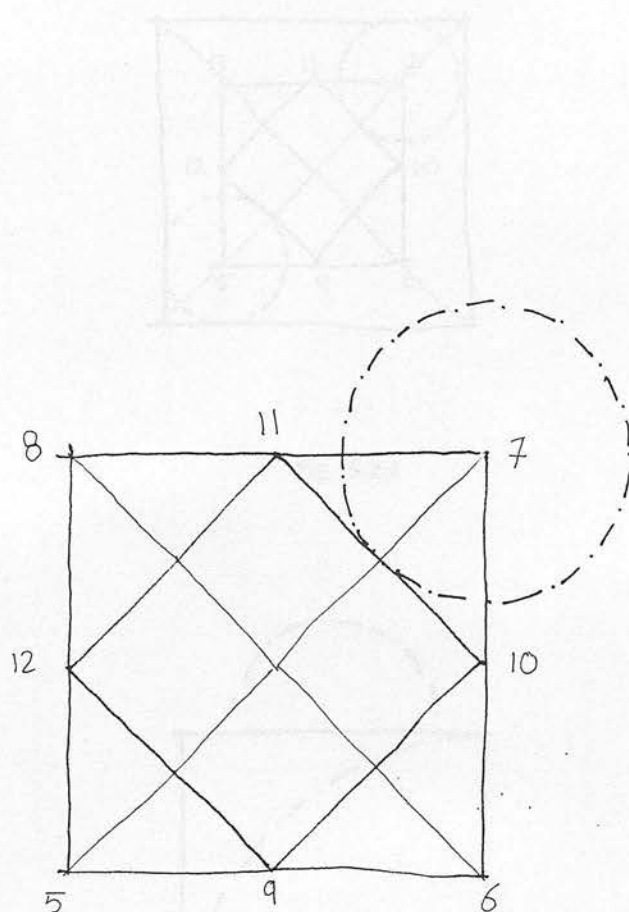


Fig. 5.2.3

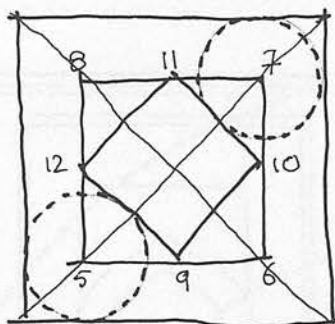


Fig. 5.2.4

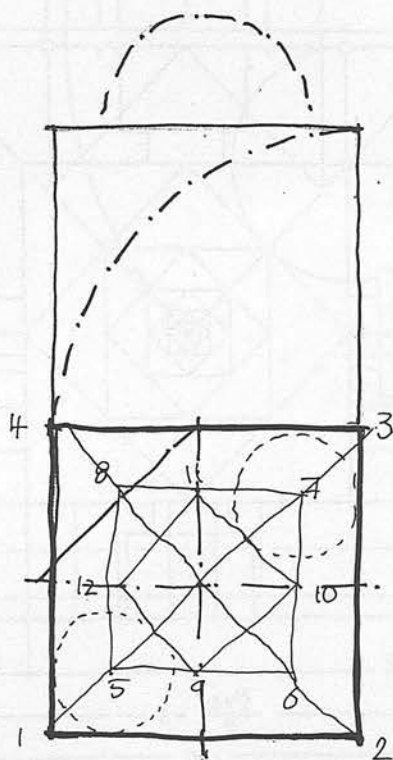


Fig. 5.2.5

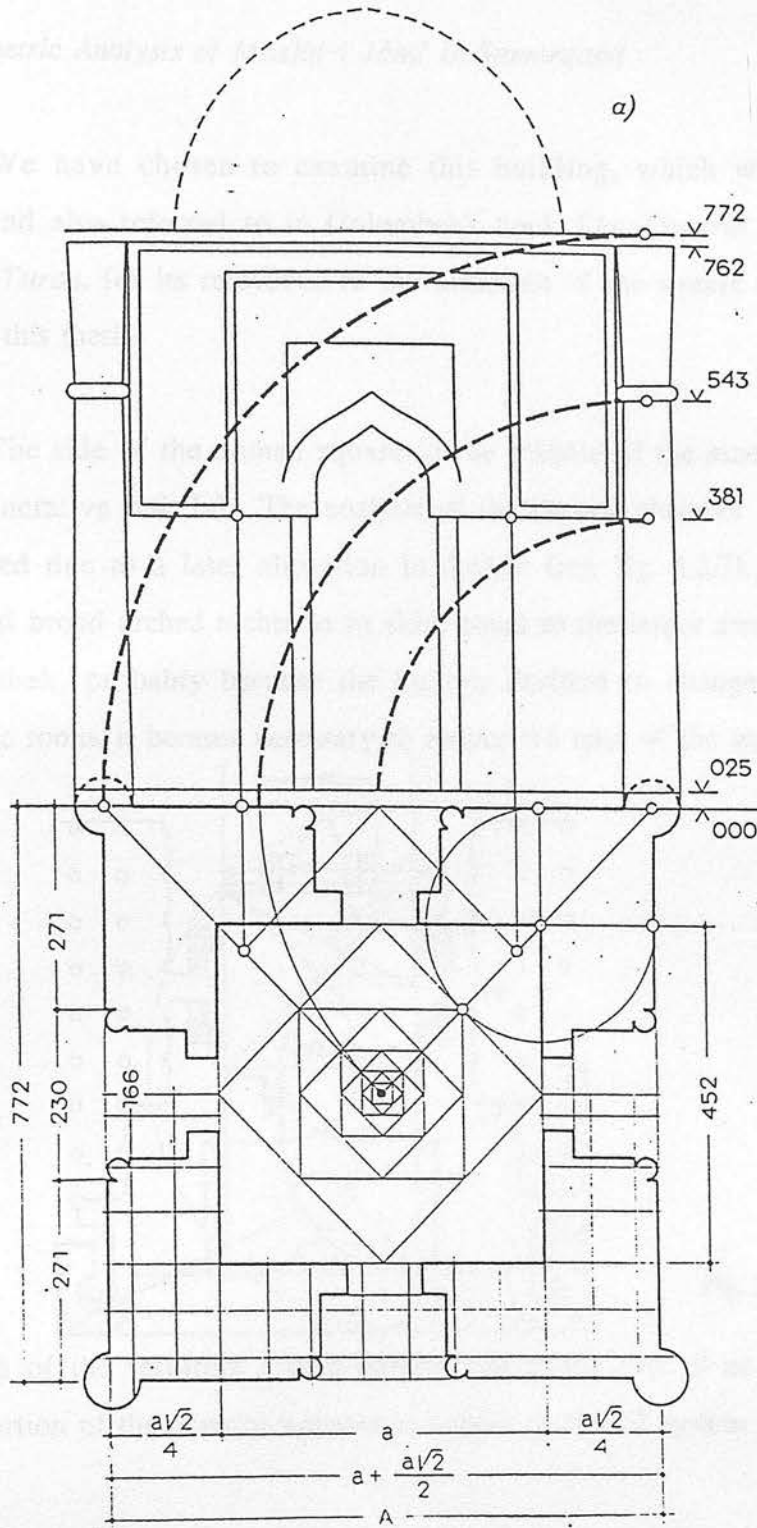


Fig. 5.2.6

The Geometric Analysis of Masjid-i Jāmi' in Samarqand

We have chosen to examine this building, which was analyzed by Bulatov and also referred to in Golombek's book *The Timurid Architecture of Iran and Turan*, for its relevance to the principle of the square and the circle as argued in this thesis.

The side of the domed square in the middle of the sanctuary appears to be the generative unit (a). The analysis of the domed chamber was made more complicated due to a later alteration in design¹ (see fig. 5.2.7). Originally, the square had broad arched niches in its sides equal to the larger area M . According to Golombek, probably because the builder decided to change the method of roofing the room, it became necessary to reduce the span of the wall arches.

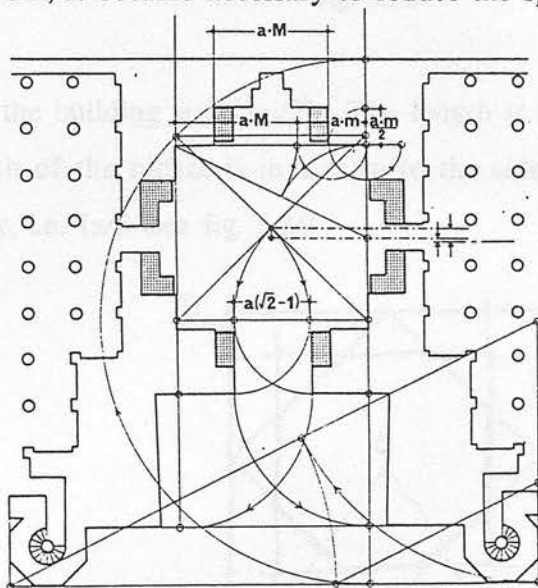


Fig. 5.2.7

The width of the resulting niches corresponds to the side of an octagon ($\sqrt{2}-1$). The proportion of the exterior appears to adhere to the $\sqrt{2}$ system (see fig. 5.2.8).

1. Golombek, *Timurid Architecture*, p.146.

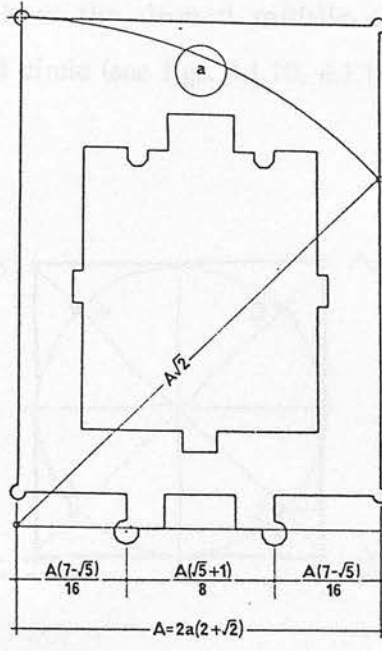


Fig. 5.2.8

The width of the building is $2a(2+\sqrt{2})$. The length is this dimension multiplied by $\sqrt{2}$. The width of the niches is in a ratio to the side of the square of the lateral dome chamber, i.e. $1:\sqrt{3}$ (see fig. 5.2.9).

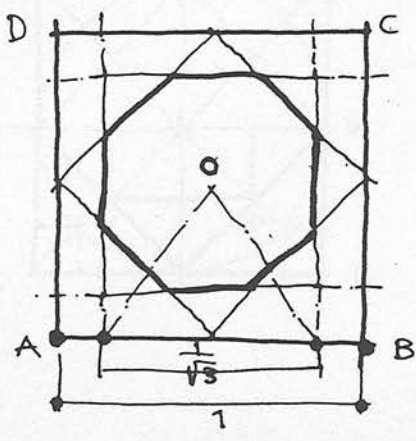


Fig. 5.2.9

One particular aspect which appears to have been overlooked in Golombek's account of Bulatov's analysis of this building is the relevance of the squared circle, and in a small diagram which appears adjacent to the mosque

plan,¹ Bulatov reveals how the domed middle section of the sanctuary is subdivided using a squared circle (see figs. 4.1.10, 4.1.11, and 4.1.12).

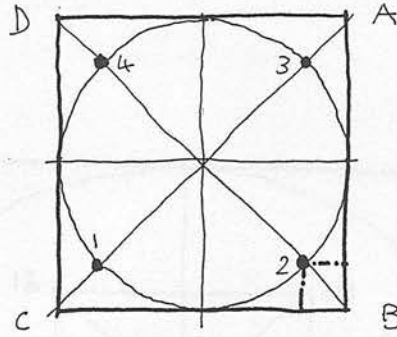


Fig. 5.2.10

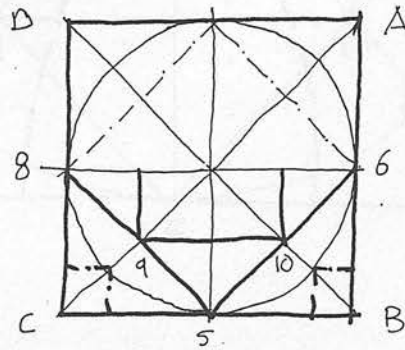


Fig. 5.2.11

1. *Geometricheskaia Garmonizatsia*, p.160.

The Geometry of Space between radiating buildings

This concept is perhaps unique to the later period of early Muslim architecture and may be peculiar to the eastern region of that architecture. Although we have viewed early examples of early Muslim architecture in chapter 3 of this book, most early examples have been shown to be related to a single underlying geometric model. Another clue to the origin of this model

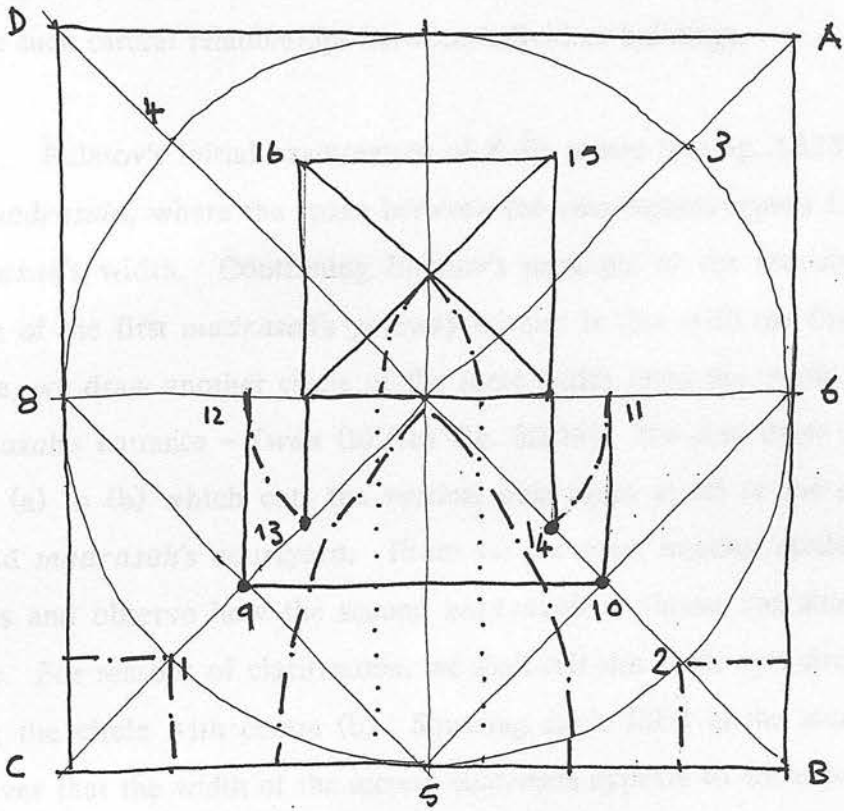


Fig. 5.2.12

The Geometry of Space between Individual Buildings

This concept is perhaps unique to the later period of early Muslim architecture and may be peculiar to the eastern region of the Islamic world. Although we have viewed early examples of 'early Muslim architecture' in chapter 3 of this thesis where separate buildings have been shown to be related in a strong unifying geometric model, nowhere else in the Middle or Near East do we see such careful relationships between individual buildings.

Bulatov's initial examination of Kalia square (see fig. 5.2.13),¹ formed by two *madrasahs*, where the space between the two façades equals $1/2$ of the first *madrasah's* width. Continuing Bulatov's principle of the semicircle from the centre of the first *madrasah's* gateway (a) and in line with the first *madrasah's* façade, we draw another circle of the same width from the centre of the second *madrasah's* entrance – *īwān* (b) (see fig. 5.2.14). We then draw a vertical axis from (a) to (b) which cuts the vertical axis again at (c) in the middle of the second *madrasah's* courtyard. From (c) we draw another circle of the same radius and observe how the second *madrasah* is almost contained within this circle. For reasons of clarification, we shall call this circle unit circle MD2, MD1 being the circle with centre (b). Squaring circle MD2 in the usual manner, we discover that the width of the second *madrasah* appears to correspond to the first points of intersection between the circle and the major diagonals. Therefore the façade of the first *madrasah* relates to the façade of the second *madrasah* in the proportion of $1:\sqrt{2}$.

Reverting to the overall geometry of the first *madrasah*, it appears that one may repeat drawing the same unit circle MS2 and MS3 (as shown in fig.

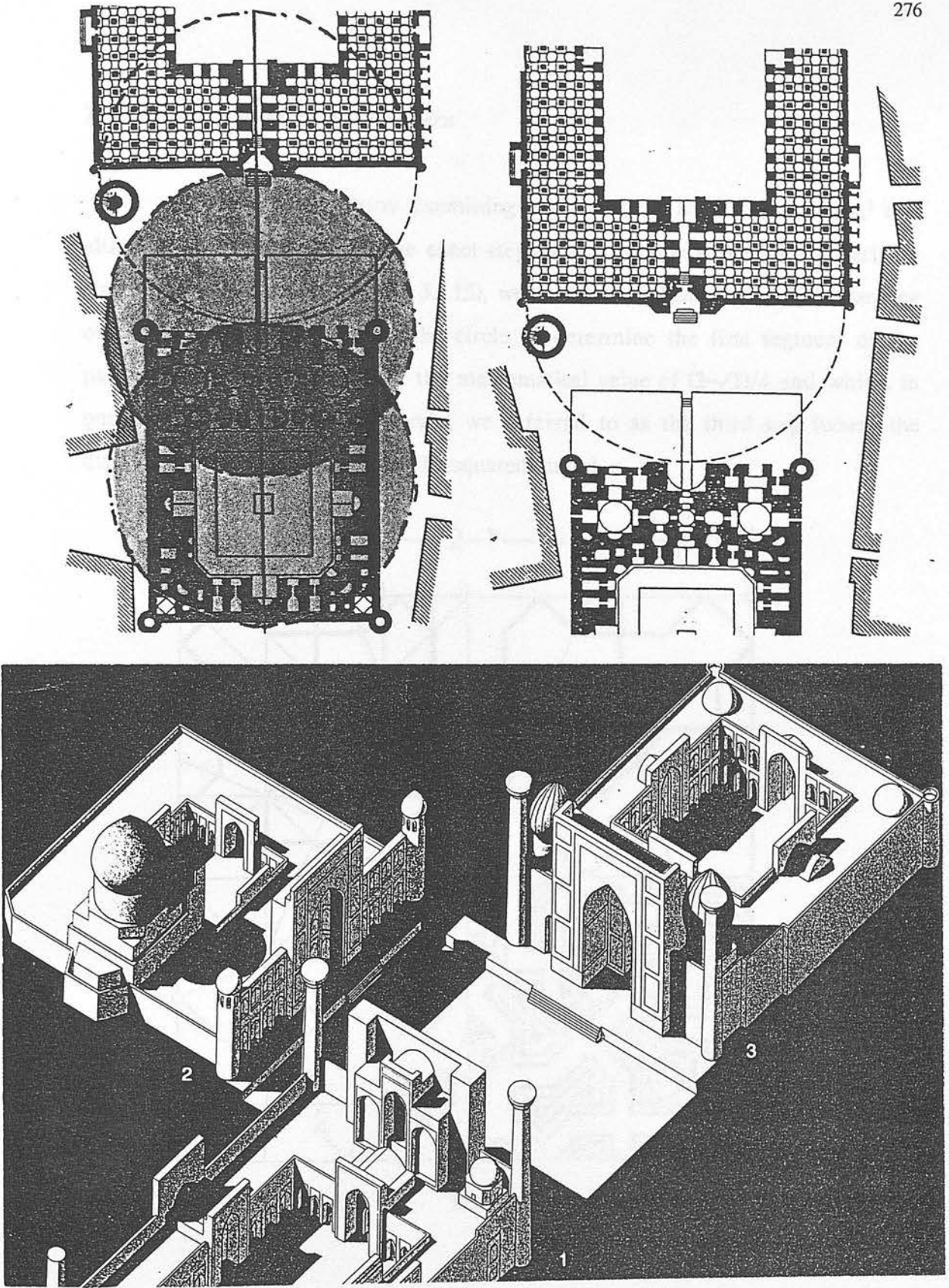
1. *Geometricheskaia Garmonizatsia*, p.210.

5.2.14) in the first *madrasah* up to the far side of the dome covering the central aisle and until just before the *mihrāb* enclosure, in a proportion of 2:3.

The individual buildings of Central Asia appear to be able to relate geometric proportion to the main elements of the courtyard – 4 *iwāns* concept (see illus. 5.2.1).¹ This relationship is between buildings of equal importance and status, set in contrast to the interdependency of the smaller buildings within major structures of early Islamic architecture (up to the 9th century AD).

Examples include the Annex of Ukhayḍir and its relation to the main abode, the racecourse of Jawsaq al-Khāqānī, the mosque within the round city of Baghdad, and the relationship of the living quarters of the main Bulkuwāra Palace to the adjacent position of the main stretch of central gardens.

1. V. Geren, *Srednya Aziia* (Moscow, 1985).



Illus. 5.2.1

The Geometry of Decorative Pattern

Here we see Bulatov examining a unit pattern from Central Asia,¹ and although he does not reveal the exact steps which produced the main underlying structure of the pattern (see fig. 5.2.15), we can nevertheless use our understanding of the principles of squaring the circle to determine the first segment of the pattern to which Bulatov gives the mathematical value of $(2-\sqrt{2})/4$ and which, in our method of squaring the circle, we referred to as the third step (where the diagonals of the square intersect the squared circle).

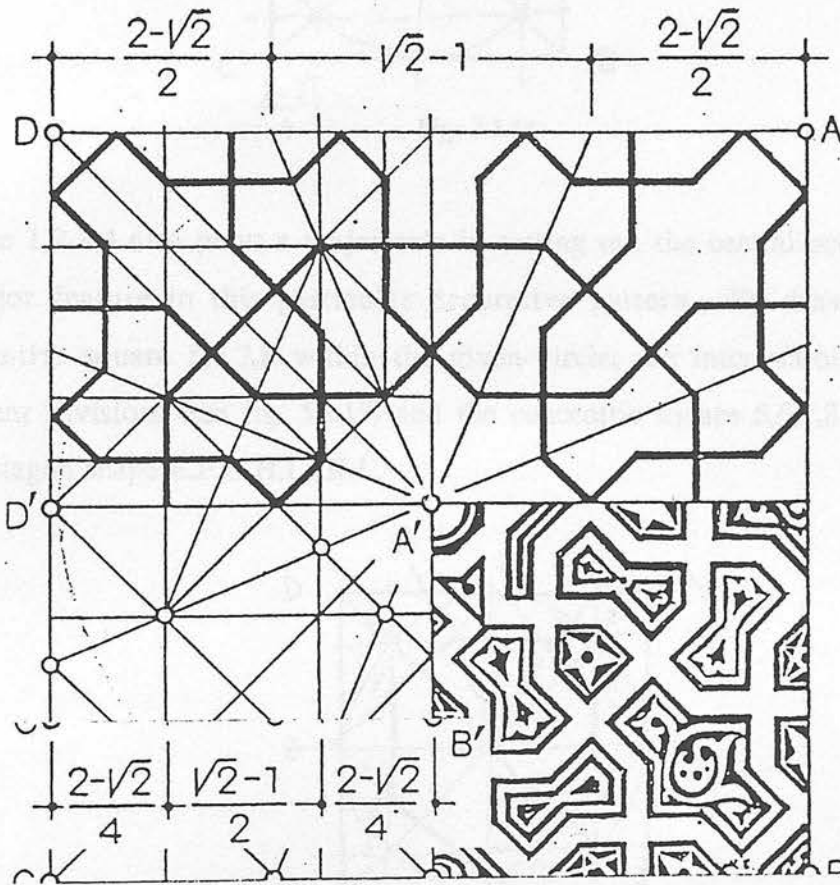


Fig. 5.2.15

1. *Geometricheskaja Garmonizatsia*, p.210.

By drawing the circle which would have been enclosed by square C,B,A,D, diagonals B-D and A-C will cut the circle at points 1, 2, 3, and 4, thus resulting in the first segment mentioned above (see fig. 5.2.16).

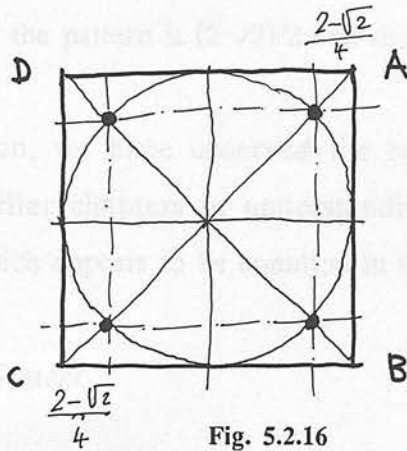


Fig. 5.2.16

Square 1,2,3,4 also plays a major role in setting out the central octagon which is a major feature in this particular decorative pattern. By drawing the first concentric square 5,6,7,8 within the given circle, the intersections of the first segment divisions (see fig. 5.2.17) and the concentric square 5,6,7,8 will result in the octagon shape E,F,G,H,I,J,K,L.

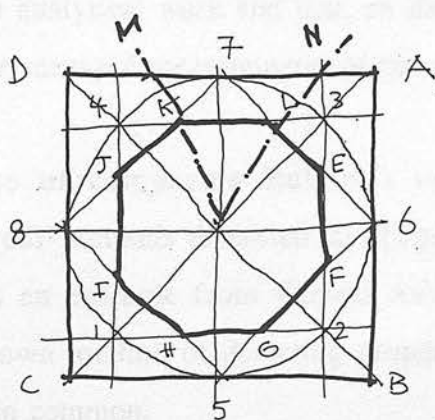


Fig. 5.2.17

The proportion of the resultant second segment 4-K, Bulatov interprets

arithmetically, as we have seen earlier, as $(\sqrt{2}-1)/2$.

If octagon diagonal O-K is extended to meet D-A at M, and octagon diagonal O-L is similarly extended to meet A-D at N, the proportion of D-M in relation to the rest of the pattern is $(2-\sqrt{2})/2$ and the proportion of M-N is $\sqrt{2}-1$.

In conclusion, we have observed the significance of the practice we discussed in the earlier chapters in understanding Bulatov's analysis of this interesting pattern which appears to be common in Central Asia.

Comparisons to our System

Bulatov's enthusiasm for interpreting geometric configurations in complex arithmetic can represent to many geometry purists a case of stating the obvious, especially when considering the fact that numerical mathematics was originally derived from geometry, which is of a much more fundamental order than manipulation of numbers which is the creation of man. However, when examining Bulatov's analysis closely, one discovers that these formulae are repeated in almost all the analytical work and that, on the whole, they serve as a checking device against the surveyed measurements of the actual structures.

For the purpose of comparing Bulatov's technique in analyzing architectural forms with our methods discussed in chapter 3 of this thesis, we choose to look closely at an example from Central Asia analyzed by Bulatov. We shall then apply our own method of detecting proportion and see if the two techniques have anything in common.

The building is the *Madrasah* of Ulugh Beg in Samarqand (see fig. 5.2.18), erected in AD 1420. Bearing in mind that Bulatov does not anywhere in

his analysis make clear the origins of each step, one is nevertheless able to follow the various stages of his analysis on the basis of the previously examined examples.

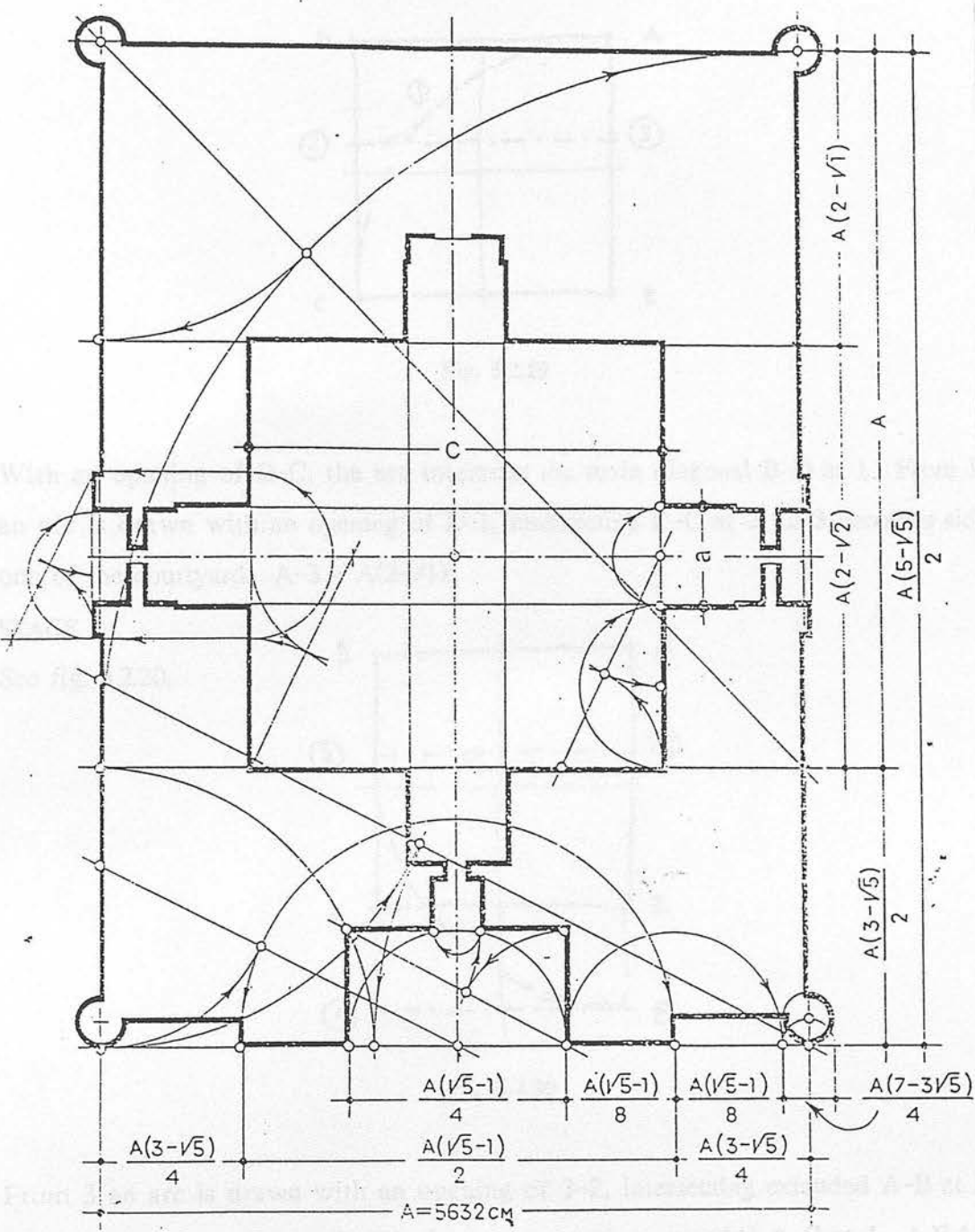


Fig. 5.2.18

STAGE 1

Assuming square A,B,C,D, in order to establish the first line which subdivides the courtyard, Bulatov draws an arc from B (see fig. 5.2.19):

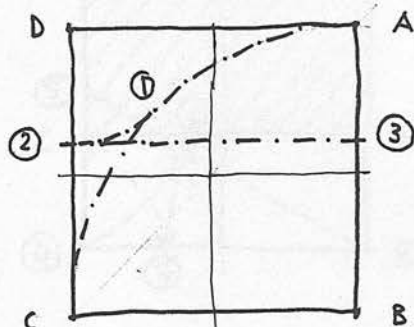


Fig. 5.2.19

With an opening of B-C, the arc intersects the main diagonal B-D at 1. From D an arc is drawn with an opening of D-1, intersecting D-C at 2. 2-3 becomes side one of the courtyard. $A-3 = A(2-\sqrt{1})$.

STAGE 2

See fig. 5.2.20.

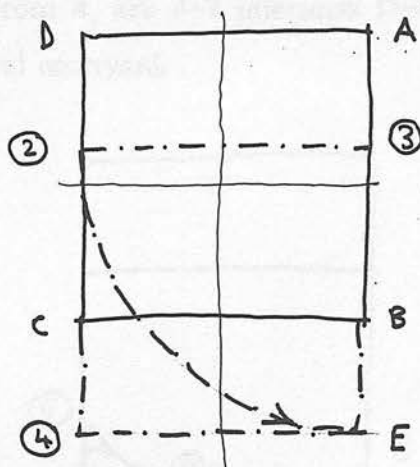


Fig. 5.2.20

From 3 an arc is drawn with an opening of 3-2, intersecting extended A-B at E. From E a line parallel to B-C is drawn, intersecting extended A-C at 4. A,E,4,D represents the overall shape of the building, where $A-E = A(5-\sqrt{5})/2$.

STAGE 3

See fig. 5.2.21

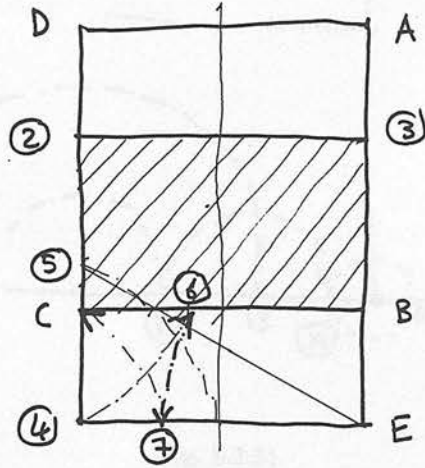


Fig. 5.2.21

To confirm geometrically the opposite side of the courtyard, an arc is drawn from 4 with an opening of 4-G, intersecting D-4 at 5. 5 and E are connected. From 5 we draw an arc with opening 5-4 intersecting 5-E at 6. From E, arc E-6 intersects E-4 at 7. From 4, arc 4-7 intersects D-4 at C. Thus C-B is the opposite side of the central courtyard.

STAGE 4

See fig. 5.2.22

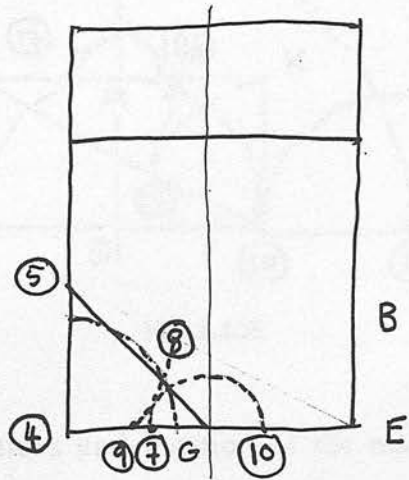


Fig. 5.2.22

Diagonal 5-G appears to intersect arc C-7 at 8. From 8 we drop a perpendicular line intersecting 4-E at 9. From G, with radius G-9, we draw a semicircle

intersecting K-L again at 17.

STAGE 8

This stage begins by determining the remaining sides of the courtyard. This is simply done by drawing a circle enclosed between 2-3 (already established) and C-B (see fig. 5.2.26).

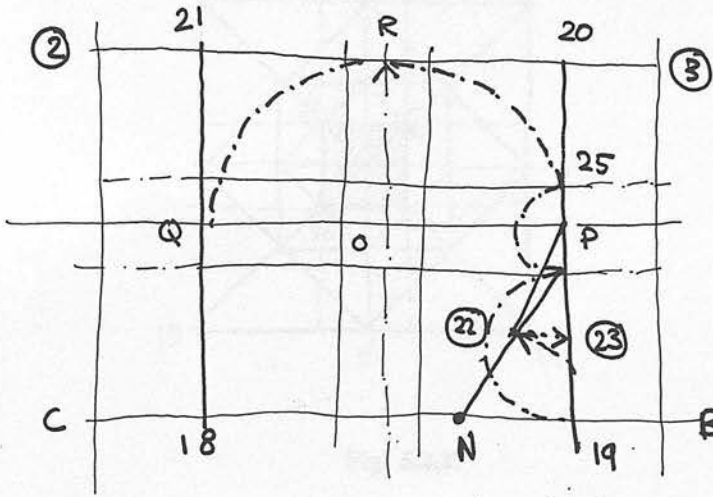


Fig. 5.2.26

Squaring this circle, sides 21-18 and 20-19 can now be located. In order to establish the location and proportions of the four internal *īwāns* leading off the courtyard, again a generative diagonal, N-P, is drawn. From N arc N-19 intersects N-P at 22. From P arc P-22 intersects 20-19 at 23. From 23, with radius 23-19, a semicircle is drawn intersecting 20-19 again at 24. From P, with radius P-24, another semicircle is drawn intersecting 20-19 at 25. Thus we determine the width and positions of *īwān* no.1 and so on for the remaining three *īwāns*.

In our method of analysis, as distinct from that of Bulatov, we basically investigate whether a building corresponds to a harmonic system of concentric

squares and circles. To superimpose this pattern on the *Madrasah* of Ulugh Beg in Samarqand, we take the square shape of the courtyard as the generative element and proceed inwards and outwards. In fig. 5.2.27 we see how, by proceeding inwards drawing concentric squares, we arrive at square 26,27,28,29, which appears to line up with the four *iwāns* leading off the courtyard.

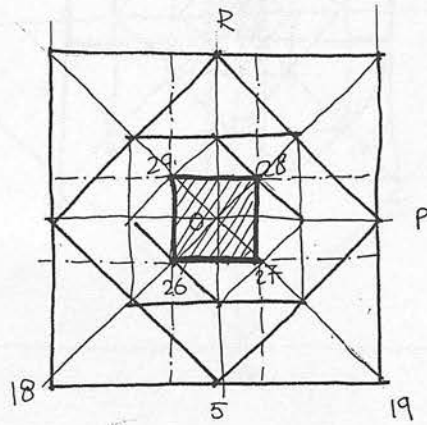


Fig. 5.2.27

The depth of the *iwāns* appears to be determined by the next step shown in fig. 5.2.28, where by circling square 18,19,20,21 and then squaring it to produce square 30,31,32,33, we in fact limit the depth of the *iwāns* by sides 32-31, 31-30, 30-33, and 33-32.

The next stage is to establish the external limits of the building. This is achieved by proceeding to draw alternate squares outwards (see fig. 5.2.29) until reaching master square 38,39,40,41.

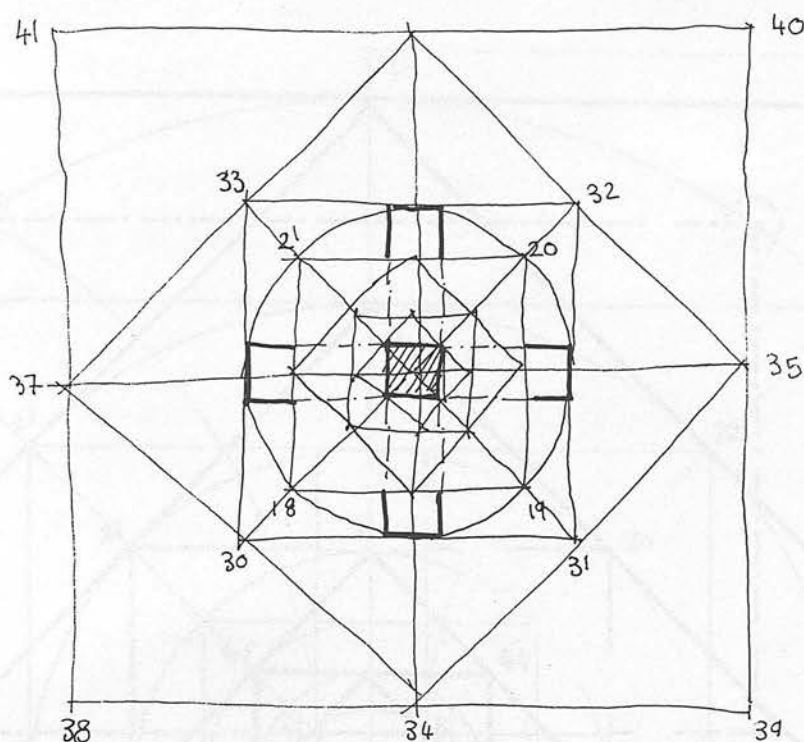


Fig. 5.2.29

This square does not appear to yield much concerning the external perimeter of this building. Only when further steps, uncharacteristic of our method of analysis, are taken do we appear to achieve relevant results. If a semicircle is drawn from 35 with a radius that passes through the corners of the smallest concentric square 26,27,28,29, the semicircle intersects the circle squared by the master square 38,39,40,41 at points A and E1. Similarly, on the opposite side a semicircle drawn from centre 37 passing through the corners of the small concentric square 26,27,28,29 will intersect at points D and 4.1 (see fig. 5.2.30), which is not as convincing as Bulatov's method.

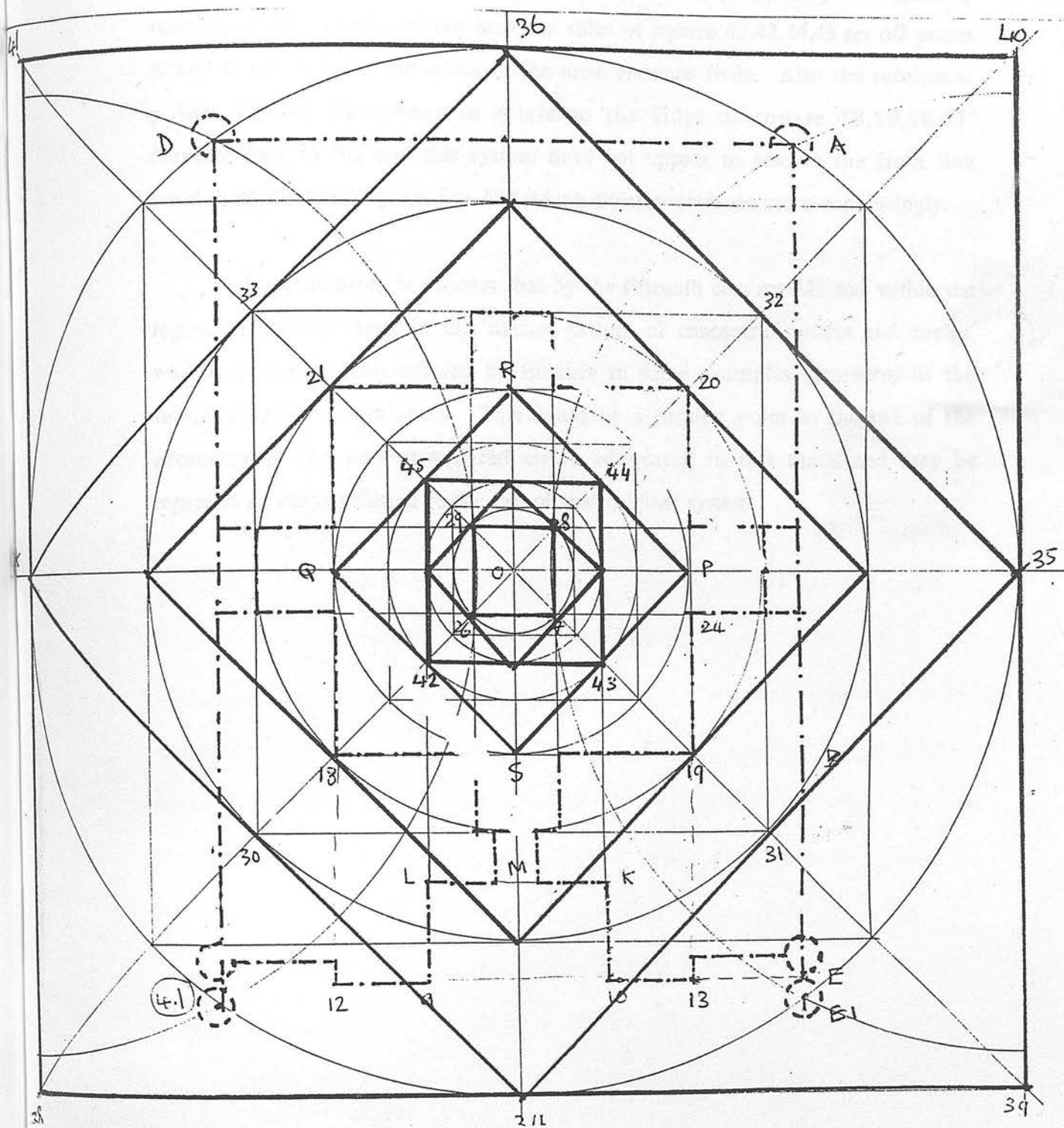


Fig. 4.1.30

However, when examining the elements which make up the façade in relation to our system, we see how the sides of square 42,43,44,45 set off points K and L which mark the width of the main entrance *īwān*. Also the recesses at points 12 and 13 appear to relate to the sides of square 18,19,20,21. Nevertheless, in the end this system does not appear to resolve the front line position of the building, i.e. line E-4 which Bulatov achieves more convincingly.

In conclusion, it appears that by the fifteenth century AD and within the region of Central Asia, in this master pattern of concentric squares and circles, which is clearly demonstrated by Bulatov in some examples, geometry of the *semi-square*¹ predominated. This could be a turning point in the use of the geometry of the perfect squared circle advocated in this thesis and may be regarded as the conclusion to the use of that original system.

1. Golombek, *Timurid Architecture*, p.141.

5.3 Lisa Golombek on Timurid Buildings

Lisa Golombek's book *The Timurid Architecture of Iran and Turan* contains a most significant section on applied geometry in the architecture of Central Asia in the fourteenth century.¹ Here Golombek deals with the nature of the geometric system, the 'working model' as found in Timurid architecture.

The architect, we are told, conceptualized the design of a building using two processes simultaneously. One was analytical, the other geometric. His parameters were function, budget, schedule, and scale. Once the architect and the patron had come to an understanding on the unit measure, the general needs, and the scale of the building, the architect could proceed to work out the details on his own. After the design was drawn up on the basis of geometric proportions, the architect returned to the analytical process. He selected one dimension within the design to serve as a module. Golombek tells us that most often in Timurid buildings the thickness of the walls served as a module. The module was then subdivided into smaller units corresponding to the size of the brick plus a joint.

The architect appears to have had at his disposal four systems of proportion or sets of ratios which he could mix. These were:

1. The square and its derivatives, particularly the diagonal $\sqrt{2}$, its half, and its double and the side of an octagon ($\sqrt{2}-1$) (see fig. 5.3.1).

1. P.141.

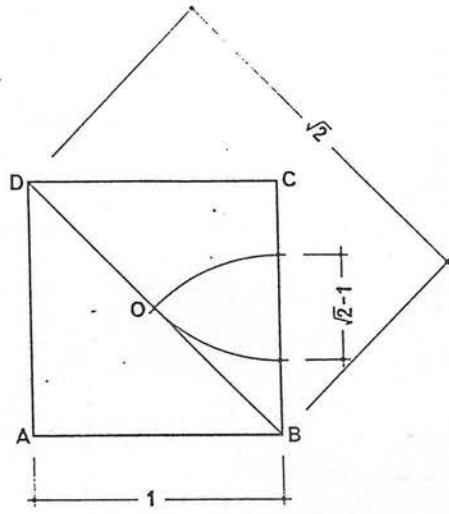


Fig. 5.3.1

2. The equilateral triangle and its derivatives (see fig. 5.3.2). Sometimes the geometry of the square and the equilateral triangle were combined, as in rectangles of $\sqrt{2}:\sqrt{3}$. The side of an equilateral triangle, whose height is half the length of the generative square, was frequently used for the width of niches in dome chambers of Central Asia, with intersecting arches as support systems.¹ The proportion $1/\sqrt{3}$ (see fig. 5.3.3) can be drawn by inscribing a hexagon and extending its radials.

1. *Timurid Architecture*, p.141.

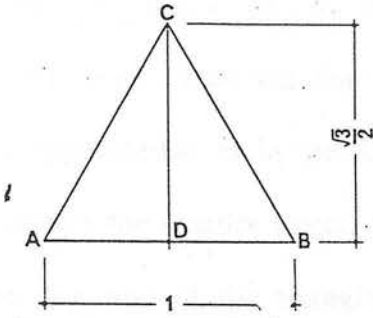


Fig. 5.3.2

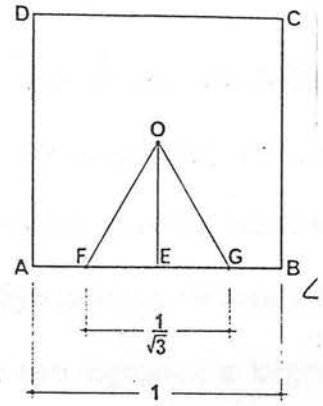


Fig. 5.3.3

3. The semi-square, usually formed by dividing the square into halves. By drawing the diagonals of the two sets of semi-squares, one arrives at a small square in the centre whose side is $1/5$ (see fig. 5.3.4). The diagonal, itself $\sqrt{2}/2$, plays an important role in the architecture of Iran and Turan, especially in determining elevational proportions.¹

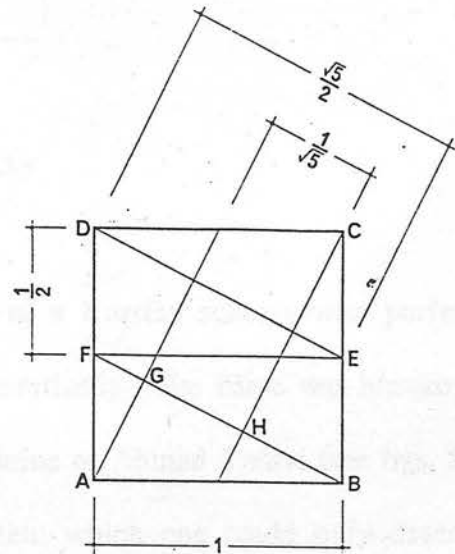


Fig. 5.3.4

1. *Ibid.*

4. The root five rectangle, using the semi-square. The base could be divided in another way. This was done by marking off an arc, the length of height, along the hypotenuse, as in the previous case, but then drawing a second arc, with its centre at the smaller angle, through the point in the hypotenuse. Where this arc cuts the base of the triangle, it divides the line into two segments: a larger one $(\sqrt{5}-1)/2$ or M, and a smaller one $(3-\sqrt{5})/2$ or 'm' (see fig. 5.3.5). Golombek believes that multiples of both segments were commonly used in Iran and Turan in designing both interior and exterior façades.¹

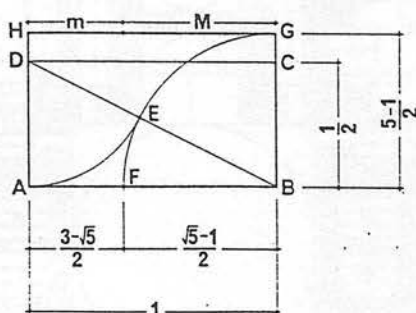


Fig. 5.3.5

Lisa Golombek also uses work of a Russian scholar who perfected a novel method of analyzing buildings geometrically. Her name was Mankovskaya and her system was used to analyse the shrine of Ahmad Yasavi (see figs. 5.3.6 & 5.3.7). The Russian scholar uses a system which one could only describe as 'analysis by numbers'. We follow this analysis step by step using diagrams only,

1. *Ibid.*

with no descriptive commentary, in order to demonstrate how easy it is to comprehend this format of analysis and proportion detection. Only in stage 20/21 do we appear to disagree with the author on the centre of the arc which produced point 22. If we had centred the arc on point 20, as indicated by the author, the result would be point B which is outside the extremity of the building, whereas if we centred the arc on point A, as shown in fig. 5.3.7, the resultant point 22 is now more convincing.

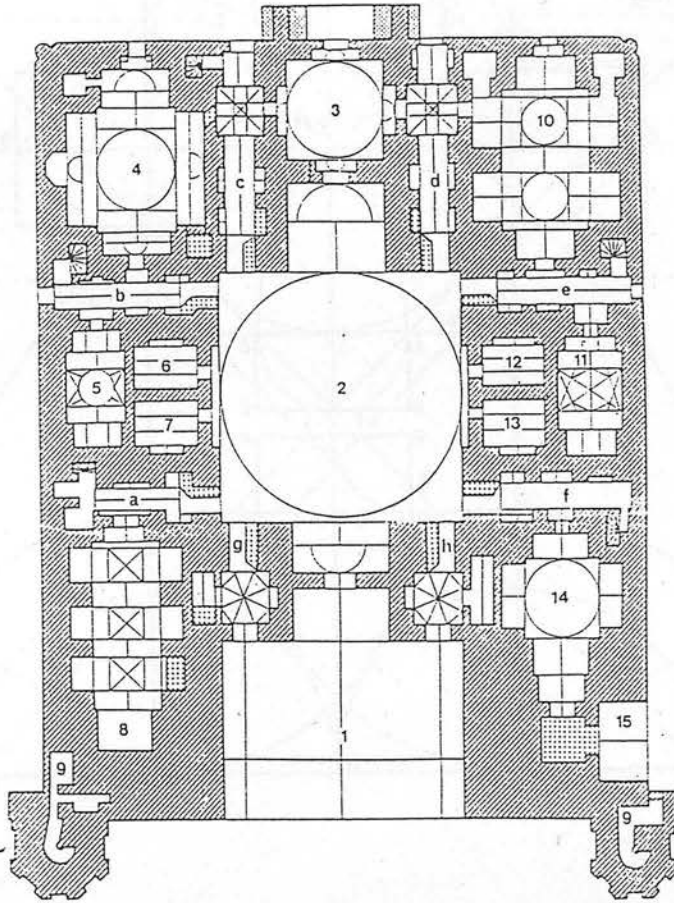


Fig. 5.3.6

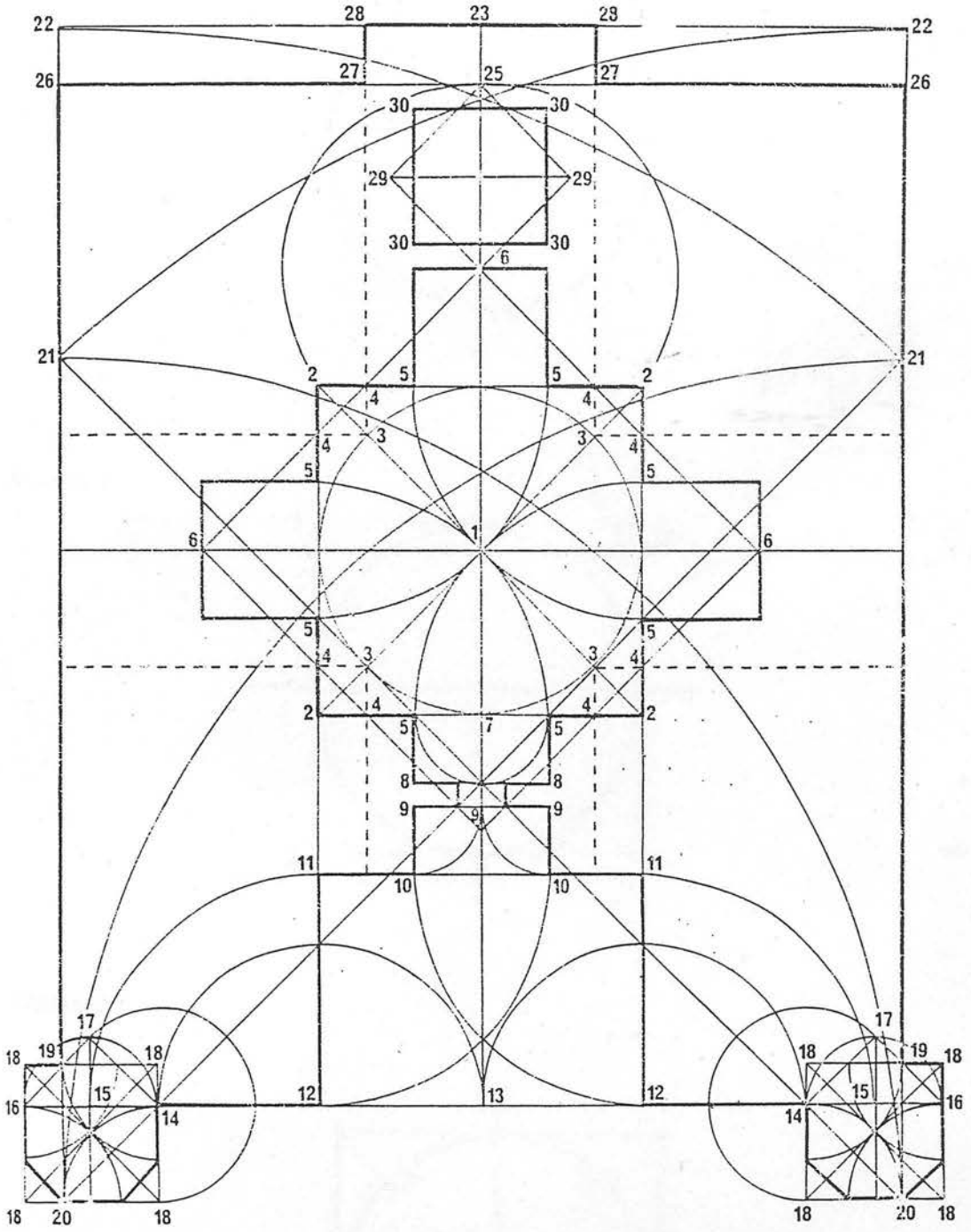
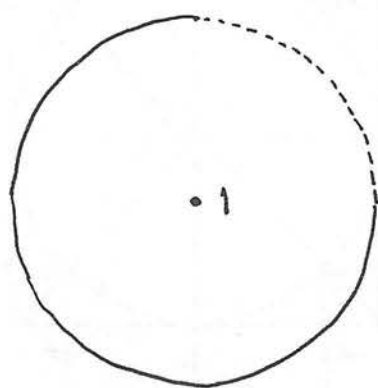
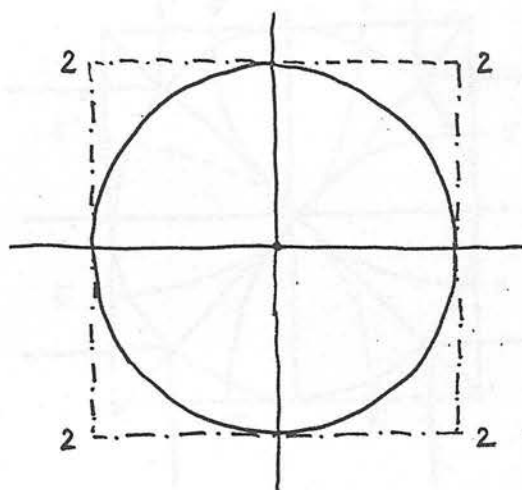


Fig. 5.3.7

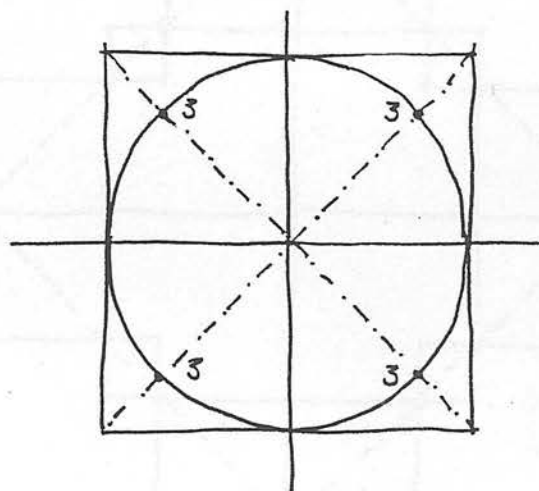
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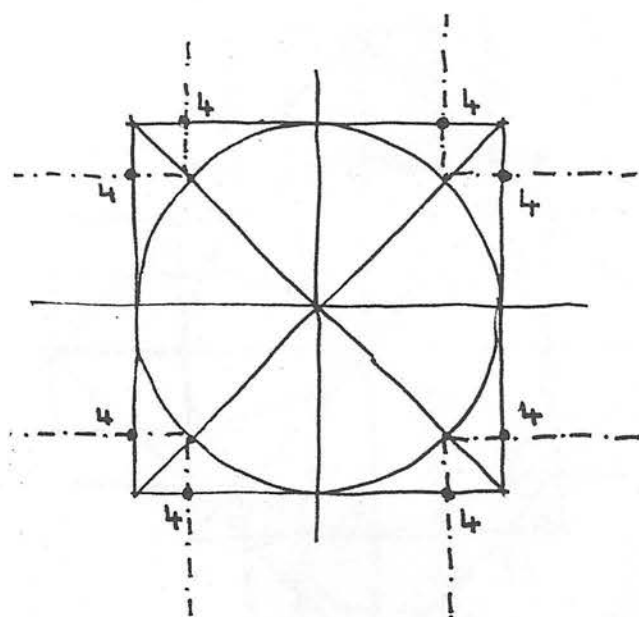
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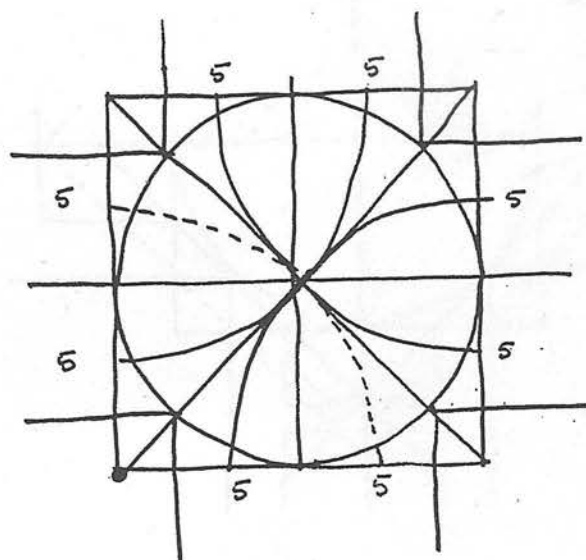
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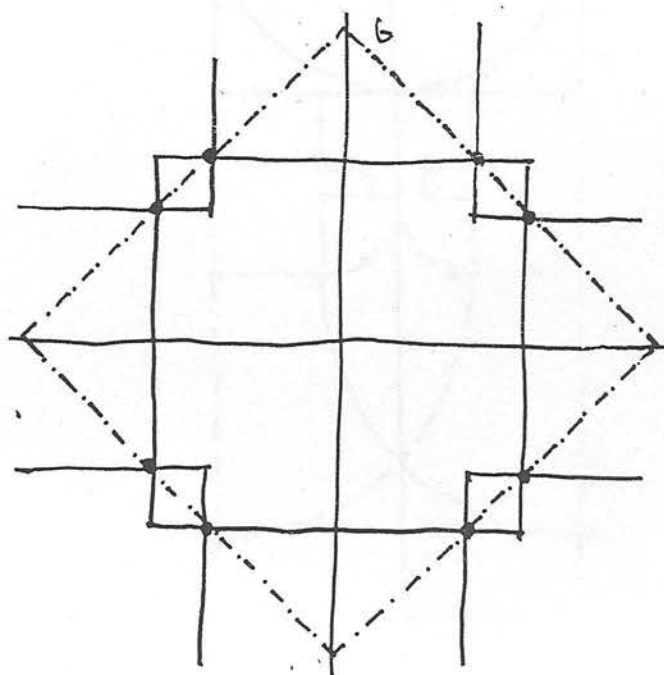
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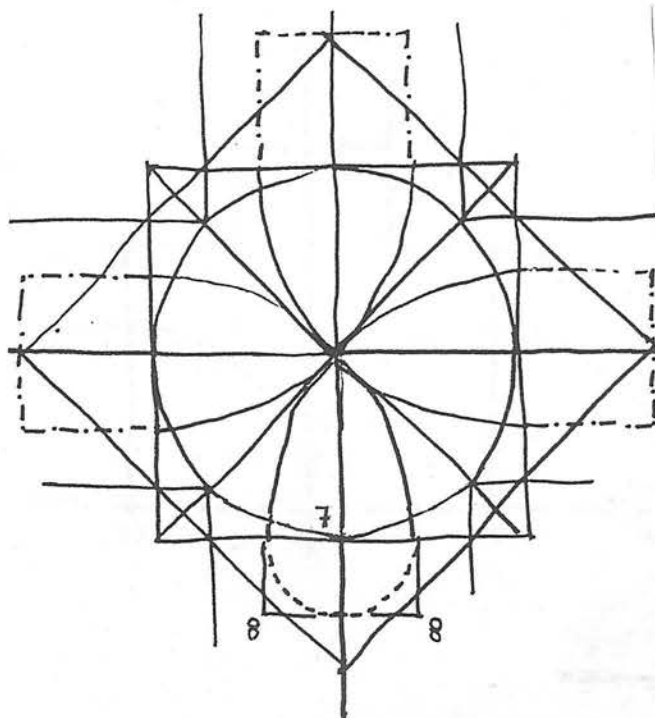
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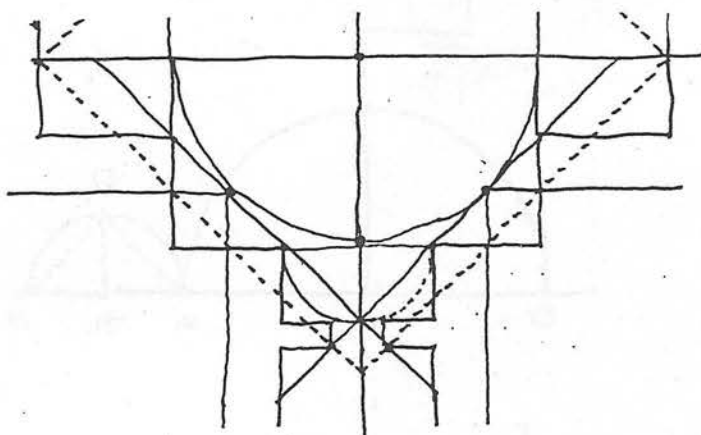
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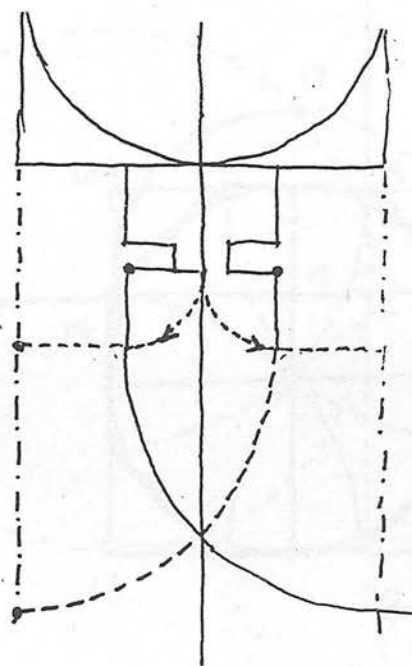
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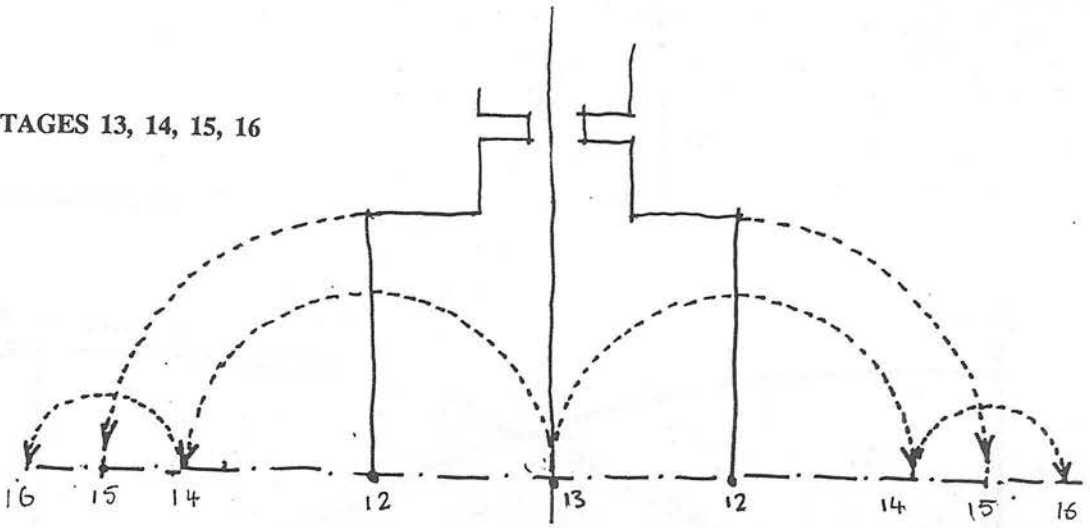
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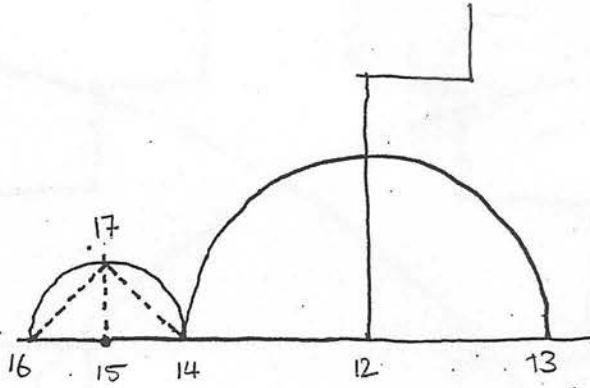
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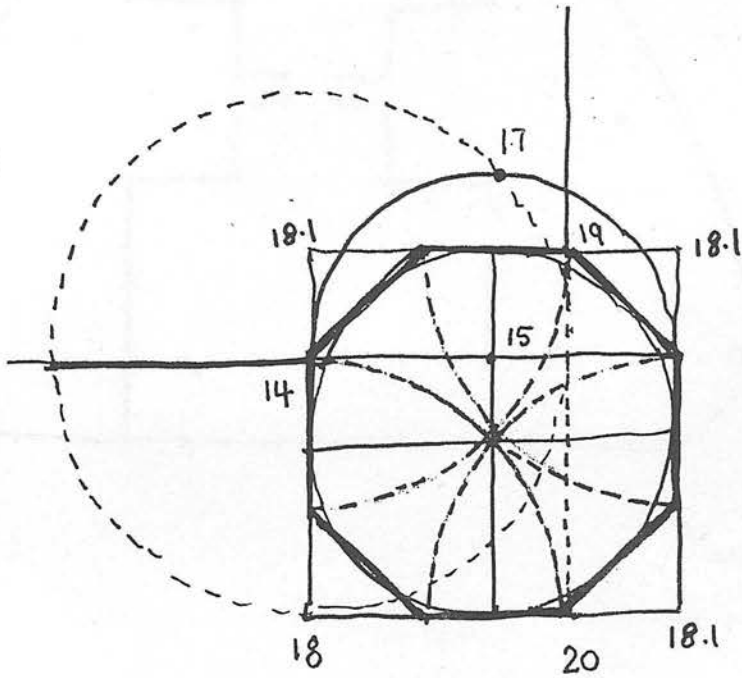
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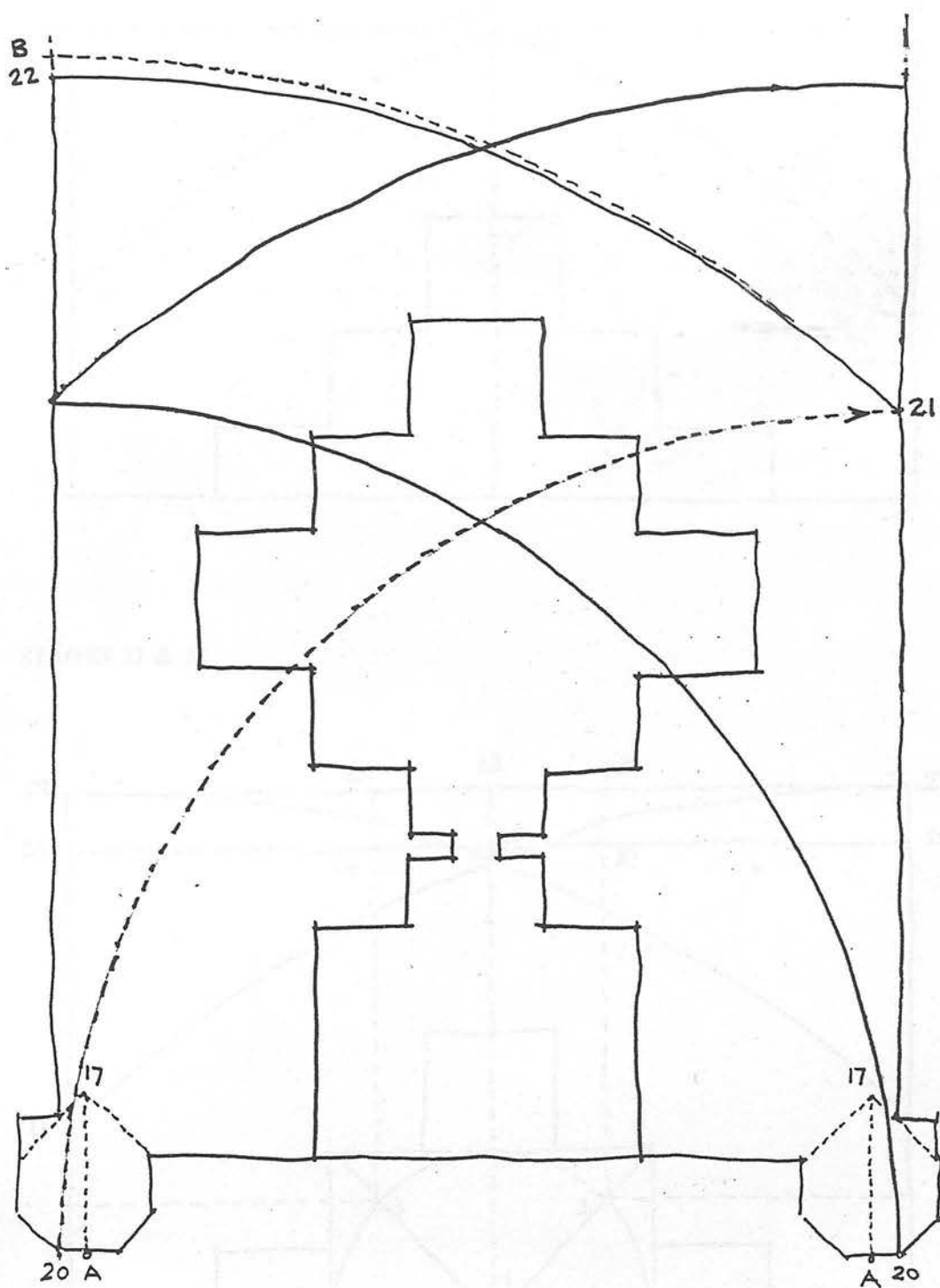
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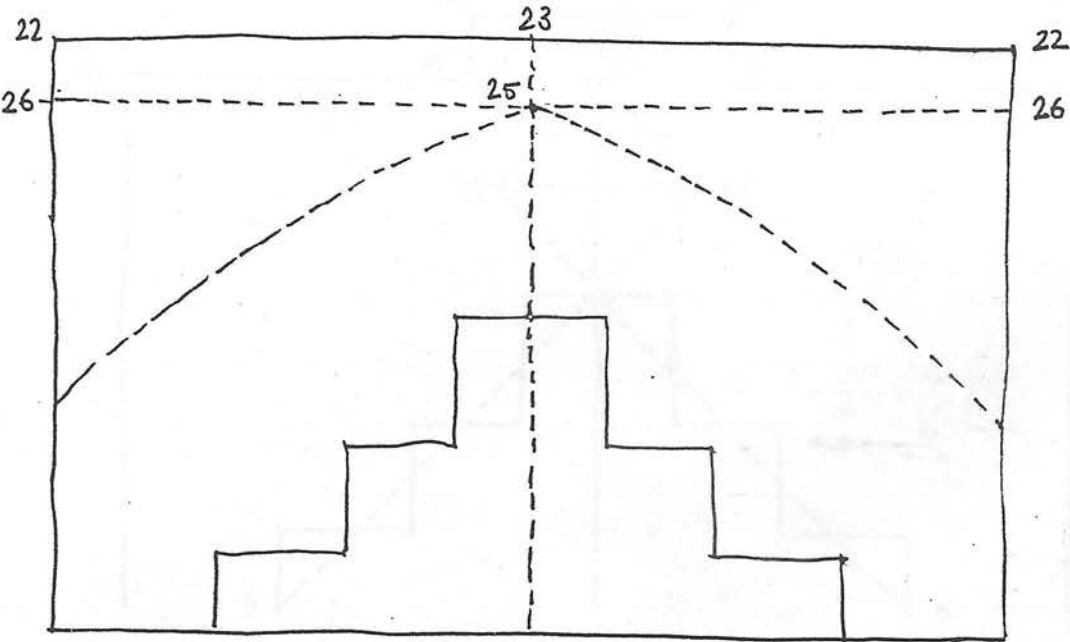
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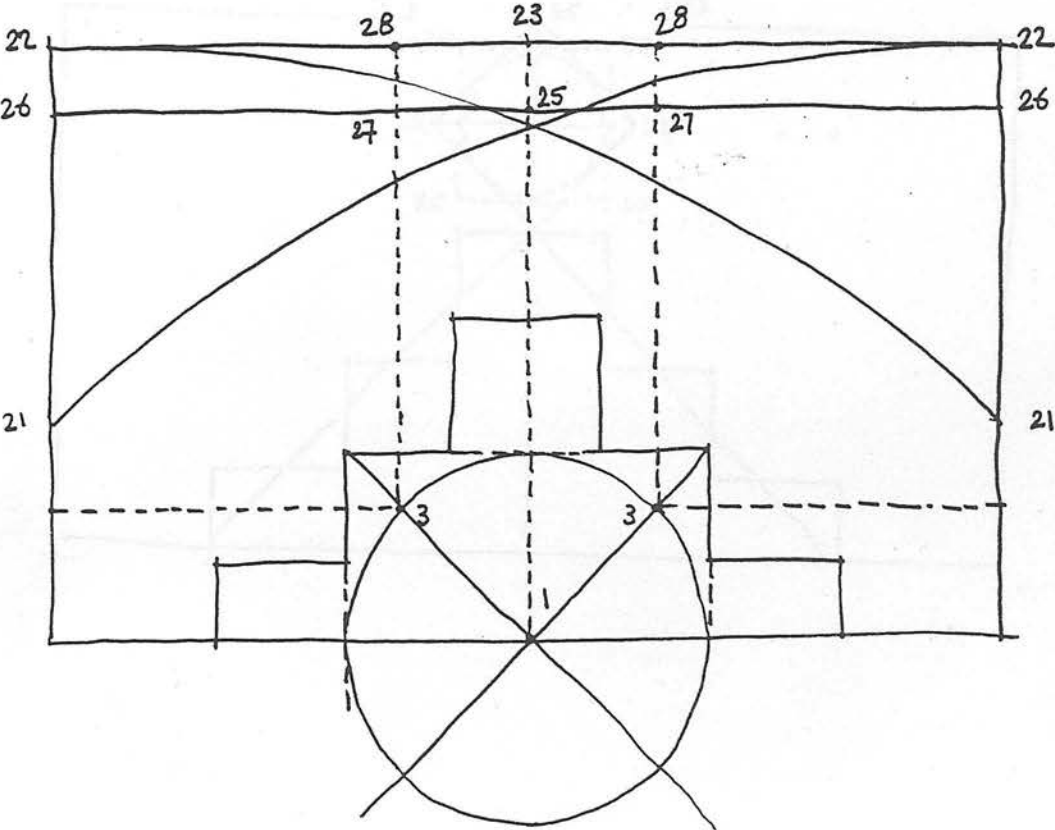
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STAGES 27 & 28



5.4 Eric Fernie on St. Anselm's Crypt, Canterbury and Ely Cathedral

At first, one may question the inclusion of Eric Fernie's work, which is largely based on Norman and Anglo-Saxon architecture, in a thesis which is primarily about early Muslim buildings. However, a closer examination of Fernie's work reveals its relevance to the subject matter of the present thesis.

In Fernie's description of a proportional system commonly used in medieval Europe to construct a two-cell building such as a church with a chancel and nave,¹ we are immediately able to equate it to the process of squaring the circle which we have discussed in Chapter III of this thesis.

Lay out a square of convenient size to provide the interior of the chancel; take the diagonal of this square and place it at right angles to the square (e.g. by rotating the square 45 degrees) to provide the internal width of the nave. Use the same diagonal length two or three times to establish the internal length of the nave.

The final stage of this sequence describes the plans of simple buildings like the

1. *A Beginner's Guide to the Study of Architectural Proportions and Systems of Length* (London, 1990).

eighth-century Anglo-Saxon church at Escomb¹ and, according to Fernie, exactly the same principle underlies many of the more complex structures of the later Middle Ages.

Fernie's work on architectural proportions is very clear and precise, paying great attention to details. His viewpoint is realistic and stripped of all myths and folk tales. As we shall see when we examine with Fernie the proportions of St. Anselm's Crypt, Fernie also confirms our view that geometry in architecture is universal and not confined to one culture or one period of history.

Although there are many systems of proportion, especially in two-dimensional architectural decoration, as we have seen in Chapter III of this thesis, Fernie also confirms our view that not only is the $1:\sqrt{2}$ system of proportion common in all cultures, but also that it appears to be particularly common in the *architecture* of most cultures.

The church begun by Abbot Simeon during his period of office between 1081 and 1093, a period parallel with the Fatimid period in Islamic architecture,² had an aisled presbytery of four bays with stilted apse, a crossing and aisled transepts of four bays each, an aisled nave of thirteen bays with a cloister

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1. See also *idem*, "Anglo-Saxon Lengths and the Evidence of the Buildings", *Medieval Archaeology*, XXXV (1991).
 2. See for example the analysis of the Ajdābiyyah Palace in this thesis, chapter IV, section 3; and of the Mosque of al-Aqmar in chapter IV, section 9. See also vol. II of thesis (analytical drawings).

flanking eight of them on the south side, and, finally, a western transept possibly with a Galilee porch. While the crossing, much of the presbytery, and the north arm of the western transept appear to have been lost, enough remained to enable Fernie to establish all the major dimensions (see fig. 5.4.1). These suggested the existence of a unit system of proportions which Fernie was able to demonstrate most clearly in the widths of the nave and aisles.

The aisles' intended size is retrievable from the fact that all the other transverse dimensions are relatively constant. The total internal width measured to the back of the blind arcading enlarges gently, from east to west, from 23.49 m. to 23.61 m. The internal width of the nave has the same consistent variation, ranging from 10.06 m. to 10.14 m., and the arcade walls vary in thickness by only a centimetre or so on either side of 1.68 m. The intended aisle widths should therefore be what is left when the nave and the arcade walls are subtracted from the total width, that is, in bay seven, for instance, 23.57 m. minus 10.06, 1.68, and 1.7 m., which equals 10.13 or two aisles of 5.06 m. each. The likelihood that this is close to the intended aisle width is supported by its being half the width of the nave (fig. 5.4.2). These figures are all reducible to the standard English foot of 0.3048 m.

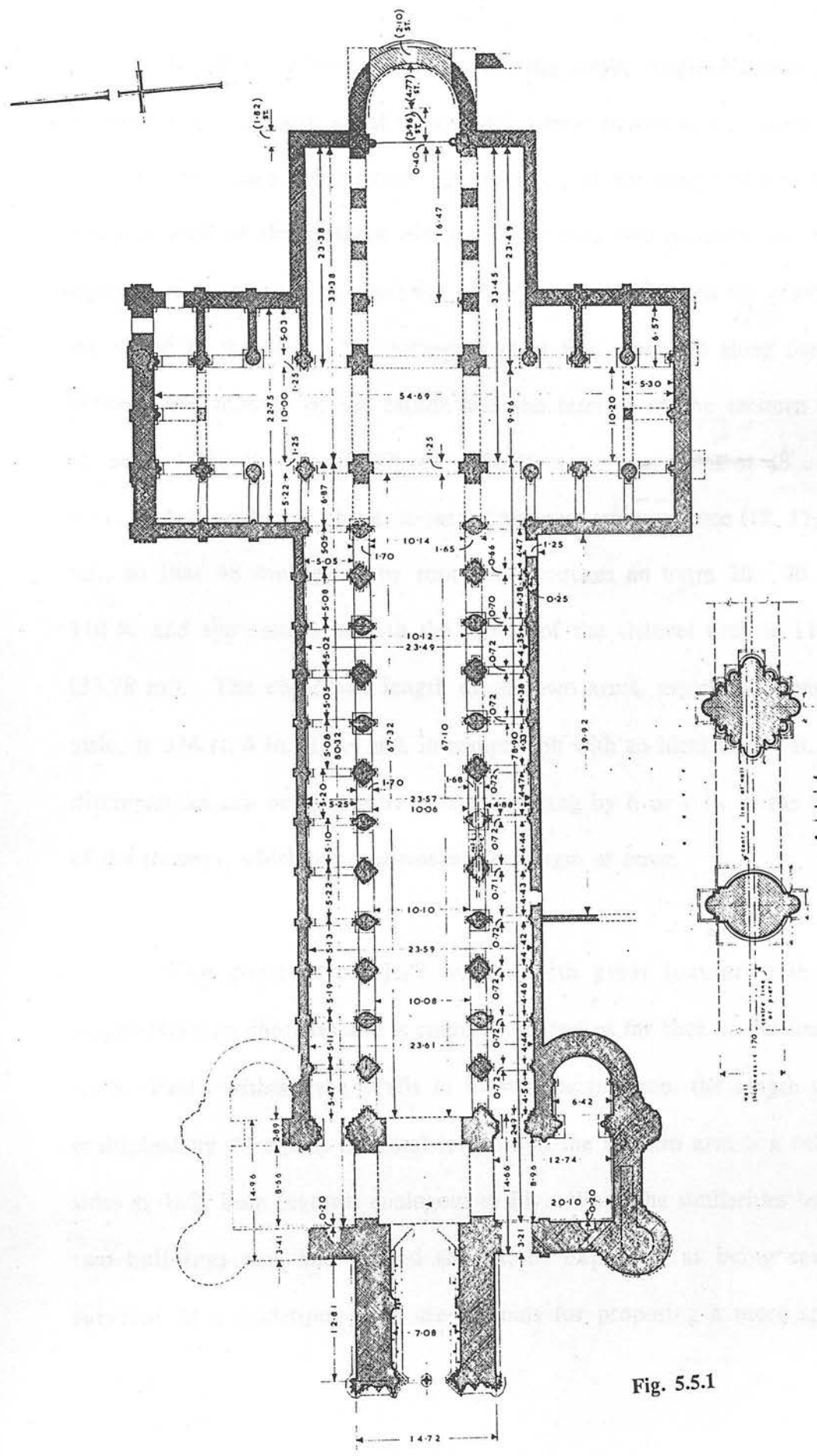


Fig. 5.5.1

DETAIL SHEWING DISPOSITION OF TYPICAL NAVE PIERS

SCALE : 1 - 50.

SCALE OF PLAN : 1 - 200m

The basic proportion found in the larger Anglo-Norman churches is between a western arm equal to one and a total length to the chord of the apse equal to the square root of two. In other words, the length of the church to the western wall of the transept multiplied by root two produces an extra length equal to the eastern arm consisting of the transept width and the presbytery up to the chord of the apse. The western arm at Ely, measured along the north aisle between the interior of the façade and the interior of the western wall of the transept is 263 ft. 6 in. (80.32 m.). 264 ft. is the equivalent of 48 units of 5 ft. 6 in., and 48, as noted above, is part of the root two sequence (12, 17, 24, 34, 48, 68), so that 48 multiplied by root two produces an extra 20. 20 units equal 110 ft. and the eastern arm to the centre of the chancel arch is 110 ft. 10 in. (33.78 m.). The combined length of the two arms, measured along the north aisle, is 374 ft. 4 in. (114.1 m.), in comparison with an ideal of 374 ft., so that the discrepancies can be explained by a misplacing by 6 or 7 in. of the western wall of the transept, which seems a reasonable margin of error.

The proportion $1:\sqrt{2}$ occurs with great regularity in the larger Anglo-Norman churches and is common at least as far back as the fourth century. At St.-Paul's-without-the-Walls in Rome, for instance, the length of the nave multiplied by $\sqrt{2}$ equals the total length and the western arm is a rectangle with sides as $1:\sqrt{2}$, both features analogous to Ely. While the similarities between these two buildings can, and indeed should, be explained as being caused by the survival of a tradition, there are grounds for proposing a more specific link,

namely that the great length of the more important Anglo-Norman churches is an attempt to emulate the size of the largest early Christian basilicas in Rome. The total internal length of St. Paul's is 128.38 m., while the total length of St. Peter's is about 120 m. without the narthex and about 133 m. with it. Ely is about 123 m. long internally, Norwich Cathedral just under 132 m., and Winchester Cathedral 143 m., even excluding its western element.

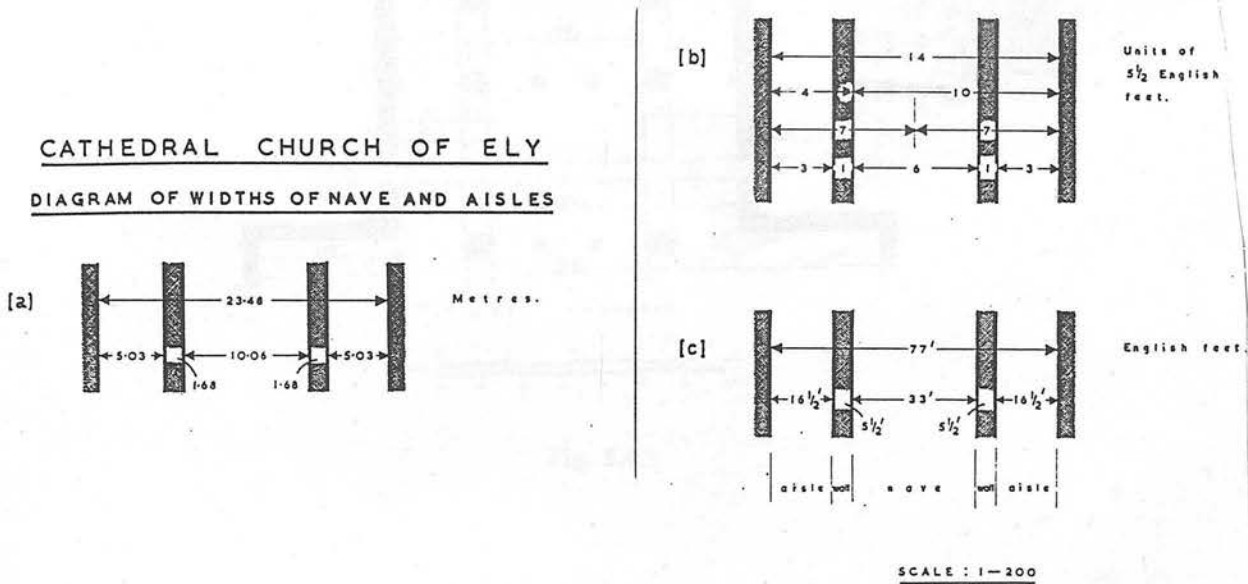


Fig. 5.4.2

St. Anselm's Crypt

Fernie's analysis of the crypt discussed its constituent parts, raising three questions concerning the designer, the proportion, and the style. Firstly, after considering numerous characteristics, Fernie concluded that the architect was either a man from the (German) Empire with a knowledge of the Anglo-Norman tradition or *vice versa*. Secondly, on proportion, Fernie found evidence of the

use of the $1:\sqrt{2}$ proportion system in all the main dimensions of the plan (see figs. 5.4.3 & 5.4.4).

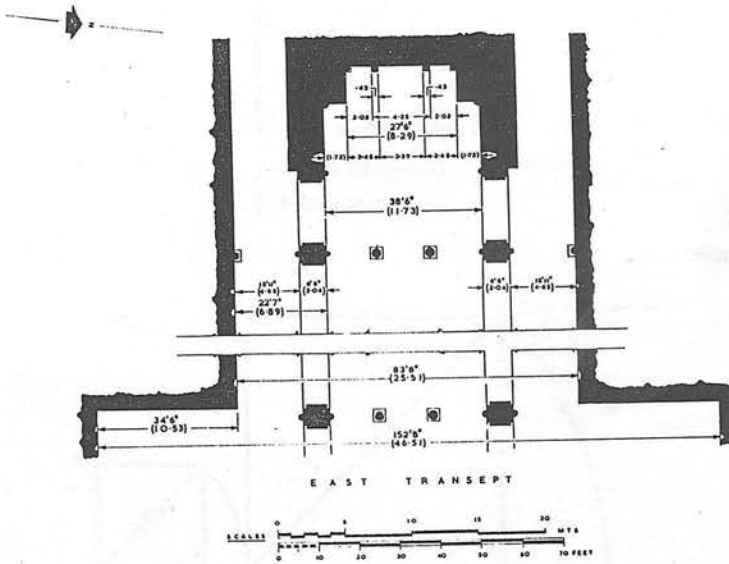


Fig. 5.4.3

The narrower, westernmost bay of the central area is 8.29 m. wide and the main section 11.73 m., which is 8.29 multiplied by $\sqrt{2}$. $11.73 \times \sqrt{2} = 16.58$, the second element of which is the average width of the aisle. Next, the width of the aisle plus the thickness of the arcade wall total 6.89 m., while $4.85 \times \sqrt{2} = 6.86$, i.e. the width of the aisle multiplied by $\sqrt{2}$ generates an extra length equal to the thickness of the arcade wall. All this produces an overall internal width between the aisle walls of 25.51 m. The initial width multiplied by $\sqrt{2}$ gives the width of the main part of the crypt. That width, multiplied by $\sqrt{2}$, generates the width of the aisle and that width similarly generates the thickness of the arcade wall.

Finally, the overall width, multiplied by $\sqrt{2}$, generates the length of the transept arm.

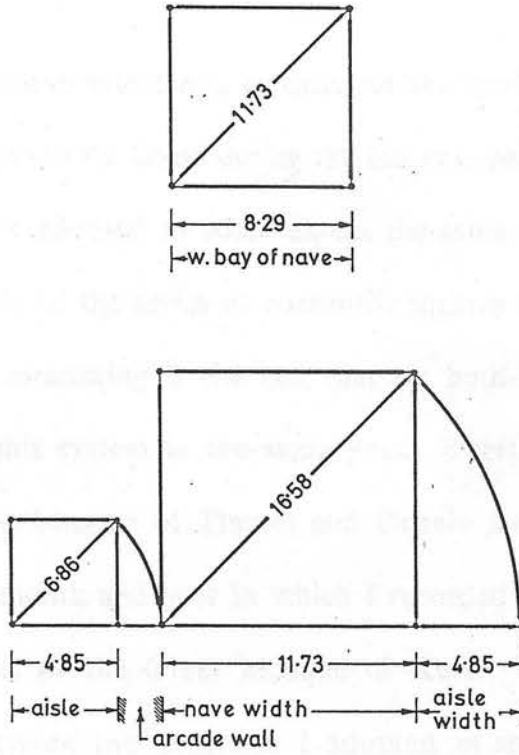
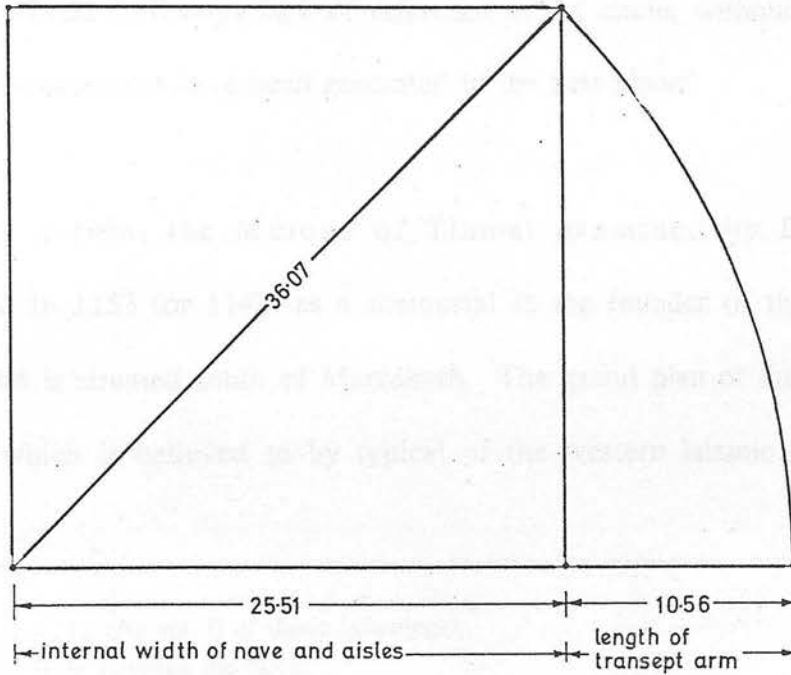


Fig. 5.4.4



5.5 Christian Ewert on the Mosque of Tinmal (Morocco) and Castelo da Lousa

It has been interesting to discover the work of Ewert on the Mosque of Tinmal and Castelo da Lousa during the last two years of my research, for he too appears to have adopted to some extent the same system of geometric analysis, i.e. the principle of the series of concentric squares alternating in their orientation by 45°. Also interesting is the fact that we both seem to have examined and worked with this system in the same year. Ewert completed and published his findings on the Mosque of Tinmal and Castelo da Lousa in April 1986, which was the same month and year in which I recorded and submitted my first results on the analysis of the Great Mosque of Kufa.¹ However, the fundamental difference between the approach I adopted in some of my analysis and that presented by Ewert is Ewert's lack of reference to the circle, without which the examined plans could not have been generated in the first place.²

Now a ruin, the Mosque of Tinmal examined by Ewert was commissioned in 1153 (or 1143) as a memorial to the founder of the Almohad movement and is situated south of Marrakesh. The grand plan of the mosque is the T-type, which is believed to be typical of the western Islamic architecture

1. See Chapter 4.10.2 and vol. II of thesis (drawings).

2. See Chapter II on squaring the circle.

and to have derived from the Great Mosque of Damascus. The central importance of the *mihrāb*-minaret block is emphasized in the layout of the grand plan (see fig. 5.5.5). Geometrically, it is included on the main axis of almost the exact square which circumscribes the entire ground floor plan of the mosque. This feature is also evident in our analysis of Qarawīyyīn Mosque in Fez.¹ Ewert's explanation for the elements projecting beyond the main building remains inconsistent with my findings. This forms the core of my comments on his paper.

Ewert states that 'the overriding importance of an exact multiplication by 100 of the unit module in the longest diagonals of the building took precedence over the ideal dimensions of this basic geometric figure.'² He bases this observation on the fact that in pre-Islamic architecture it was already the custom to fix the critical dimensions of a building by diagonals. In support of this, Ewert presents his and J. Wahl's analysis of Castelo da Lousa in Portugal, a Roman fortified farmstead of the late Republican period (see fig. 5.5.1).

The dimensions and proportions of the plan are, according to Ewert's findings, determined by a square with diagonals of exactly 100 Roman feet. The square Ewert adds accounts for three of the four exterior faces of the building. All the essential faces of the interior articulation are also based on the two main diagonals. The basis, Ewert argues, is a diagonal scale of $10 \times 10 = 100$. The

1. See section 4.10.5 and vol. II of thesis (drawings).

2. *Proceedings of the British Academy* (1986) p. 127. See also *Madriider Mitteilungen* (1985) p. 150-176.

geometric skeleton is amazingly simple: starting from the inner faces of the outer walls, three concentric squares were placed orthogonally, each side half the length of the next (see fig. 5.5.1)

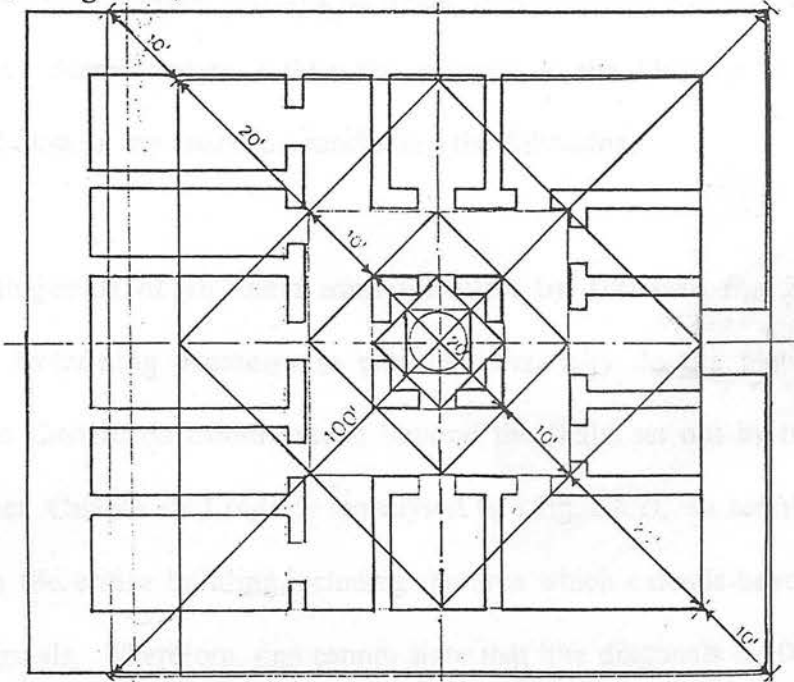


Fig. 5.5.1

Ewert also expressed his belief that this same scheme, based on the square and found in Portugal, can also be found in the Hellenistic Empire, where the main diagonals are found to measure 100 Attic feet. This led him to propose that the quadratic scheme, for which the constituent interior measurements are determined by the diagonals, was adopted for the Syrian Umayyad desert castles and thus, he adds, in the Tinmal Mosque, after more than four centuries, it is no more than an echo of this pure quadratic scheme. This may be correct, but in the Tinmal Mosque we can see that the longest diagonal of the building does not coincide with the diagonals of the theoretical square which surrounds it. Therefore, there

2. In the fortified farmstead at Castelo da Lousa, the thickness of the outer walls was not explained geometrically in Ewert and Wahl's diagram, although they posited the unit of 10 Roman feet as a diagonal measure. The problem becomes clear if we try to reconstruct the plan geometrically to the exact proportional relationships of its elements.

3. Finally, in the case of Castelo da Lousa, it is more likely that the architect started out with a central unit circle measuring 10 Roman feet in diameter and that he progressed outward, squaring circles and alternating squares, in my usual manner, until reaching the inner face, where the outer wall was to be. Then he would select 10 Roman feet as the original unit measure. The result of this approach resolves both the geometry and the arithmetic of the dimensions.

In order to comment on Ewert's analysis of the Tinmal Mosque, I chose to reconstruct the plan of the mosque, using my 'squared circle' method of analysis.

A. Starting from a plain sheet of paper and regardless of any dimensions or scale, we draw a circle of a given size, showing both the vertical and horizontal axes (see fig. 5.5.3).

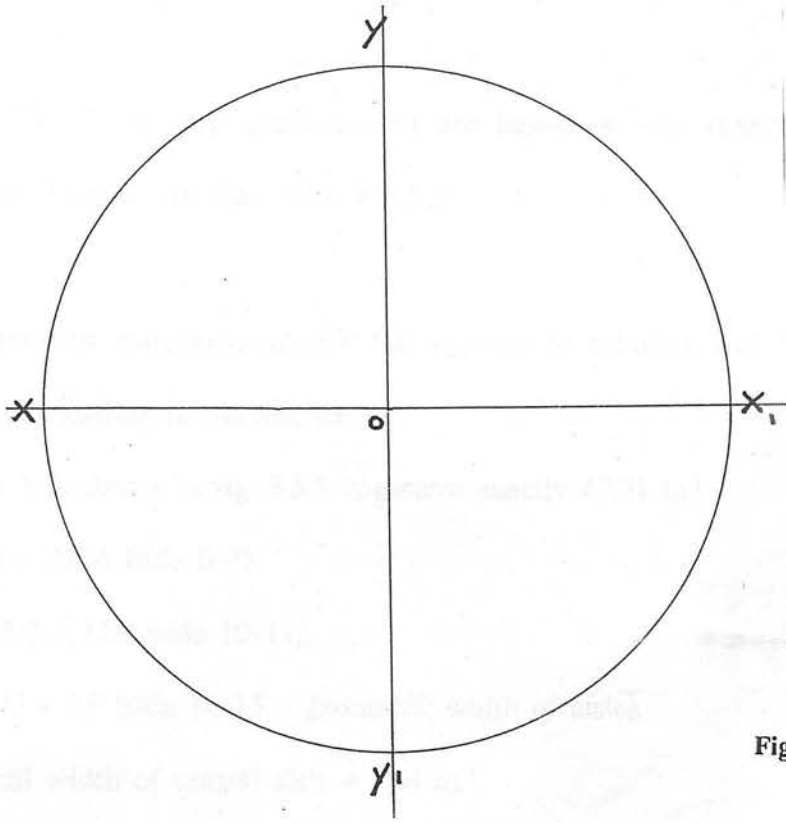


Fig. 5.5.3

B. Having squared the circle and having drawn seven concentric squares, each side being half the length of the next, we observe the following.

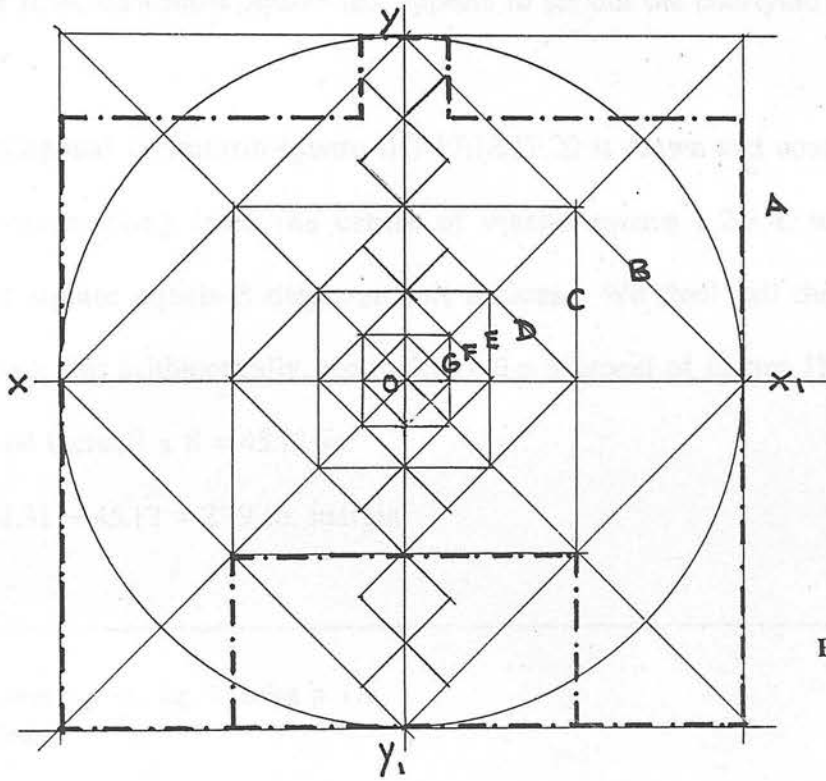


Fig. 5.5.4

The following observations are based on our reanalysis of Ewert's Mosque of Tinmal (see figs. 5.5.4 & 5.5.5).

1. The seventh concentric square (G) appears to coincide with the width of the central aisle leading to the *miḥrāb*.

Side 2-3, shown in fig. 5.5.5, measures exactly 47.31 m.¹

$47/2 = 23.65$ (side 6-7).

$23.65/2 = 11.8$ (side 10-11).

$11.8/2 = 5.9$ (side 14-15 – geometric width of aisle).

Actual width of central aisle = 5.64 m.²

$5.9 - 5.64 = 0.26$ m. margin of error from geometric theory to actual dimensions.

2. The fifth, concentric square (E), appears to set out the courtyard's south *riwāq*.

3. If diagonal concentric square (H) 17,18,19,20 is drawn and considered as a unit measure, starting from the centre of master square 1,2,3,4, we find that the master square equals 8 diagonal unit squares. We shall call the unit square K.

To check this arithmetically, we multiply the diagonal of square 17,18,19,20 by 8.

5.64 (actual) $\times 8 = 45.12$ m.

$47.31 - 45.12 = 2.19$ m. margin.

1. Ewert, *op. cit.*, fig. 1 facing p. 116.

2. *Ibid.*

However, if we use our geometric dimension for the diagonal of square 17,18,19,20 (width of aisle),

$$5.9 \times 8 = 47.2 \text{ m. (geometric)}$$

$$47.31 \text{ (actual width of side 2-3)} - 47.2 = 0.11 \text{ m. margin,}$$

which is a better result. This could therefore mean that the actual measurement is less accurate than the geometric scheme originally drawn by the architect.

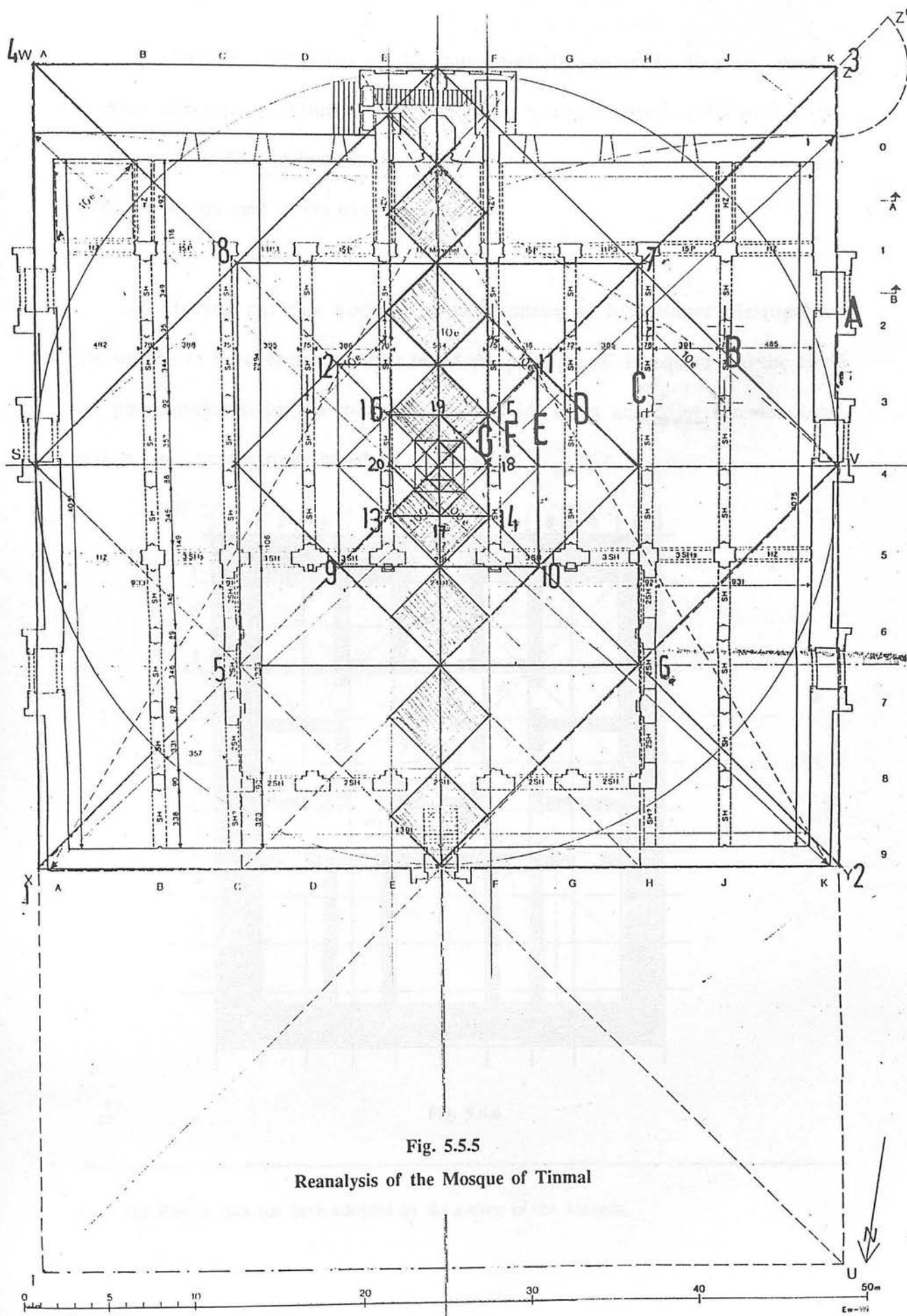
4. Geometrically, the depth of the mosque's courtyard appears to measure two diagonal squares (K), including the piers' nibs.

Depth (including piers' nibs) is

$$12.33 - 0.8 = 11.53 \text{ (actual).}^{1} 11.53 - 11.28 = 0.25 \text{ m.}$$

5. In establishing the line of the *qiblah* wall in relation to the geometry of the mosque as a whole, which includes the projection of the *mihrāb*, leaving the *qiblah* wall set further back, we find Ewert's system of drawing an arch from an imaginary corner U applaudable. However, when we tried this principle on one of the mosques analysed in Chapter IV (Al-Qarawīyyīn Mosque), the only mosque with a substantial *mihrāb* projection, the result was far from satisfactory.

1. *Ibid.*



Further subdivision of the squared-circle geometric diagram, used to analyse the plan of Tīnmāl, will lead us to a simple square grid with a unit measure of 10 Roman feet (see fig. 5.5.6).¹ The inevitable question is, what came first, the square grid or the overall geometry?

Having put this question to my students at Manchester Metropolitan University as an exercise, we first tested the principle of the square and the circle on plans prepared by the modern architects M. Botta and Mies van der Rohe, whose plans appear to be based on square grids.

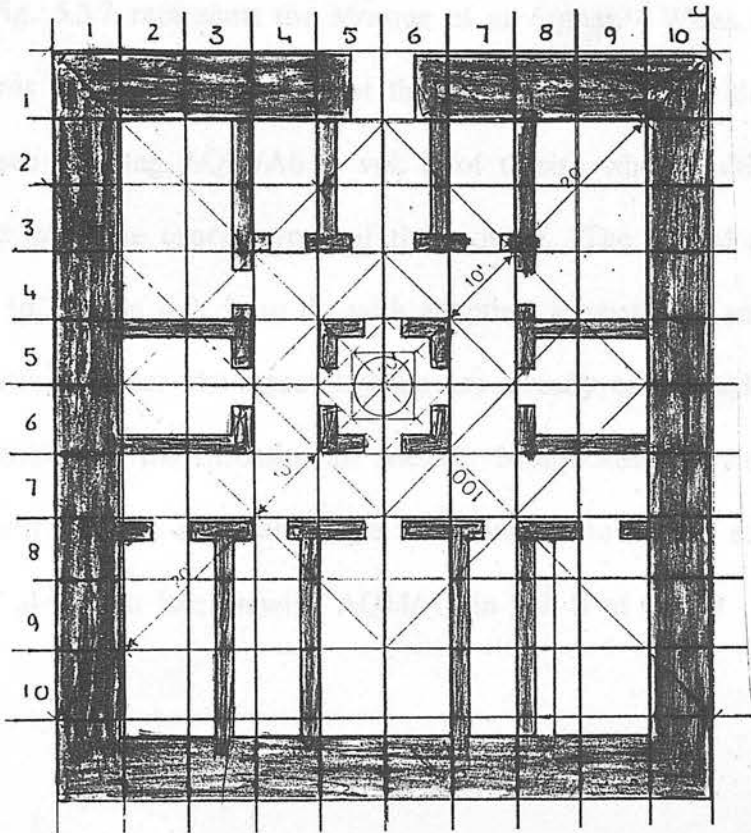


Fig. 5.5.6

1. The Roman foot has been adopted by the author of the analysis.

The result was negative. We could not establish the principle of the squared circle as the generator of the overall plan which would subsequently have given the unit square from which the plan developed. However, in examples of medieval buildings in which the principle of the square and the circle is clearly reflected, almost all would subdivide into small unit squares which, when added up, make up the overall plan. This indicates that modern architects are not making full use of such systems, an opportunity more valued by our predecessors.

Fig. 5.5.7 represents the Mosque of al-Aqmar.¹ When analysed further, the last unit square at the centre of the concentric squares within the geometric diagram (see drawing AQM/A6 in vol. II of thesis), when multiplied, appears to correspond with the exact layout of the mosque. The shaded area, as we have explained in section 4.9, is to do with adapting geometry to an already existing urban fabric in which the street layouts are already established and in order to reconcile this with the direction of Mecca. Mosques in this situation turn on a pivotal point or with a certain angle, but nowhere so clearly achieved as in the Mosque of al-Aqmar (see drawing AQM/A7 in vol. II of thesis).

1. See thesis, vol. I, section 4.9; and vol. II (drawings). See also D. Hoag, *Islamic Architecture*, p. 146.

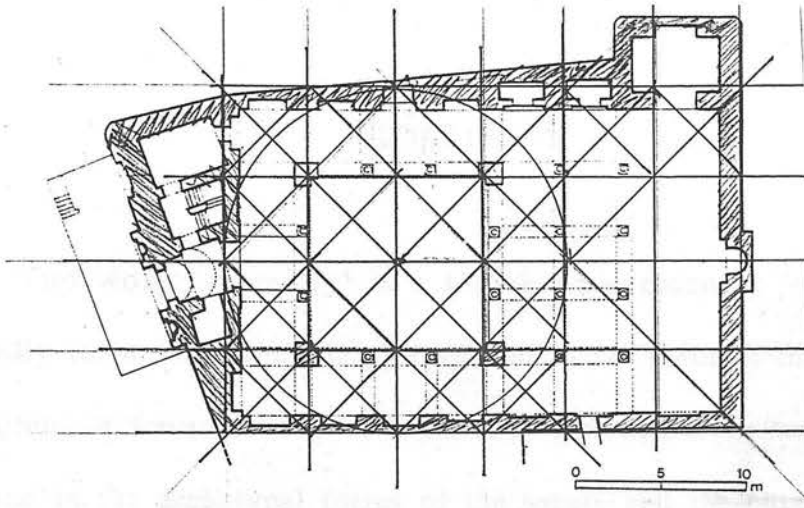


Fig. 5.5.7

* * *

It is clear therefore from our discussion in this chapter that the convention of the square and the circle is at the heart of nearly all systems of proportion used in the examples of architectural planning. Furthermore, this was not apparently limited to one culture or to one period of history.

CONCLUSION

The word 'geometry' is a Greek term meaning 'earth-measure', principally to do with the study of *spatial order* through the measure and relationships of forms: a body of knowledge inherited from ancient Egypt and embodied in the archetypal forms of the square and the circle. The Arabic equivalent to the word 'geometry' is *handasah*, or '*ilm al-handasah*', derived from the Persian verb *andākhtan* (or also *andāzīdan*), meaning 'to throw', 'to project', or 'to measure'. Prior to being considered solely as a purely theoretical science, geometry once exemplified a body of practical knowledge which proved to be most useful to many ancient cultures, such as those of Egypt, Mesopotamia, and India.

Islamic culture encouraged learning and the study of the past, and Muslim scholars soon became acquainted with the achievements of previous cultures. This was most evident in the area of Byzantine Palestine, where the architectural planning of the Near East prior to the coming of Islam clearly influenced the architectural aspirations of the new religion, for the geometric scheme we find underlying the design of the Dome of the Rock, for example, can easily be detected in numerous Byzantine buildings constructed between the fifth and seventh centuries AD.

As a body of science, geometry (known in Arabic as [*ilm*] *al-handasah*, i.e. 'the science of proportioning') became familiar to medieval Muslim scholars through the *Elements* of Euclid, first translated by al-Ḥajjāj ibn Yūsuf ibn Maṭar in AD 790. Their acquaintance with applied geometry appears to have reached them through the works of Archimedes, Hero of Alexandria, and the compendium of an Indian text known as the *Siddhantā*. A medieval treatise by Naṣīr al-Dīn al-Ṭūsī on Archimedes's work on the squaring of the circle (*Tarbī' al-Dā'irah*) confirms that Muslim scholars were familiar with such geometric terms.

As a method of achieving harmony and proportion in three-dimensional design, medieval Muslim architects appear to have made use of a proportional geometric system based on a circle inscribed within a square, and by inscribing a smaller square *within* the circle and rotating it at angles of 45 degrees, an octagonal star is composed, thus creating a 'master grid' made up of a series of concentric octagonal stars relating in a proportion of 1:2 and $1:\sqrt{2}$. As this thesis demonstrates, the resulting diagram seems to underlie many early Muslim buildings, controlling both the plan and the elevation. However, as far as two-dimensional decorative art is concerned, the use of these geometric principles is still evident in many parts of the Muslim world, particularly in Morocco and Tunisia.

This thesis explains the system which medieval Muslim architects seem to

have used to organize the plan as well as transfer the design concept from a two-dimensional drawing to full scale without losing any of its proportions in the process. The author has applied this system to a number of early Muslim monuments constructed between AD 692 and 1125, and the results suggest that such geometric schemes were indeed used. The process has been analysed in the examples selected, here using actual measurements and to within a margin of error of one decimal point.

The fourteen examples of early Muslim architecture examined in this thesis establish beyond doubt that the architecture of this period derived the most significant proportions of its buildings from the harmonious divisions of the circle and the square. These results suggest that the plans analysed with reference to the principle of squaring the circle could not easily have been actualized without the use of such techniques.

In considering alternative systems of proportion, I was able to determine that one proportional system appears to have been overwhelmingly more popular than any other in the designing of buildings, namely the ratio of the side of a square to its diagonal, which is one to the square root of two, or 1:1.4142.

As the main objective of this research has been to examine the evidence for the use of geometric calculations in the construction of early Muslim architecture, I hope to have proved this convincingly.

I hope to have carried on from where Issam El-Said and Ayşe Parman left off in terms of their work on geometry in two-dimensional Islamic art and to have been able to correlate this with the work of Creswell in respect of the numerous measured plans of early Muslim buildings. Supporting evidence for the theories presented in this thesis is provided by the fact that the same system which organized surface decoration also played a major role in setting out the buildings on which these designs were assembled.

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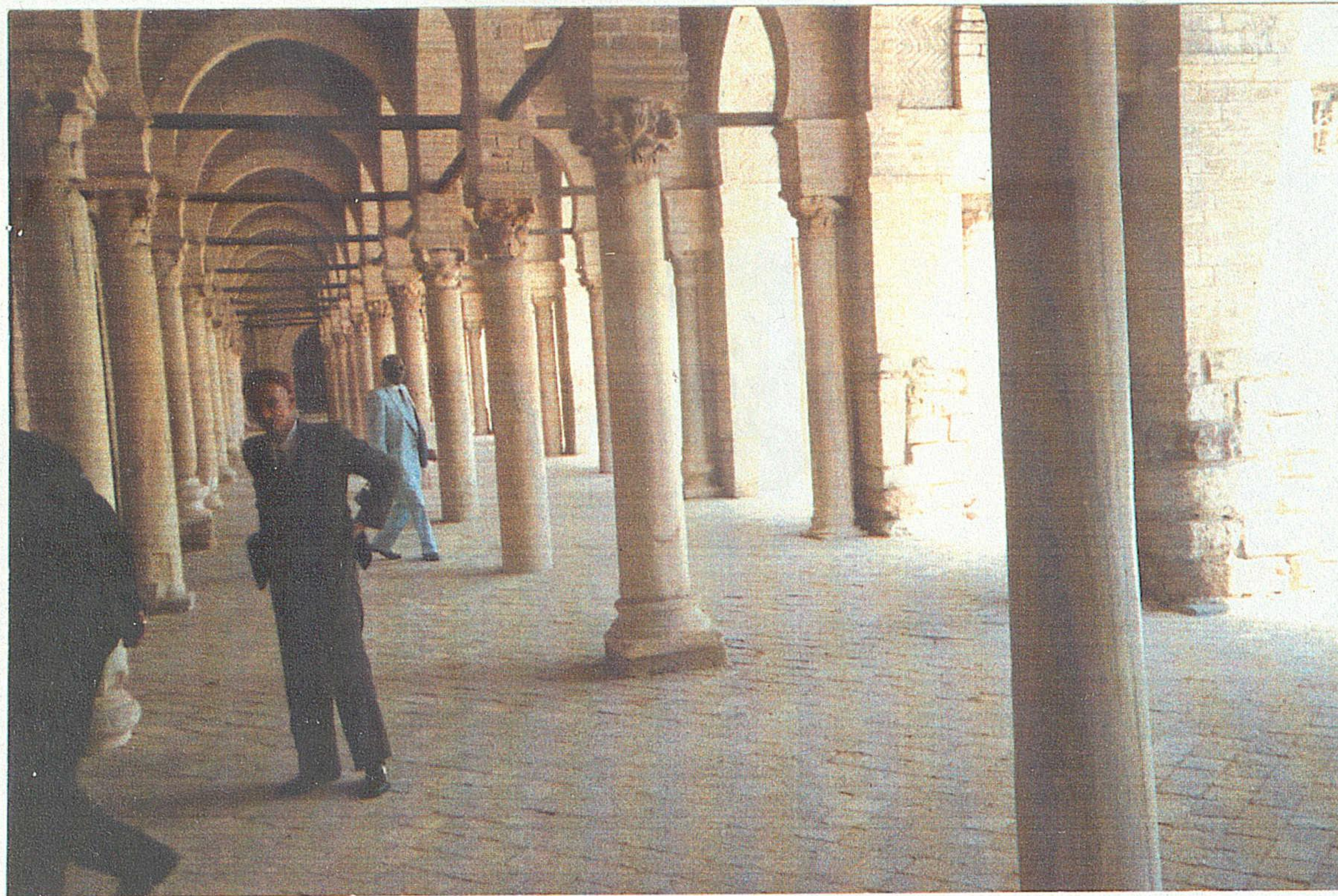


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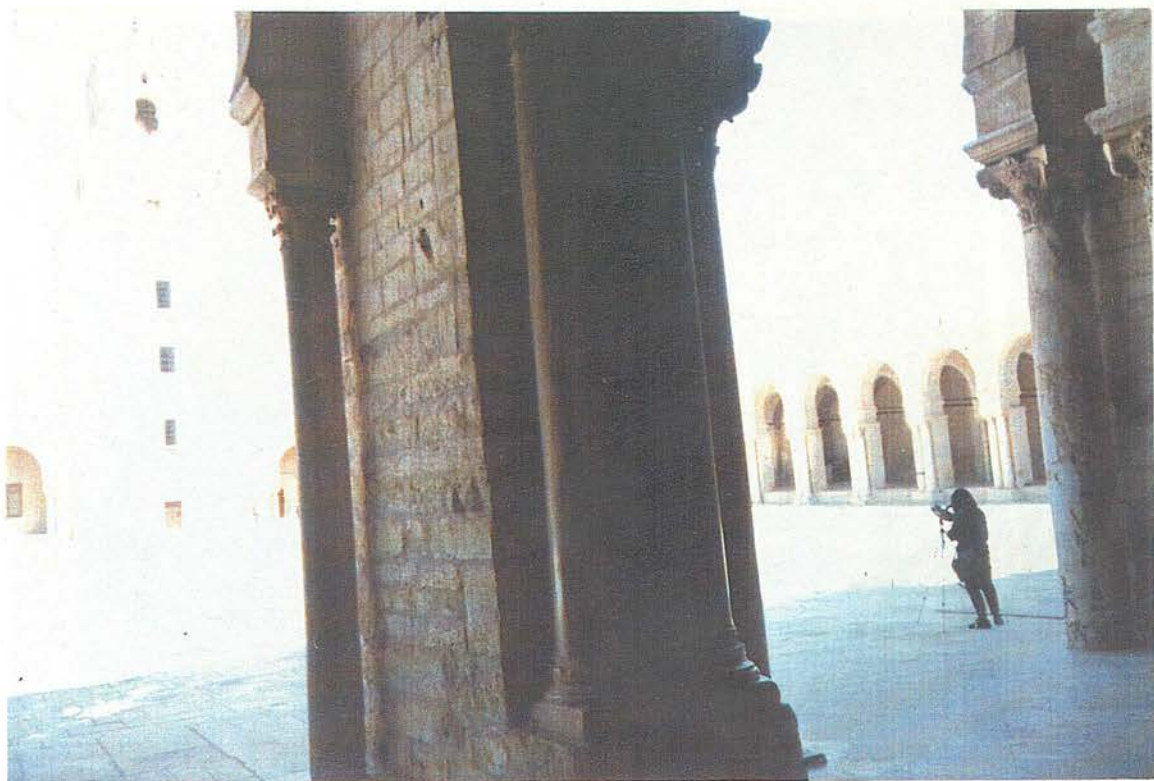


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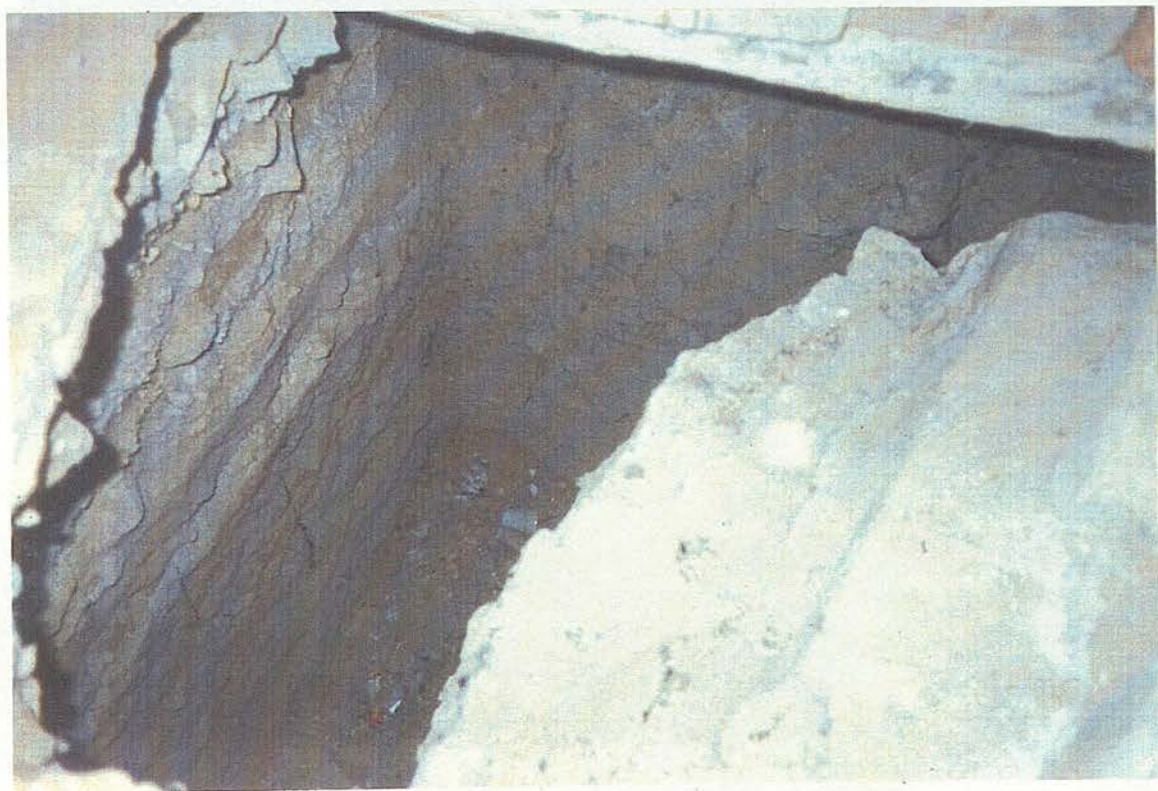


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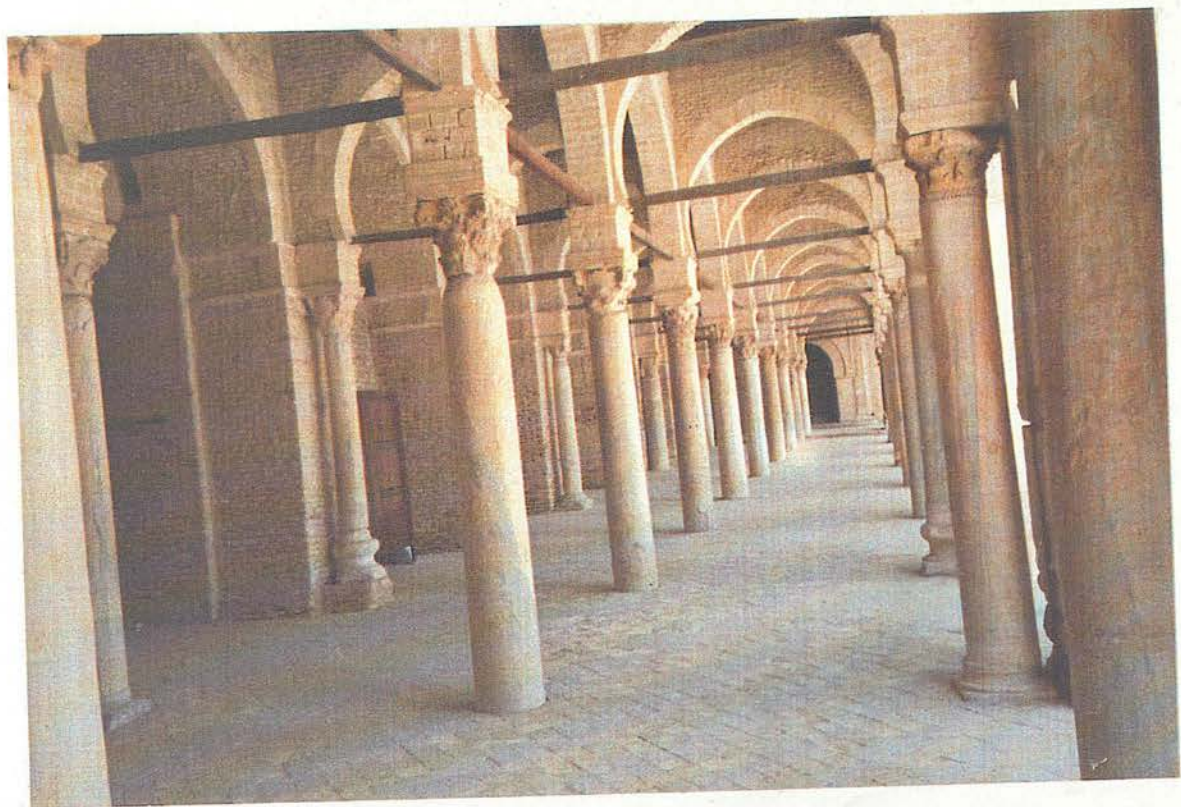


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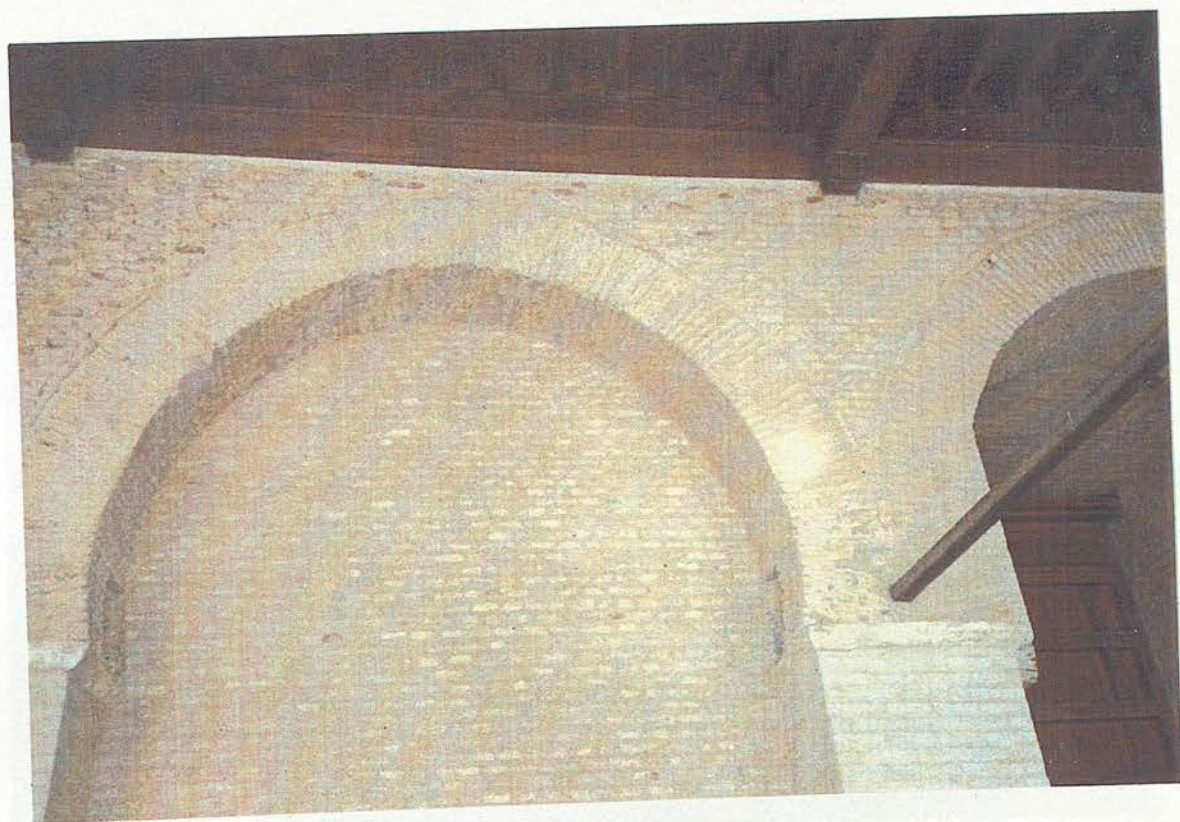


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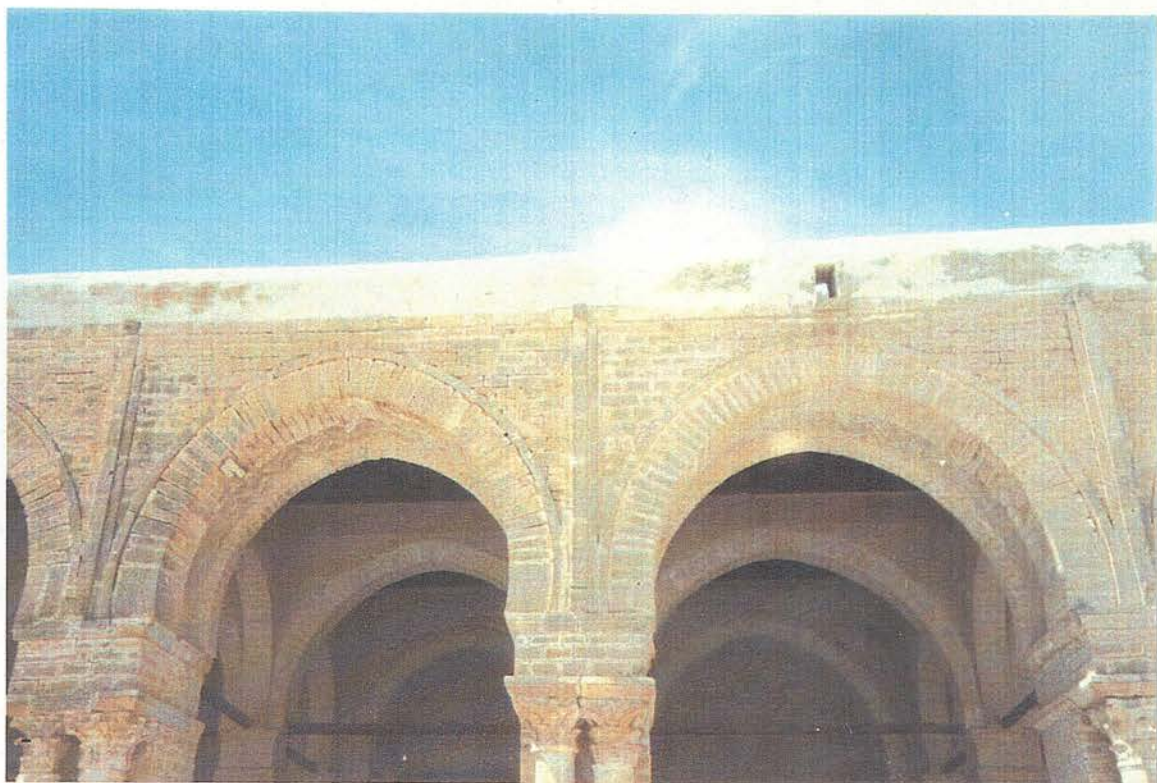


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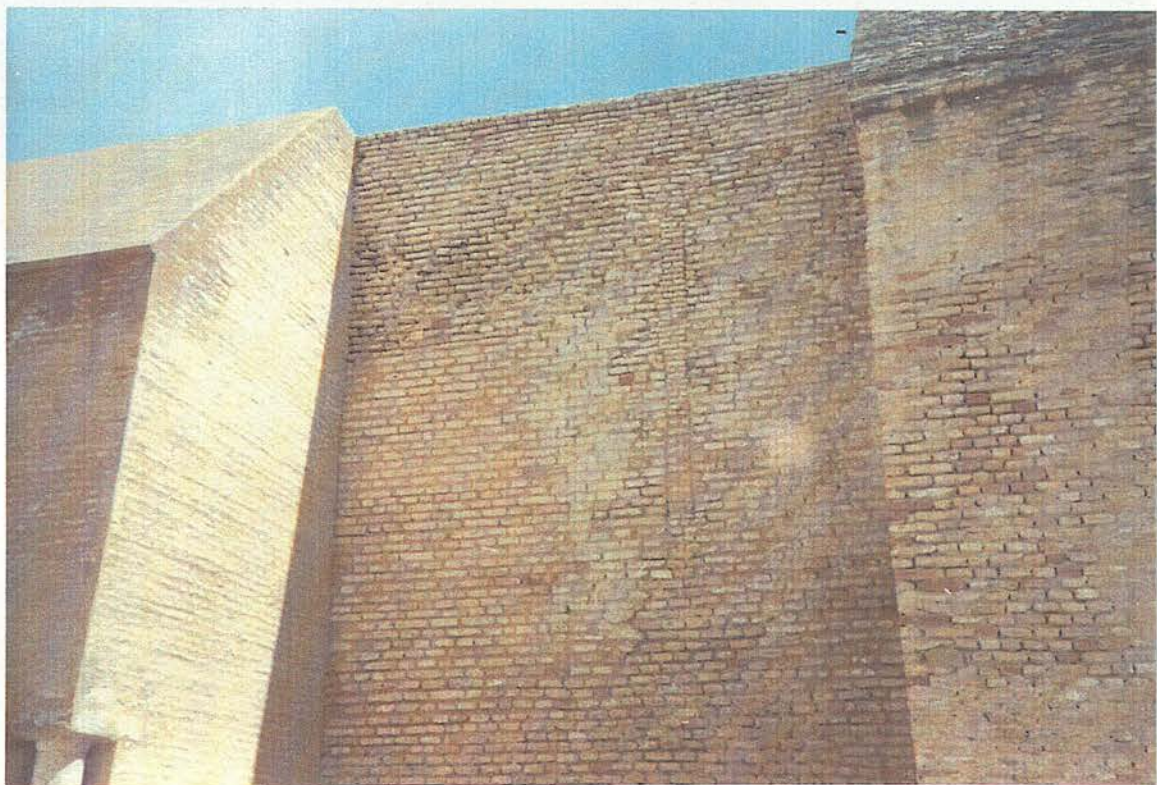


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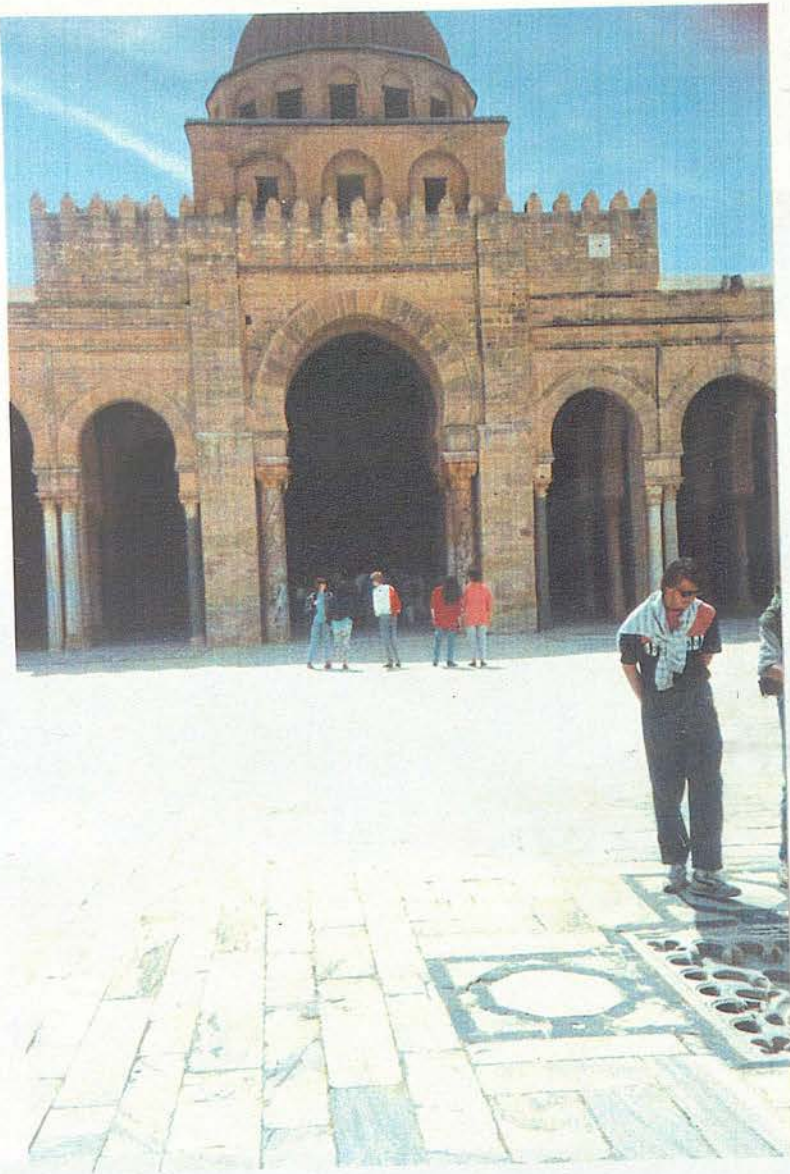


PLATE 10



PLATE 11



PLATE 12



PLATE 13



PLATE 14



PLATE 15



PLATE 16



PLATE 17



PLATE 18

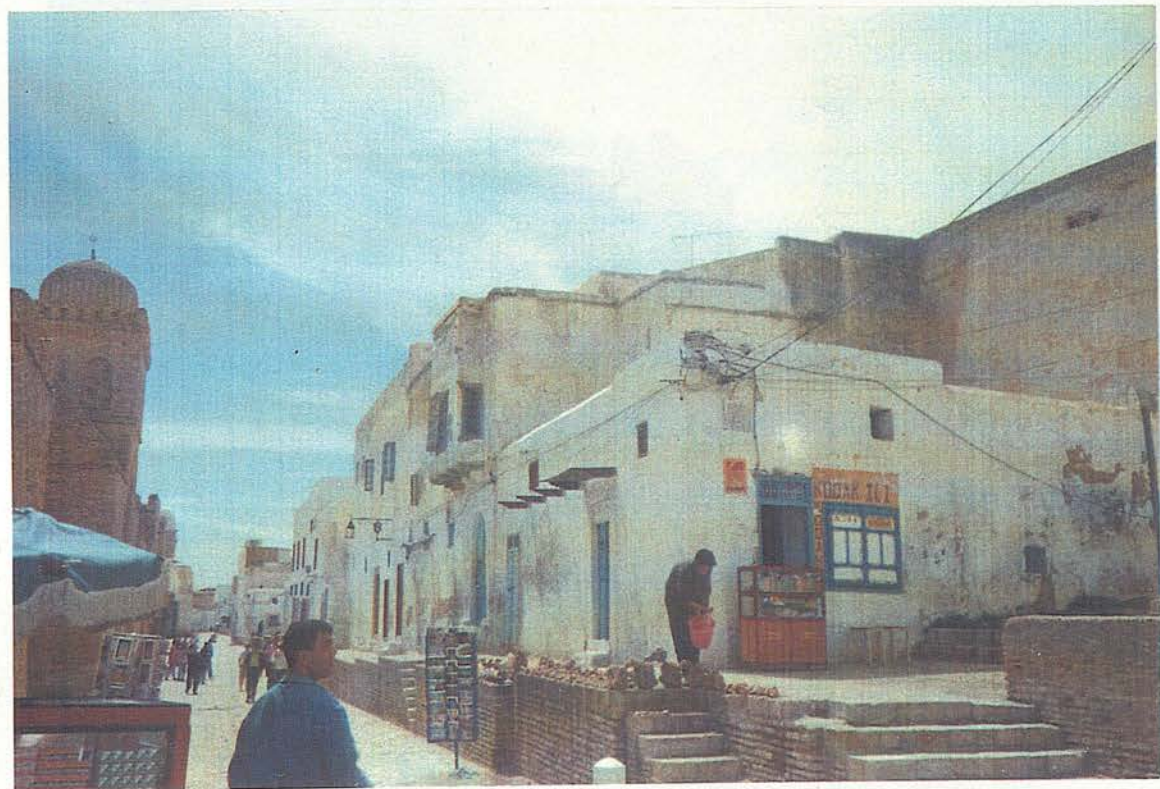


PLATE 19



PLATE 20

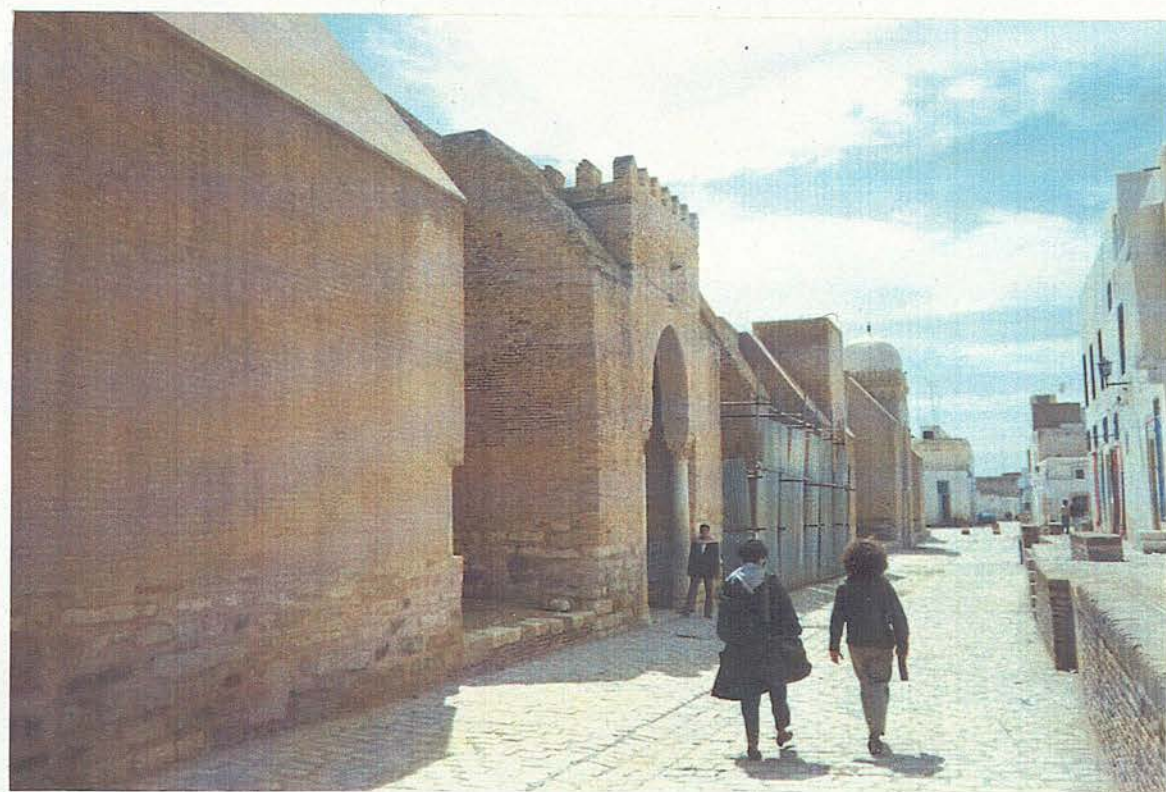


PLATE 21



PLATE 22

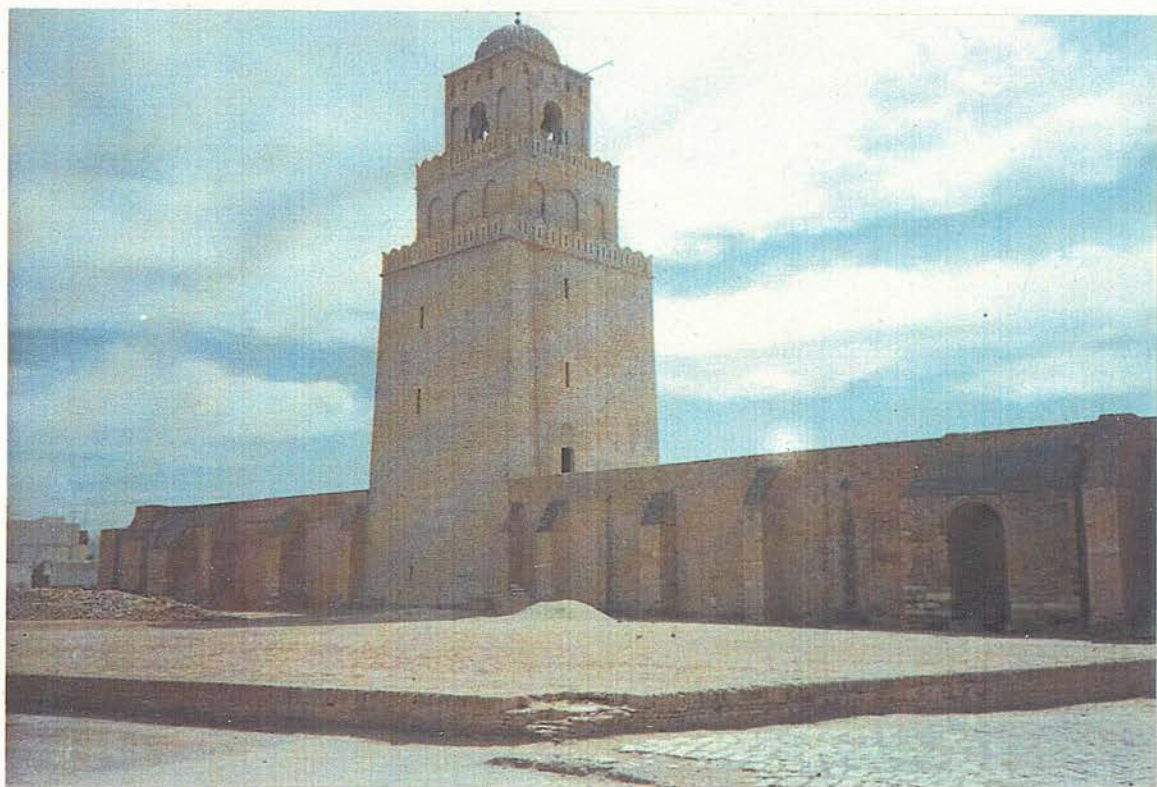


PLATE 23